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FAST IDENTIFICATION OF PLASMA BOUNDARY AND X-POINTS IN ELONGATED TOKAMAKS

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ABSTRACT

A new method for identifying the plasma boundary and X-points in elongated tokamaks is presented. The method uses a finite-element representation of the plasma current. It is applied to a number of computed equilibria with various shapes and elongation up to b/a = 3. Errors in the flux, magnetic field and coil current measurements are simulated by adding random perturbations to the values obtained from the equilibrium code. The method appears to be sufficiently accurate and fast for real-time shape control.

1. INTRODUCTION

The great dilemma of elongated tokamaks is that, on the one hand, the plasma-wall distance (or, equivalently, the distance between the plasma boundary and the passive coils) must be small to ensure axisymmetric stability, and on the other hand, this distance must be as large as possible to avoid plasma contamination and edge cooling. There is very litte margin between these two conflicting requirements, hence the position of the plasma boundary must be accurately controlled. One of the key elements in any plasma shape control system is a method for identifying the plasma boundary. Several schemes, based on magnetic measurements close to the wall, have been proposed [1-10]. The simplest of these schemes consists of expanding the flux function locally into a Taylor series [1, 2] and then computing flux values at a number of points inside the vacuum vessel, close to the desired plasma boundary. This method was used in many tokamaks and gives good results if the plasma boundary is relatively close to the wall, i.e. if the plasma-wall distance is everywhere a small fraction of the minor radius. However, in an elongated tokamak, this condition cannot always be satisfied, especially during the start-up phase, when the plasma occupies only a small fraction of the vacuum vessel. In addition, the Taylor expansion method is unable to localize X-points. In order to overcome these difficulties, more sophisticated methods for identifying the plasma boundary have been developed. One method [2-4] uses a finite number of filaments to simulate the toroidal plasma current. The currents in these filaments are then computed such as to obtain the best fit to the measured fluxes and magnetic fields. Another scheme [5] assumes a toroidal current sheet inside the plasma, whose current density is expressed as a sum of Fourier modes. A third method is based on an expansion of the flux function in terms of toroidal eigenfunctions [1, 6] or an expansion of the plasma current density in terms of multipole moments [7].

All of these techniques can give good results in cases where the Taylor expansion fails, and in particular, they are able to localize X-points. However, these methods have been developed and tested for near-circular or moderately elongated plasmas. The application to highly elongated tokamaks may be difficult for the following reasons: In the case of the filament approach, the definition of appropriate positions for the current filaments requires some a priori knowledge of the approximate position of the plasma boundary, since the filaments must be placed inside the plasma volume, not too close to the edge. The evaluation of radial and vertical moments of the plasma current distribution does not give sufficient information for defining the filament positions. The same problem is encountered when applying the current sheet method. Multipole moment methods, on the other hand, are ideally suited to configurations with nested flux surfaces, having a single magnetic axis. The application of moments methods to more complex shapes, such as doublets or triplets, has not yet been demonstrated. It is likely that a large number of terms will be required in the expansions, and this may not be compatible with the ill-conditioned nature of the problem.

In this Letter, we present an alternative method, based on a finite element representation of the plasma current. The method is closely related to the filament approach. Instead of delta-function current densities at each filament position we now have finite element basis functions at each node. In this way, the model function representing the plasma current is made free of singularities. As a consequence, the choice of node positions is not critical. In fact, the method gives accurate results even when some of the nodes lie outside the plasma boundary.

