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Abstract

A new way is discussed to handle numerically the ion-ion hybrid resonance appearing in the cold plasma model for plasma heating in the ion-cyclotron range of frequency (ICRF). It is shown that the resonance can be resolved without introducing artificial and unphysical damping outside the resonance domain. This feature is particularly important for computations in two dimensions. This note supplements and corrects a recently published conference contribution [1].

1. Introduction

In the last few years, a considerable effort has been made to solve numerically the linear wave equations in the plasma and the vacuum together with adequate boundary conditions at the interfaces plasma-vacuum, vacuum-antenna-vacuum and vacuum-conducting shell [1 - 5]. This approach is known as the "global wave approach". For computations in one dimension both hot and cold plasma models have been developed whereas in two dimensions only the cold model has been extensively used up to now. In the cold model a singularity appears in the equations at the ion-ion hybrid resonance where, in the hot model, the mode conversion of the fast wave into the slow wave takes place [2]. To resolve this resonance one adds a small imaginary part to the resonant denominator which introduces a damping of the fast wave at the resonance replacing the mode conversion mechanism. The formulation of the numerical algorithm can be done in different ways. After introducing the physical problem, we shall present the two numerical models used so far and show how they can be modified to eliminate their defficiencies and yield a faithful numerical picture of the physical problem.

2. Physical problem

Throughout this paper we shall consider a slab geometry identical to the one used in Ref. [2] but the considerations made here are relevant to toroidal geometry as well. We suppose the plasma to be inhomogeneous along the x-direction and the magnetic field to be parallel to the z-direction. Assuming an $\exp(-i\omega t)$ time dependence and neglecting both the displacement current and the mass of electrons the Maxwell's equations in the cold plasma model can be written as follows :

$$\underline{\nabla}\times(\underline{\nabla}\times\underline{E}) - \underline{\underline{\varepsilon}}\cdot\underline{E} = 0 \quad , \tag{1a}$$

where

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & i\varepsilon_{xy} \\ -i\varepsilon_{xy} & \varepsilon_\perp \end{pmatrix} \quad ,$$

$$\varepsilon_\perp = \sum_\ell \frac{\omega^2{}_{p\ell}}{c^2} \frac{\omega^2}{\omega^2{}_{c\ell}-\omega^2} \quad , \tag{1b}$$

$$\varepsilon_{xy} = -\frac{\omega^2{}_{pe}\,\omega}{|\omega_{ce}|\;c^2} + \sum_\ell \frac{\omega^2{}_{p\ell}}{c^2} \frac{\omega\omega_{c\ell}}{\omega^2{}_{c\ell}-\omega^2} \quad . \tag{1c}$$

The summation extends over all ion species ℓ, $\omega_{p\ell}$ and $\omega_{c\ell}$ being the plasma and ion-cyclotron frequencies. The ion-ion hybrid resonance, defined by the zero of the denominator of the fast wave dispersion relation for $k_x{}^2$, is then expressed as :

$$\varepsilon_\perp - k_\parallel{}^2 = \varepsilon_\perp - k_z{}^2 = 0 \; .$$

In toroidal geometry $k_\parallel$ becomes a differential operator but the feature of the resonance and thus the method to resolve it remains the same as in 1-D geometry [3, 4]. The singularity of the dielectric tensor, eqs. (1b) and (1c), at $\omega = \omega_{ci}$ does not engender a singularity of the solution of eq. (1a) as can be seen by inspecting the dispersion relation for $k_x{}^2$. In this relation both the numerator and denominator have the same dependence in $(\omega - \omega_{ci})$. Apart from a condition on the wave polarization one does not expect any special feature at the point $\omega = \omega_{ci}$ and in particular no wave damping. The numerical method should not therefore introduce any artificial damping outside the ion-ion hybrid resonance domain. We exclude from our discussion strongly collisional plasmas where the wave damping peaks at $\omega = \omega_{ci}$.

3. Numerical model

So far, there have been two ways to get around the resonance :

1) by adding an imaginary part to the mass, $m(1+i\nu)$, or equivalently to the frequency, $\omega(1+i\nu)$, which corresponds to introducing an artificial collisional damping [4,5];
2) by adding an imaginary part directly to $\varepsilon_\perp$, $\varepsilon_\perp + i\delta$, [1 - 3].

