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ABSTRACT

For a high poloidal beta value, the access to the second stability region of the ballooning modes can be obtained in the low shear region of a tokamak plasma. The first and the second stability boundaries are obtained for the different profiles of the safety factor. Dependence of the accessibility on the profile of the safety factor is also studied.

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1. INTRODUCTION

One of the critical issues in the design of a tokamak reactor is to improve the maximum beta value of a plasma, which is the ratio of the volume-average plasma pressure, $\langle p \rangle$, to the magnetic pressure, $B^2/2\mu_0$, i.e., $\beta = 2\mu_0 \langle p \rangle / B^2$. The maximum value of B is limited by the engineering constraints such as the current density of the toroidal coil or the electro-magnetic force, etc. The beta limit, therefore, becomes the limitation of the capable thermal energy in a plasma. In a tokamak plasma, the beta value is considered to be limited by the macroscopic instabilities caused by the global quantities such as the plasma current or the gradient of the plasma pressure. As the first approximation, the maximum value of the beta is theoretically evaluated by an ideal MHD stability analysis. The results can be approximately summarized as $\beta(\%) = gI_p(MA)/a(m)B_t(T)$, where g takes 3 through 4 and I_p, a and B_t denote the total plasma current, the horizontal minor radius and the strength of the toroidal magnetic field at the center of the plasma. The constant, g, depends on the profiles of the plasma pressure and the current density, and the shape of the cross section. One of the methods to enhance g is the access to the second stability region of the ballooning modes. The direct access without passing the unstable region has been shown for the indented cross section /1/ and for the low shear equilibria with the non-indented cross-section /2,3/. For the non-indented cross section, the almost unlimited pressure gradient can be maintained in the low shear region and the pressure gradient is limited by the first stability boundary in the higher shear region. Across a certain magnetic surface, the high pressure region and the low pressure region coexist. It is not clear if the smooth transition from the first stability boundary to the second one across the magnetic surface is possible. In the report, we study the first and the second stability boundary of the ballooning modes by changing the profile of the safety factor. Dependence of the accessibility on the profile of the safety factor is also studied.

2. NUMERICAL METHODS AND PROFILES

The ideal MHD beta limit due to $n = \infty$ ballooning modes (n is the toroidal mode number) is obtained by optimizing the plasma pressure profile for a given safety factor profile, $q(\psi)$, where ψ is the normalized poloidal flux to $0 \leq \psi \leq 1$ ($\psi = 0$ and 1 denote the magnetic axis and the plasma surface, respectively). The $n = \infty$ ballooning mode equation /4/ with zero growth rate and the Grad-Shafranov equation are solved iteratively for a fixed q-profile to obtain the pressure profile of a marginally stable state. In this analysis, we choose the following q-profile:

$$q(\psi) = q_0(1+a\psi^b) \tag{1}$$

where q_0 is the safety factor at the magnetic axis, and the parameters a and b determine the safety factor at the plasma surface, q_s, and the magnetic shear. In this study, we use the circular cross section with the aspect ratio of A = 5.

The beta value is increased by using the marginal pressure profile to the ballooning modes until it reaches the first stability boundary. Figure 1(a) shows the given pressure gradient, $dP/d\psi$, (the solid line) and the marginal one (the dashed line) obtained by the ballooning equation for $\beta_t = 0.77\%$ and $\beta_J = 2.15$, where

$$\beta_t = \frac{2\mu_o\langle p\rangle}{B_t^2}, \qquad \beta_J = \frac{8\pi\langle p\rangle}{\mu_o R_o I_p^2} \tag{2}$$

and B_t is the toroidal magnetic field at the plasma center. The pressure gradient coincides with the marginal one in the high shear region while there is the capability to increase it in the low shear region. After $dP/d\psi$ reaches the first stability boundary in the high shear region, the pressure gradient is increased by multiplying a numerical factor and by keeping the profile. There appears the unstable magnetic surfaces in the high shear region and every surface again enters the stable region (Fig. 1(b)). During this process, the marginal pressure derivative (the dashed line) is always above a given one (the solid line) in the low shear region.