2. FINITE ELEMENT METHOD

Let us consider a set of two-dimensional, rectangular elements with bilinear basis functions of the form

$$F_n = \left\{ 1 - \left| \frac{R - R_n}{\Delta R} \right| \right\} \left\{ 1 - \left| \frac{Z - Z_n}{\Delta Z} \right| \right\} \tag{1}$$

where R_n, Z_n are the coordinates of the n-th node and ΔR, ΔZ are the element half-widths. F_n is defined within the rectangle $|R-R_n| < \Delta R$, $|Z-Z_n| < \Delta Z$. We assume that the geometrical parameters of the elements are fixed, and that the same elements are used to describe all imaginable plasma shapes. Each current element is defined by a single free parameter, i.e., the current density at the central node. These nodal current densities (or element amplitudes) are computed in such a way as to obtain the best possible fit to the measured fluxes and magnetic fields. This is achieved by a standard least-squares analysis, using weighting coefficients derived from estimates of the measurement errors. The solution can be written in the form

$$\vec{X} = A_1 \vec{\Psi} + A_2 \vec{B} + A_3 \vec{I} \tag{2}$$

where the vector $\vec{X}$ contains the element amplitudes, and the vectors $\vec{\Psi}$, $\vec{B}$ and $\vec{I}$ represent the flux, magnetic field and coil current measurments, respectively. The matrices A_1, A_2 and A_3 contain Green's functions, connecting currents and probe positions. They depend only on the geometry of the current elements and on the coordinates of external currents and magnetic probes, and hence they need be calculated only once. It should be noted that there are several ways to treat the contribution of external coils to fluxes and fields at the probe positions. Here, we assume that the currents in the external coils are measured and we compute the resulting fluxes and fields, using Green's functions [3,7]. Induced currents in the vacuum vessel are not taken into account explicitly. It would be easy to add these currents if measurements are available. If the vessel currents are not measured, it might be advantageous to use one of the fitting procedures [4,5] which do not make use of coil current measurements. In the present study, measurements are generated by an MHD equilibrium code [11] and errors are simulated by random perturbations. These errors will appear in all three terms on the right hand side of eq. (2), as will be discussed in the following section.

The optimum number of current elements to be used in a particular application depends somewhat on the accuracy of the measurements: As the measurements become more accurate, or as the number of probes is increased, more elements can be used. It is clear, however, that the number of current elements must always be small in order to avoid non-physical solutions. In an elongated tokamak, for example, we find that one cannot use more than two elements in the radial direction, and 6 to 8 elements in the axial direction. Once the element amplitudes are known (Eq. 2), one can then compute the flux values at a number of points on the desired plasma boundary, using precalculated Green's functions. Alternatively, it is possible to compute the shape of the plasma boundary, $\Psi=\Psi_{lim}$, by assuming an arbitrary limiter point within the vacuum vessel. If one suspects the occurrence of an X-point, a search routine is used to localize it. The flux value at that point then defines the plasma boundary.

3. RESULTS

The method outlined above has been applied to a wide variety of computed equilibria [11]. Figure 1 shows reconstructed plasma current distributions for a D-shaped and a triplet-shaped plasma. Fourteen current elements were used in both cases. The corresponding original equilibria are shown in Figs. 2(A) and 2(G). Simulated measurements were obtained from these equilibria by assuming 32 flux loops and 32 magnetic field probes, oriented parallel to the wall. Probe positions are indicated in Fig. 2(A). Measurement errors were taken into account by adding random perturbations to the values obtained from the equilibrium code. The maximum amplitude of these random errors was assumed to be constant and equal to ± 1% of the highest measured signal. The same scheme was applied to the flux, magnetic field and coil current measurements. Random perturbations of this magnitude correspond to typical measurement errors in real experiments [12].

Figure 1A shows that the D-shaped plasma occupies only a fraction of the available cross-section. The elements which lie outside the plasma boundary should ideally have zero current. In practice, however, the amplitudes of these vacuum elements are not exactly zero.

This is because, in the least-squares analysis, we make no distinction between plasma and vacuum elements, assuming that there is no a priori information available on the position and shape of the plasma. Nevertheless, current densities in the vacuum region are very small, typically two orders of magnitude smaller than in the plasma. It would, of course, be possible to eliminate the vacuum currents completely by performing a second iteration. We find, however, that such a second step produces only a negligible gain in accuracy of the reconstructed plasma boundary.