To understand the effect of both methods, let us look at the power deposition density in the plasma $P_d = -1/2 \, \mathrm{Re}\, (\underline{\nabla} \cdot (\underline{E}^* \times \underline{B}))$, which can be expressed in the following ways :

$$P_d = \frac{1}{2\omega} \left(\mathrm{Im}\, \varepsilon_\perp |\underline{E}|^2 - 2\, \mathrm{Im}\, \varepsilon_{xy}\, \mathrm{Im}(E_x^* E_y) \right) \tag{2a}$$

$$= \frac{1}{2\omega} \left(\mathrm{Im}\, \varepsilon_\perp |E_+|^2 + 2(\mathrm{Im}\varepsilon_\perp - \mathrm{Im}\varepsilon_{xy})\, \mathrm{Im}(E_x^* E_y) \right) \tag{2b}$$

$$= \frac{1}{2\omega} \left(\frac{1}{2} (\mathrm{Im}\varepsilon_\perp + \mathrm{Im}\, \varepsilon_{xy}) |E_+|^2 + \frac{1}{2} (\mathrm{Im}\varepsilon_\perp - \mathrm{Im}\, \varepsilon_{xy}) |E_-|^2 \right), \tag{2c}$$

where $E_+ = E_x + iE_y$ is the left-hand component of the field and

$E_- = E_x - iE_y$ is the right-hand component of the field.

Let us consider the first method. If we analyse $\mathrm{Im}\varepsilon_\perp$ and $\mathrm{Im}\varepsilon_{xy}$ replacing ω by $\omega(1+i\nu)$ in eqs. (1b) and (1c) we remark that :

a) $\mathrm{Im}\varepsilon_\perp$ and $\mathrm{Im}\varepsilon_{xy}$ are equal at $\omega=\omega_{ci}$ and of the same order and sign when ω is of the same order as ω_{ci}.

b) They are proportional to ν for $\omega\neq\omega_{ci}$ and to $^1/_\nu$ at $\omega=\omega_{ci}$.

Remark a) together with eq. (2c) shows that the absorption of $|E_-|^2$ tends to cancel and that of $|E_+|^2$ tends to add up, a fact which has been pointed out by Jaeger [6]. Thus P_d tends to be proportional to $|E_+|^2$. As $|E_+|^2$ is small outside the ion-ion hybrid resonance $(|E_+|/|E_-| \sim (\omega-\omega_{ci})/(\omega+\omega_{ci}))$ and large at the resonance, so will be the power deposition profile, which is exactly what we want. Remark (b) shows that the imaginary parts of $\varepsilon_\perp$ and ε_{xy} have a peak at ω_{ci} due to the term $1/(\omega_{ci}{}^2 - \omega^2)$. Thus in conjunction with an eventually unprecise numerical approximation of $E_+=0$ at $\omega=\omega_{ci}$ it can lead to an unphysical wave damping localized around ω_{ci}. This artificial peak of $\mathrm{Im}\varepsilon_\perp$ and $\mathrm{Im}\varepsilon_{xy}$ is even worse in toroidal current-carrying plasmas where the ion-ion hybrid resonance intersects with $\omega=\omega_{ci}$ [7]. This feature of the two-dimensional problem makes it difficult to separate the damping due to mode conversion from the unphysical damping at $\omega=\omega_{ci}$ [8].

The second method takes care of this last problem as it introduces a constant imaginary part δ in $\varepsilon_\perp$ and none in ε_{xy} [8]. In this case, however, a problem of convergence and smoothness of the solution arises because δ must be taken very small compared to the number of points [1] for physical reasons : with a larger δ

most of the power is absorbed by the fast wave outside the resonance, which is again unphysical. This is due to the fact that, with $\mathrm{Im}\,\varepsilon_{xy}$ $=0$ and eq. (2c), P_d is proportional to $(|E_+|^2 + |E_-|^2)$. As seen before, $|E_-|^2$ is much larger than $|E_+|^2$ outside the resonance and much more power is absorbed there than in the first method.