3. FIRST AND SECOND STABILITY BOUNDARIES

For a non-indented cross section, a plasma can enter the second stability region of the ballooning modes in the low shear region near the magnetic axis and stay at the first stability boundary in the high shear region. There appears a "transitional magnetic surface", ψ_{cr}, across which the second and the first regions co-exist. A smooth transition across the transitional magnetic surface was shown by Seki et al. /2/. On the other hand, Todd et al. showed the unstable layer across the surface /3/. We study the dependence of the first and the second stability boundaries on q_0 and the global shear, $S = (\psi dq/d\psi)/q$. We choose the following parameters in the q-profile (1): (a) $q_0 = 1.5$, $q_s = 3.2$, (b) $q_0 = 2.0$, $q_s = 4.2$ and (c) $q_0 = 2.5$, $q_s = 4.2$ for $b = 2$. For the cases (a) and (b), the global shear S takes almost the same value. For the cases (b) and (c), q_0 changes by fixing q_s, i.e. S changes as well as q_0. Figures 2, 3 and 4 show the stability boundary for the cases (a), (b) and (c), respectively. In these figures, the lower and the upper lines denote the first and the second stability boundaries, respectively. Two boundaries are merged each other at a magnetic surface, ψ_{cr} (a transitional magnetic surface).

Within $\psi < \psi_{cr}$, the ballooning modes are always stable and the pressure gradient can be increased indefinitely. For the same S, the transitional magnetic surface appears at the same place (Figs. 2 and 3) and it is shifted to the plasma surface as S becomes smaller (Fig. 4). In a smaller region of $S(\psi)$, the local shear can be negative easily as is shown in Ref. 2. As q_0 increases, the second stability boundary for $\psi > \psi_{cr}$ approaches the first stability boundary. When two boundaries become close, $dP/d\psi$ can be changed between the first and the second stability regions near ψ_{cr} during the iteration. Once $dP/d\psi$ enters the second stability regions during the iteration, then it increases indefinitely to cause a drastic change near ψ_{cr}/3/. For the smooth transition in the case of a small S, a high resolution and a relaxation procedure in the iteration near ψ_{cr} are required.

4. CONCLUSION

We have studied the stability region of the ballooning modes in a $dP/d\psi$ - ψ space and have shown the first and the second stability boundaries. The global shear determines the position of the transitional magnetic surface. As q_0 increases, the second stability boundary approach the first boundary. When two boundaries become close each other, $dP/d\psi$ can enter the second stability region during the iteration and a large jump in $dP/d\psi$ may appear.