Figure 2 shows four examples of original equilibria and reconstructed Ψ-surfaces. Probe positions and current element geometry are identical in all cases. A detailed analysis of these results shows that the deviation between the original and reconstructed plasma boundaries is everywhere less than 2% of the horizontal minor radius of the plasma (except in the vicinity of the X-point). The position of the reconstructed X-point (Fig. 2(F)) agrees with that of the original X-point (Fig. 2(E)) to within the same precision. However, when, we compare the inner flux surfaces we find progressively larger errors as we approach the magnetic axis. This is not simply a consequence of inaccuracies of the finite element model but it reflects the fact that it is rigorously impossible to compute the inner flux surfaces by a purely magnetostatic approach, i.e. without invoking a self-consistent equilibrium solution.

We have tested the sensitivity of this method with respect to large errors in the measurements. Fig. 3 shows four cases of original and reconstructed plasma boundaries. The first case (Fig. 3A) is identical with the one presented in Figs. 2A and 2B. In the remaining three cases (Figs. 3B, 3C, 3D), the amplitude of the random perturbations increases from ± 2% to ± 8%. It should be remembered that, for each type of measurement (flux, magnetic field, coil current) the maximum amplitude of the random errors is assumed constant, and that the relative errors quoted are only valid for the maximum signal. The relative errors on the average signal are approximately twice the values quoted. We conclude from Fig. 3 that the finite element method is quite insensitive to measurement errors.

Finally, let us consider the question whether this method could be used for real-time plasma control. For this purpose, one does not need to reconstruct the Ψ-surfaces as was done in Fig. 2. One only needs to calculate the flux errors at a certain number of points on the predetermined plasma boundary. These flux errors, $\overrightarrow{\Delta\Psi}$, can be expressed as linear functions of the current element amplitudes, $\vec{X}$, and the external currents, $\vec{I}$,

$$\overrightarrow{\Delta\Psi} = A_4 \vec{X} + A_5 \vec{I} \tag{3}$$

where the matrices A_4 and A_5 contain precalculated Green's functions. Equations (2) and (3) may be combined into a single equation,

$$\overrightarrow{\Delta\Psi} = A_6 \vec{\Psi} + A_7 \vec{B} + A_8 \vec{I} \tag{4}$$

Assuming 16 flux errors, 32 flux measurements, 32 field measurements and 16 external currents, the evaluation of Eq. (4) involves a total number of 1280 multiplications and 1280 additions. These operations would take approximately 100 μs, if executed on a typical array processor. This would certainly be acceptable for plasma shape control, and may even be sufficient for feedback control of the vertical instability.

In conclusion, we have shown that a finite element model can be used successfully to reconstruct the plasma surface and to localize X-points in elongated tokamaks. Accuracy and speed of the method appear to be sufficient for applications in real-time plasma shape control. The main advantage of this method, as compared to other methods using filaments or current sheets, is that the model function representing the plasma current is continuous, without any singularities. As a consequence, the choice of node positions is not at all critical. In fact, it is possible to use the same current model for all plasmas in a given vacuum vessel. This is particularly useful in the start-up phase of an elongated tokamak where the plasma position and shape undergo drastic changes.

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FIGURE CAPTIONS

Fig. 1: Reconstructed plasma current distributions for D-shaped (A) and tripled-shaped (B) plasmas.

Fig. 2: Original equilibria (A, C, E, G) and reconstructed flux surfaces (B, D, F, H). Positions of flux loops and magnetic field probes are indicated by closed circles and open circles, respectively.

Fig. 3: Original (dotted lines) and reconstructed (solid lines) plasma boundaries. Random erros on probe measurements are ±1%, ±2%, ±4% and ±8% in case A, B, C and D respectively.