This last point combined with eq. (2b) shows us how to correct the second method. We have to add an imaginary part to ε_{xy} as well, so that :

$$\mathrm{Im}\,\varepsilon_{\perp} - \mathrm{Im}\,\varepsilon_{xy} = 0 \quad ,$$

that is to replace ε_{xy} by $\varepsilon_{xy} + i\delta$ as we did for $\varepsilon_{\perp}$. Let us notice that $\mathrm{Im}\,\varepsilon_{\perp}$ and $\mathrm{Im}\,\varepsilon_{xy}$ have now the same relation as in the first method (remark (a)). In this way P_d is exactly proportional to $|E_+|^2$ over the whole plasma and hence is small outside the ion-ion hybrid resonance. It has no artificial structure around $\omega=\omega_{ci}$. This corresponds to the physical solution we wanted and yields a good numerical model of the resonance. We shall see in the next section that we can considerably enhance the value of δ and thus have much better smoothness and convergence properties.

4. Results

It has already been shown in Ref. [8] that the artificial structure at $\omega=\omega_{ci}$, occuring in the first method, can be eliminated by using constant imaginary parts in $\underline{\underline{\varepsilon}}$. We shall therefore compare our new method only with the results obtained using the second method [1]. We shall use the same parameters as in Ref. [1].

Let us first look at the power deposition versus frequency obtained by means of the second method, that is $\mathrm{Im}\varepsilon_{\perp} = \delta$ and $\mathrm{Im}\varepsilon_{xy} = 0$, and using hybrid elements for the finite element method of discretizing the variational problem (Fig. 1, dotted line). We see that the curve is not smooth at all and that numerical and unphysical peaks appear. This curve was obtained using a δ, normalized by ω^2/c^2, of 1. The continuous line is the result obtained using the hot plasma model which can be taken as a reference. This is dramatically improved with the new method, that is $\mathrm{Im}\varepsilon_{\perp} = \mathrm{Im}\varepsilon_{xy} = \delta$, as can be seen in Fig. 2. There are no numerical and unphysical peaks and the curve is very smooth. Moreover the curve matches well the results of the hot plasma model, showing that the cold model gives now the right physical description of the mode conversion mechanism. This result was obtained using a value of δ of 40 which is, as expected, a much larger value than in the other method. This improvement can also be seen in the convergence study (Fig. 3). With the second method (dotted line) it barely seems to converge while with the new method (continuous line) it converges well for N larger than 500 points. However, in two dimensions we are so far unable to take more than about 400 points across the plasma cross-section using also hybrid elements. We cannot therefore have a well converged solution. Nevertheless the solution is still much smoother with the new method as can be seen in Fig. 4 where we plot the power deposition versus the wave frequency. The dotted line shows the results obtained by means of the second method while the continuous line shows those obtained using the new method. The result is more credible and the multitude of fine-structure peaks (dotted line) have been identified as being of purely numerical origin.

5. Conclusion

We have discussed the advantages and the main defficiencies of the numerical methods used so far to resolve the ion-ion hybrid resonance. We have devised a new method which combines their advantages and eliminates their defficiencies. We have shown how this method dramatically improves the smoothness of the numerical solution.

In two dimensional plasmas, it has eliminated resonances with the modes of numerical origin and allows us to analyse better the mode conversion mechanism using the cold plasma model.

Acknowledgement

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References

[1] K. Appert et al., Comput. Phys. Commun. (1986) 43, 125

[2] K. Appert et al., Comput. Phys. Commun. (1986) 40, 73

[3] L. Villard et al., Comput. Phys. Reports (1986) 4, 95

[4] E.F. Jaeger et al., Comput. Phys. Commun. (1986) 40, 33

[5] K. Itoh et al., Nucl. Fusion (1984) 24, 13

[6] E.F. Jaeger, Private Communication

[7] T. Hellsten and E. Tennfors, Physica Scripta (1984) 30, 341

[8] K. Appert et al., Plasma Phys. and Contr. Fusion (1986) 28, 133

Figure captions

Fig. 1 : Power deposition versus frequency as obtained using the cold plasma model in 1-D geometry on a coarse equidistant mesh with N=512 points and with the second method : $\mathrm{Im}\varepsilon_{xy}=0$, $\mathrm{Im}\varepsilon_{\perp}=\delta$ with $\delta c^2/\omega^2=1$ (dotted line). The continuous reference line is obtained using the hot plasma model.