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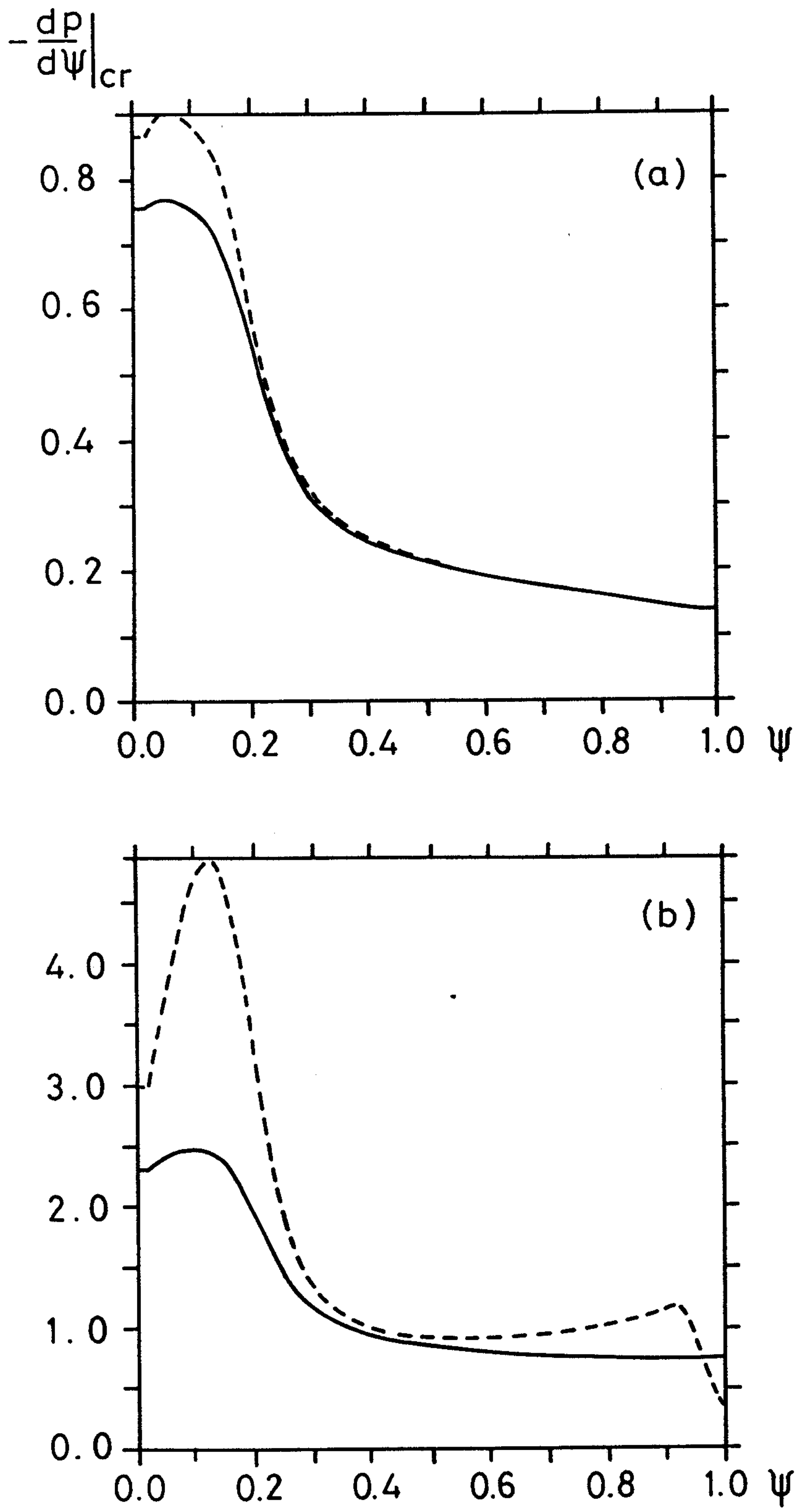
FIGURE CAPTIONS

Fig. 1 Pressure gradient vs. ψ for q_0 = 2, q_s = 4.2, (a) β_t = 0.77% and β_J = 2.15 and (b) β_t = 3.46% and β_J = 4.58. The broken line denotes the marginal pressure gradient obtained by the ballooning equation.

Fig. 2 Stability region in $dP/d\psi$ - ψ plane for q_0 = 1.5 and q_s = 3.2. The symbols S and U denote the stable and the unstable regions, respectively. The lower and the upper lines are the first and the second stability boundaries of the ballooning modes, respectively.