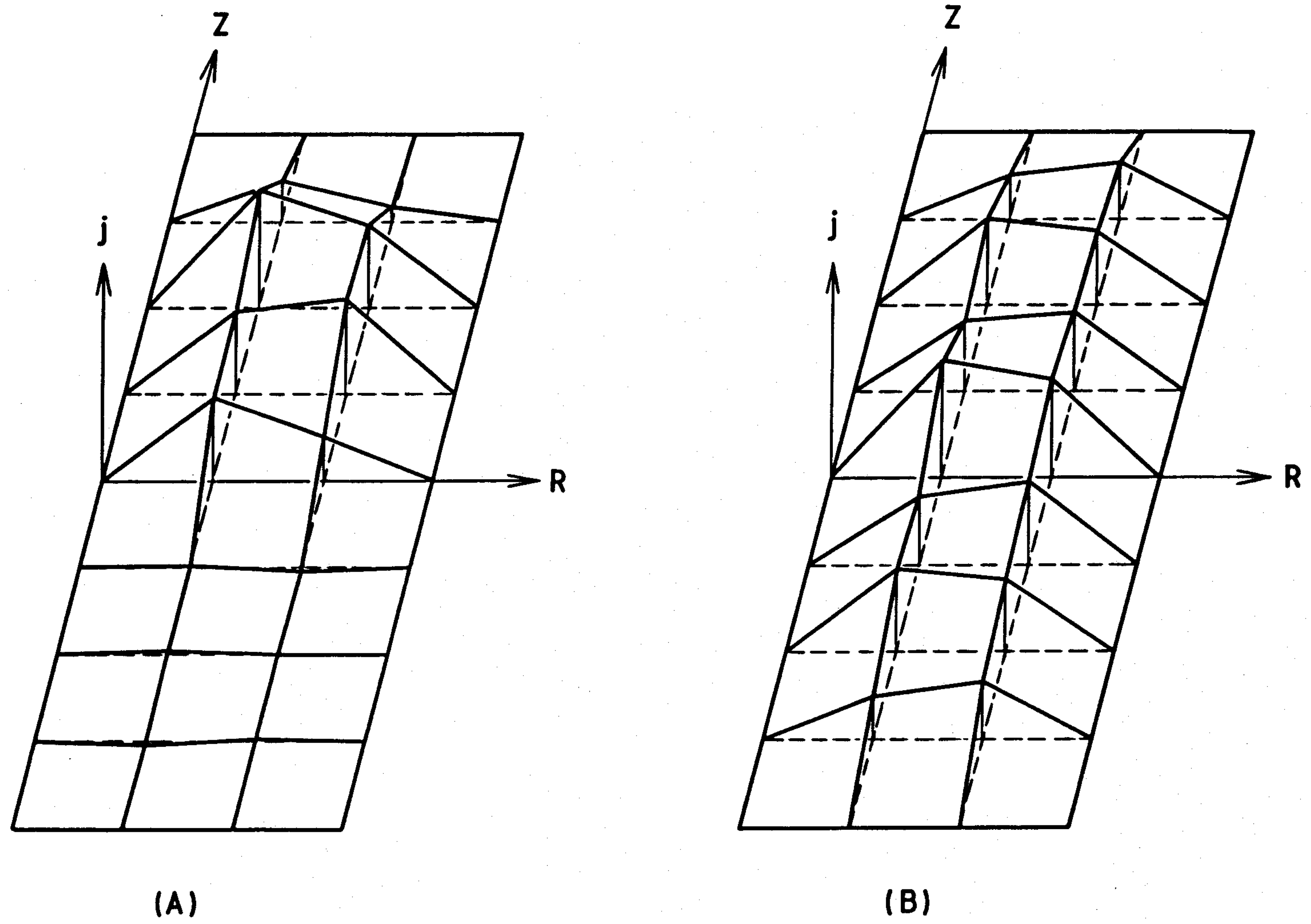

FIG.1