Fig. 2 : Power deposition versus frequency as in Fig. 1 but with the new method of modelling the cold ion-ion hybrid resonance : $\mathrm{Im}\varepsilon_{xy}=\mathrm{Im}\varepsilon_{\perp}=\delta$ with $\delta c^2/\omega^2=40$ (dotted line). The continuous reference line is the same as in Fig. 1.

Fig. 3 : Convergence behaviour of the power deposition versus mesh size N for the second method (dotted line) and the new method (continuous line), the other parameters being the same as in Fig. 1 and 2 respectively.

Fig. 4 : Power deposition versus frequency as obtained using the cold plasma model in 2-D geometry on a $N_\psi \times N_{pol}$ = 160×80 mesh. The dotted line corresponds to the second method (with $\delta c^2/\omega^2=2.5$) and the continuous line to the new method (with $\delta c^2/\omega^2=60$).

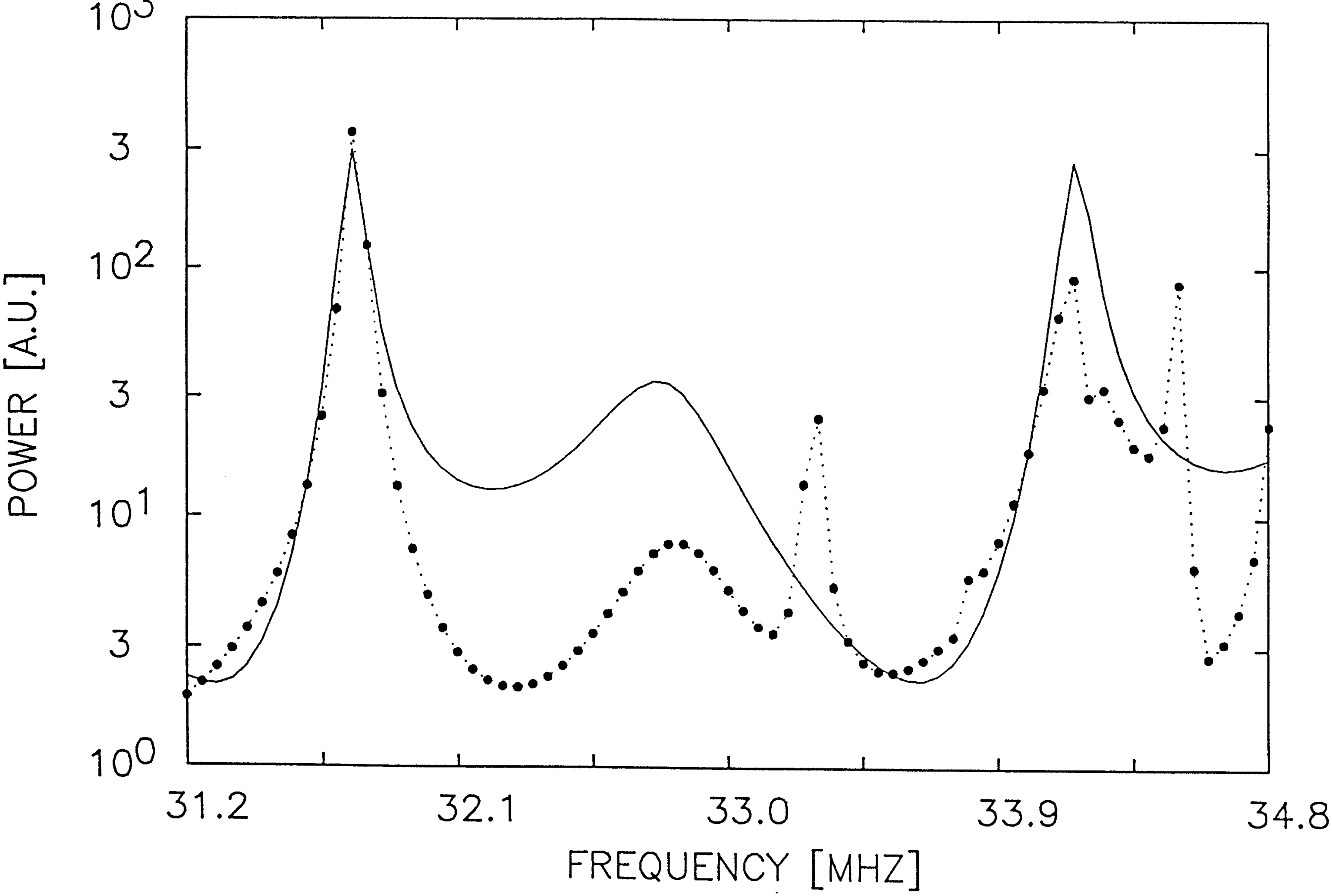

Fig. 1

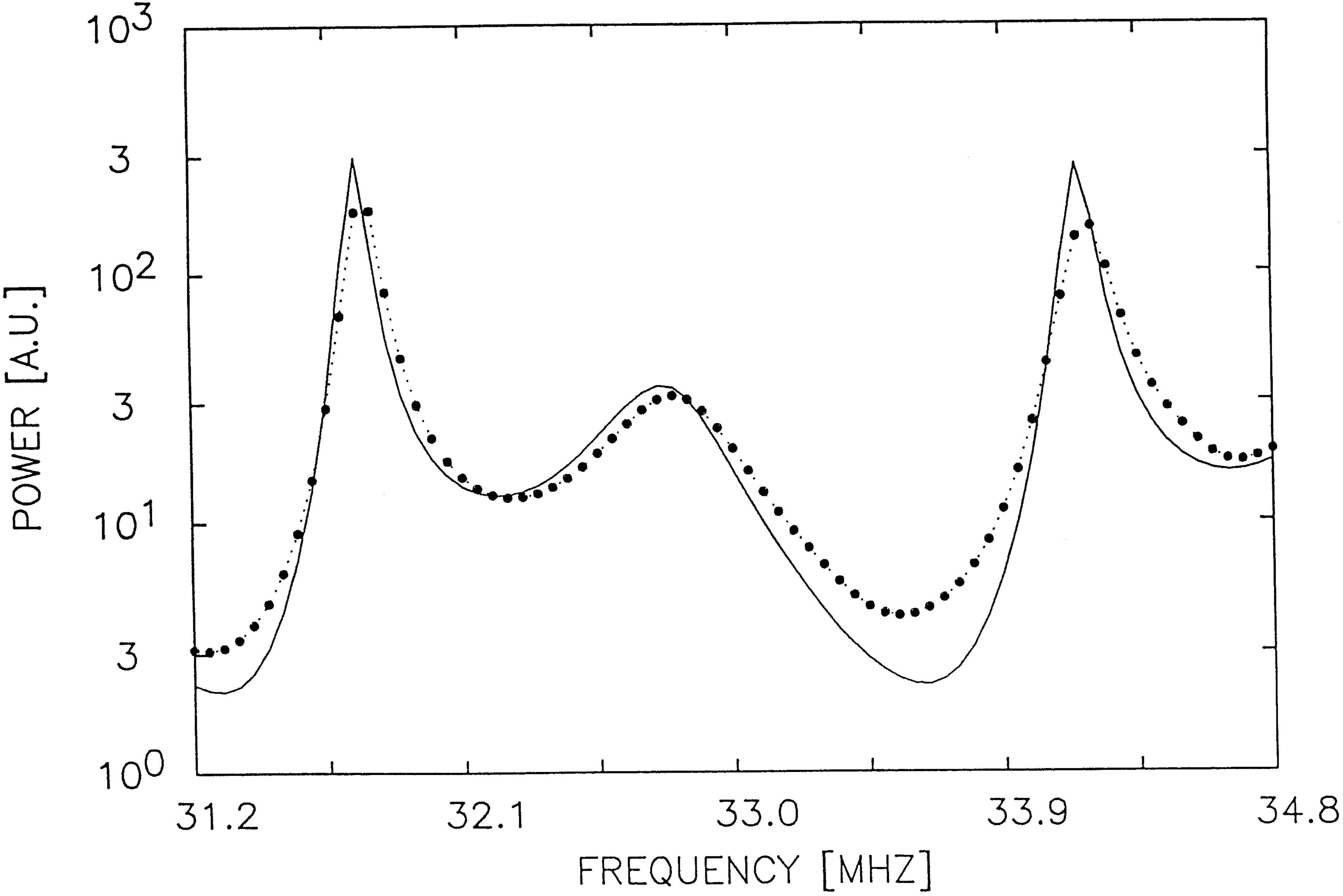

Fig. 2

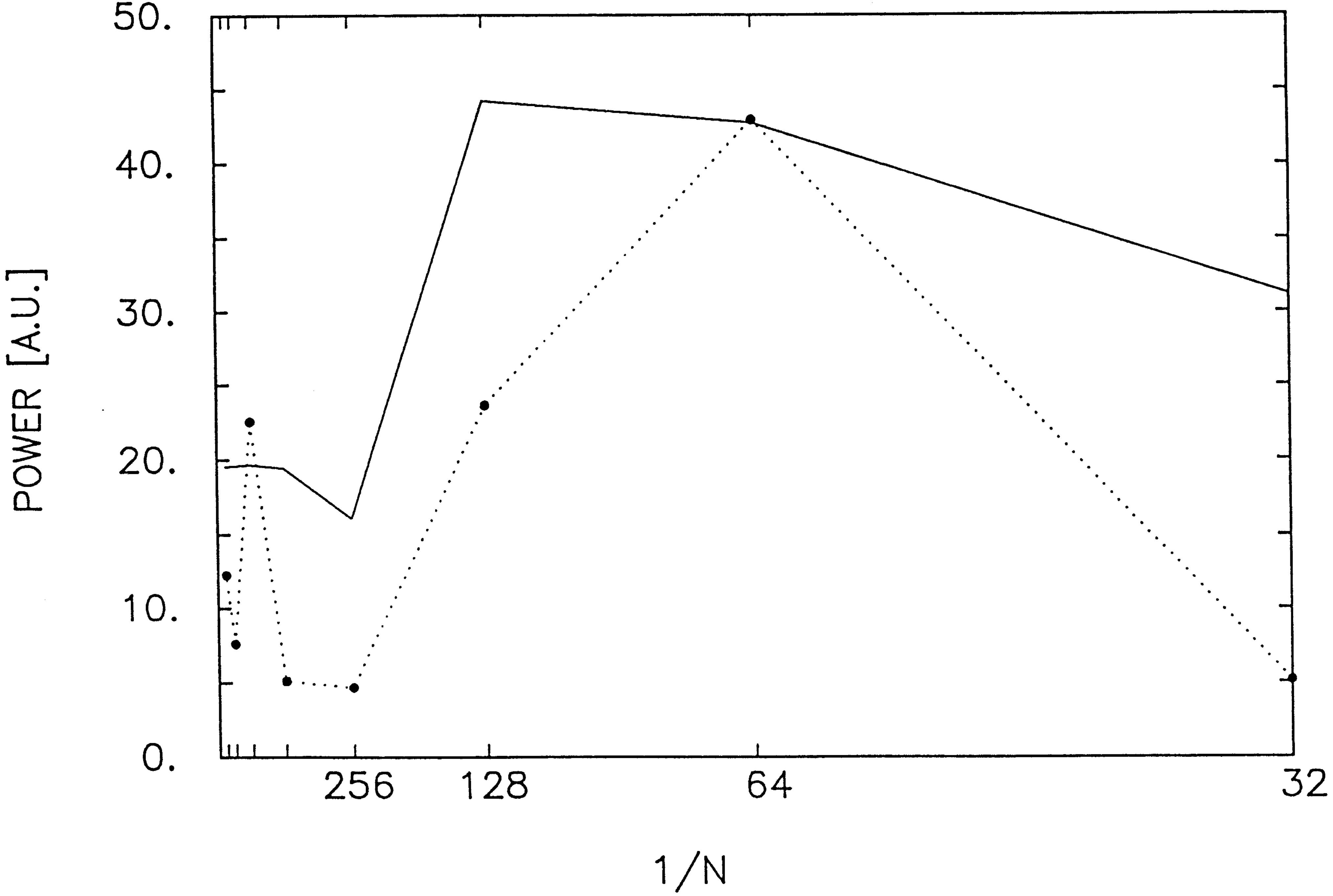

Fig. 3

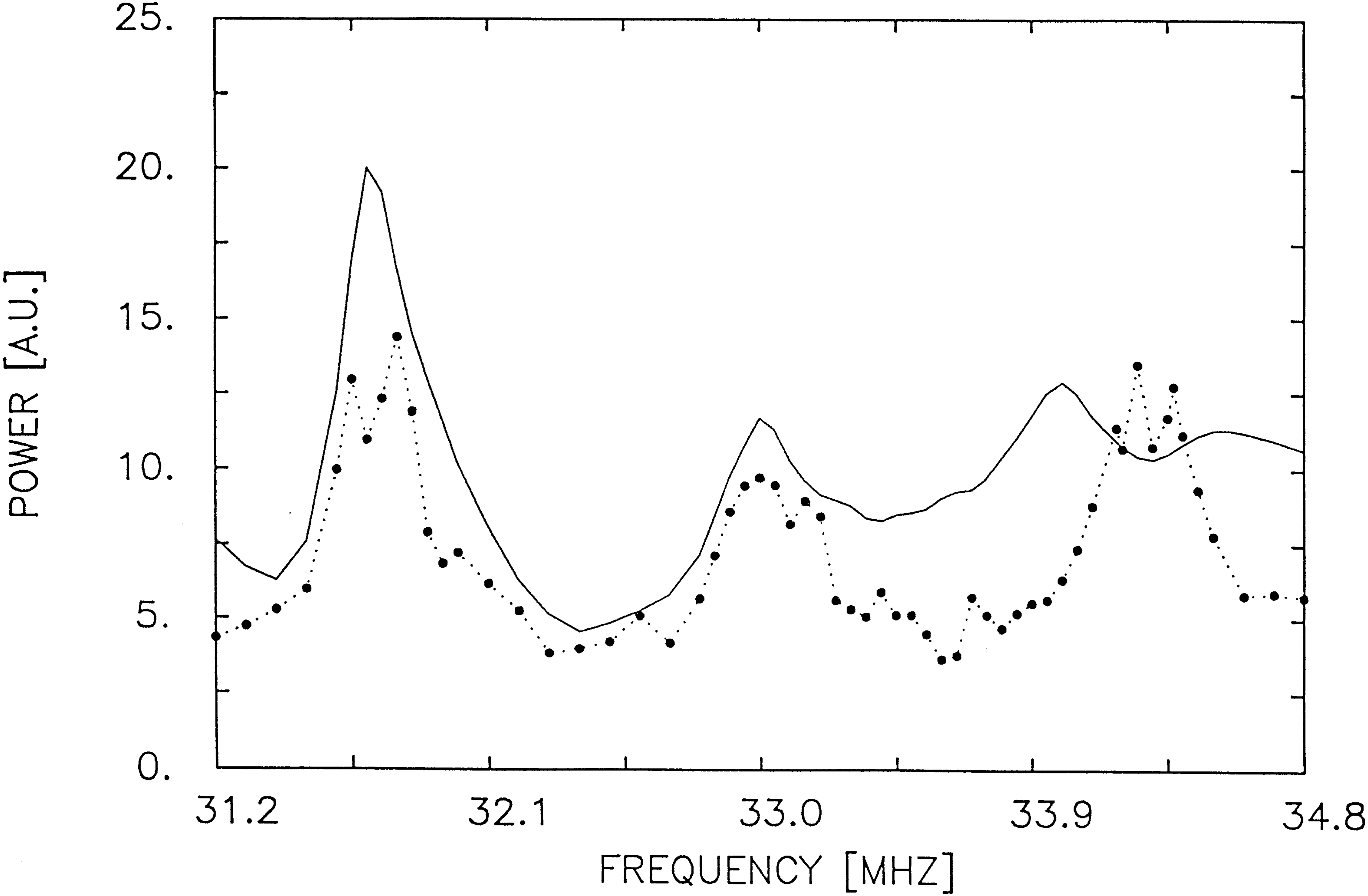

Fig. 4