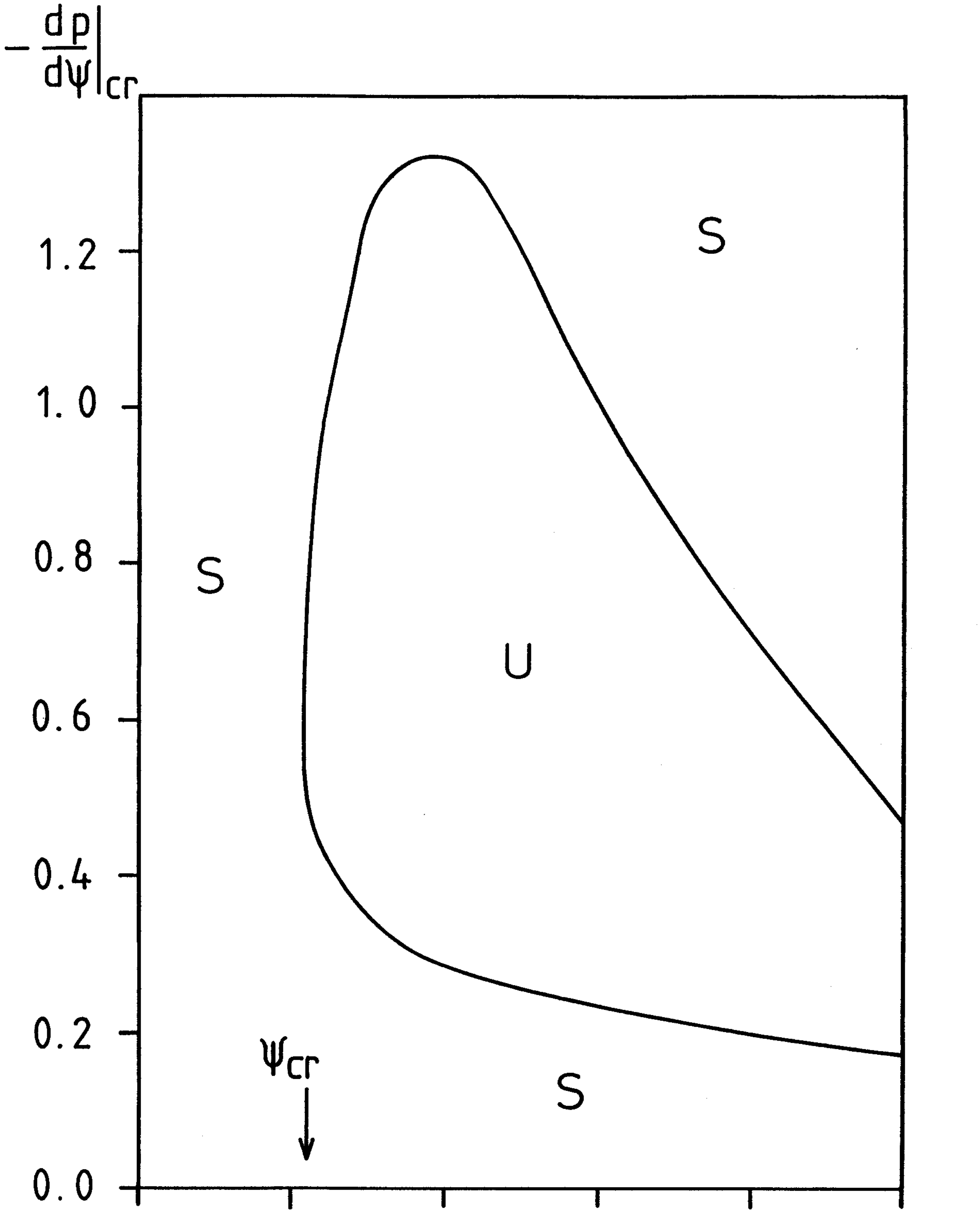
Fig. 3 Stability region for q_0 = 2.0 and q_s = 4.2.