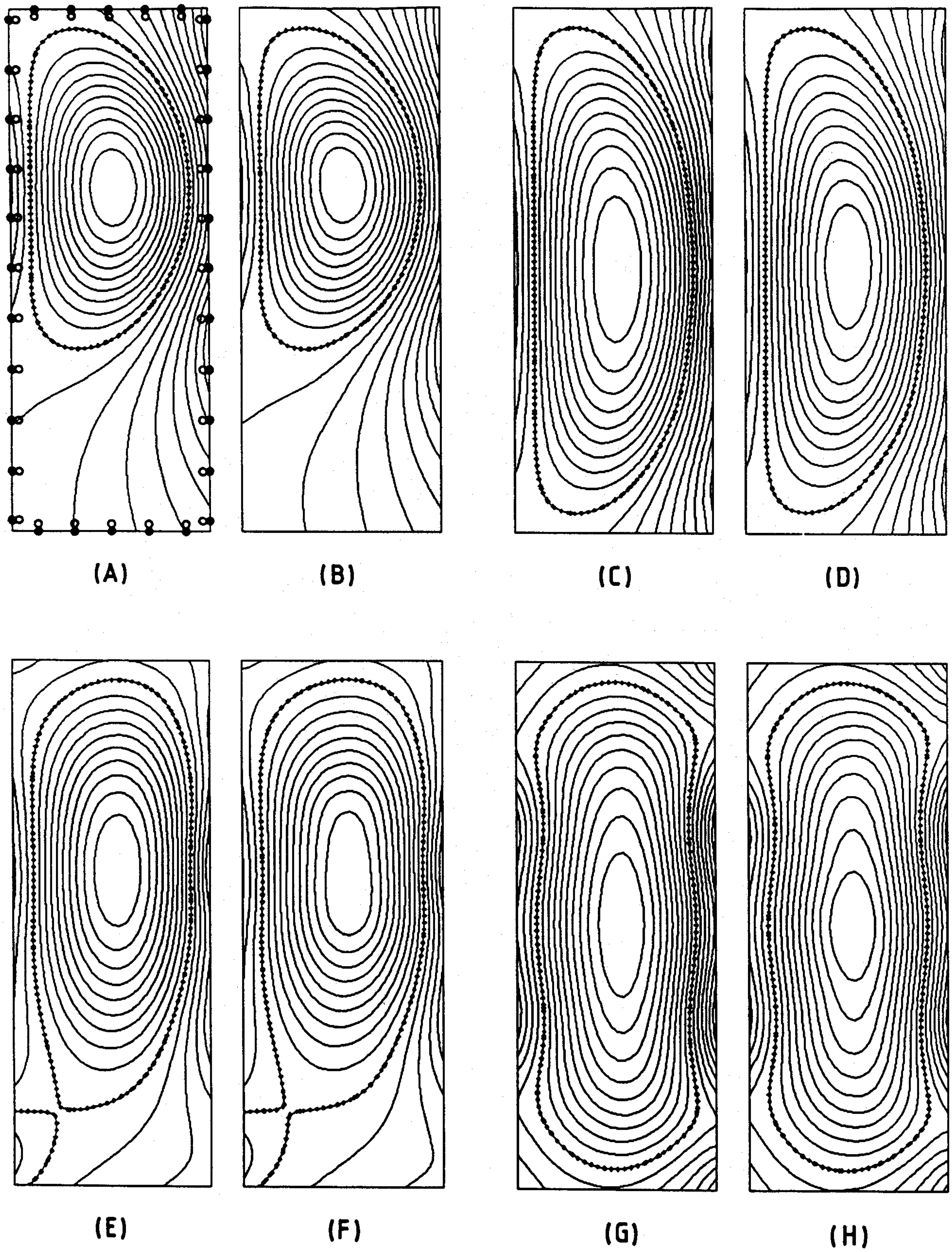

FIG.2