Fig. 4 Stability region for q_0 = 2.5 and q_s = 4.2



$-\frac{dp}{d\psi}\Big|_{cr}$
(a)
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ψ
(b)
4.0
3.0
2.0
1.0
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ψ

FIG. 1



$-\frac{dp}{d\psi}\Big|_{cr}$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
S
S
U
S
ψ_{cr}
0.0
0.2
0.4
0.6
0.8
1.0
ψ

FIG. 2

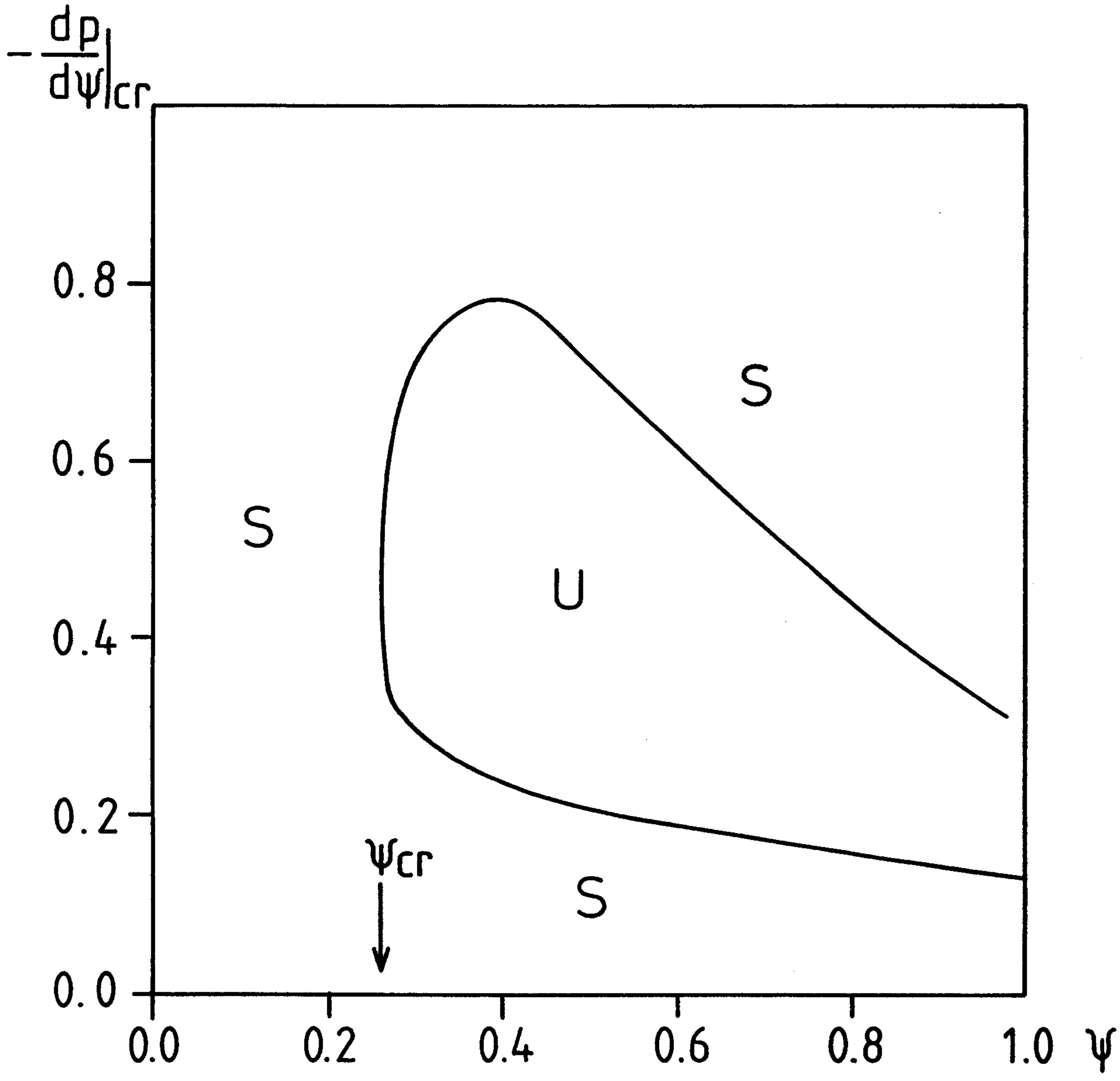


Fig. 3

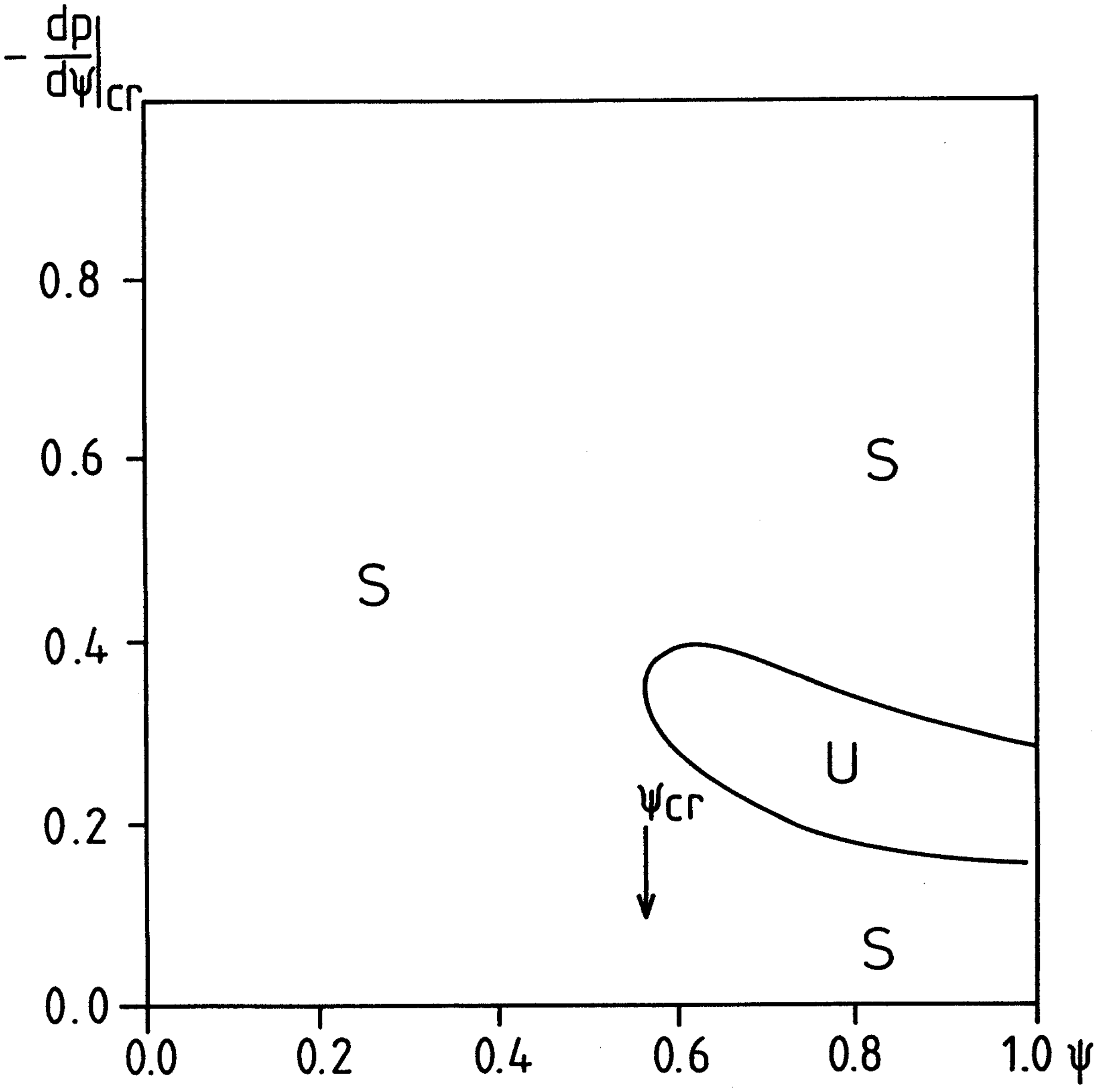


FIG. 4