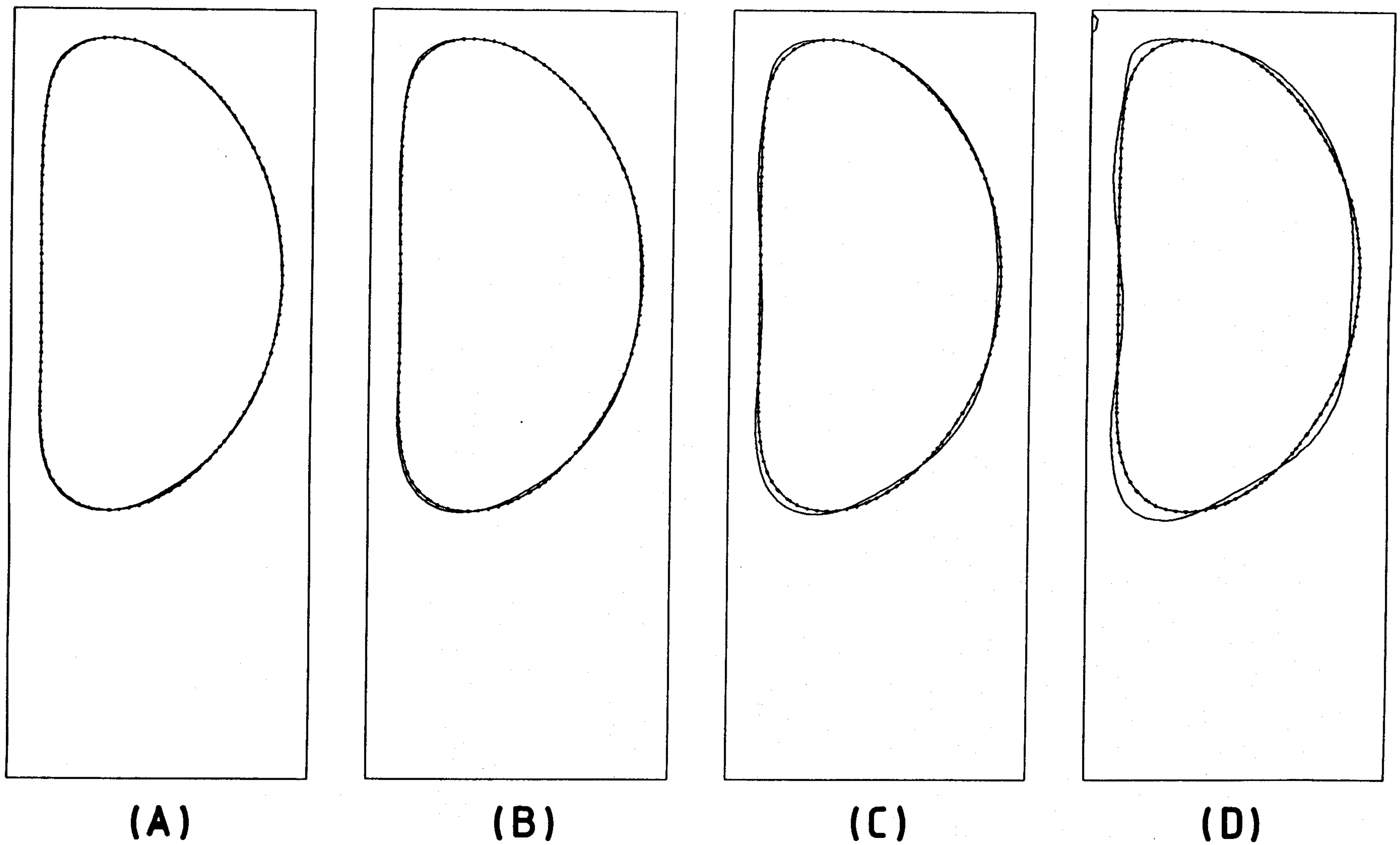

FIG.3