LOW POWER MEASUREMENTS OF THE QUALITY FACTOR
OF AN OPEN RESONATOR WITH STEPPED MIRRORS

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ABSTRACT

Open resonators in which both mirrors have a central step have been predicted to exhibit mode selection properties. This is advantageous for the design of quasi-optical gyrotrons. Quality factor measurements of a 36 cm resonator at 120 GHz are presented and compared with theory. Although the effect of the step is clearly seen, the quality factor Q of the resonator is lower than that predicted.

Key words: Open resonator, quasi-optical gyrotrons, mode selection, stepped mirrors
1. INTRODUCTION

The main advantage of quasi-optical gyrotrons over conventional ones is the use of a larger resonant device, a Fabry-Pérot resonator, which reduces the constraints associated with the ohmic losses. Nevertheless the larger the resonator, the more severe the problem of competition among longitudinal modes becomes. To our knowledge, two theoretical solutions compatible with the removal of large heat flux have been so far proposed to overcome this drawback. Firstly, in numerical simulation the shaping of the magnetic field has been shown to influence the multimode behavior and to favor single mode operation when the magnetic field seen by the electron beam in the interaction region decreases along the trajectory [1]. Secondly, the resonator can be made mode selective by the implementation of "central steps" [2] so that the resonator can be viewed as comprising, in fact, two coaxial Fabry-Pérot cavities of different lengths. It is the purpose of this article to verify experimentally the selectivity of such a resonant device at 120 GHz.

2. EXPERIMENTAL SETUP

The resonator is made of two identical mirrors facing each other (Fig. 1). The output coupling is achieved via annular slots, a solution similar to that adopted in the Naval Research Laboratory quasi-optical gyrotron experiment [3].
The design parameters are:

\[
d = 360 \text{ mm} \text{ (can be varied experimentally from 100 to 900 mm)}
\]

\[
R = 288 \text{ mm} \quad a_m = 70 \text{ mm}
\]

\[
a_0 = 43.50 \text{ mm} \quad a_1 = 25.7 \text{ mm}, \quad a_e = 12 \text{ mm}
\]

\[h, \text{ the step height can be varied from } -2 \text{ to } +10 \text{ mm.}\]

The radius \(a_e\) is such that about half of the circulating power is incident on the step. A small coupling iris has been pierced in the center of the left mirror so that the cavity can be excited by an external Backward Wave Oscillator (BWO). The size of the hole is critical and results from a trade-off between the power transmitted to the cavity and the e.m. field perturbation. We have chosen a diameter of 0.6 mm which lowers the resonator \(Q\) by only 11\%, for an estimated insertion loss of 13 dB.

The microwaves coupled out of the slots of the right-hand mirror are detected by a harmonic mixer (Fig. 2). This method avoids drilling a second hole and, therefore, does not influence the e.m. field inside the cavity. The measurement is somewhat more delicate however, since the field diffracted through the slots presents spatial minima and maxima. The position of the detecting horn has to be adjusted at each measurement to achieve a signal with sufficient dynamic range on the spectrum analyser. The local oscillator (LO) is phase-locked by a quartz frequency counter. The natural linewidth of our BWO is about 2 MHz. The frequency is externally swept by 5 - 20 MHz so that the resonator \(Q\) is directly given by the standard method of dividing the resonant frequency by the resonance width at -3 dB.
The experimental setup (Fig. 3) allows us to slide the right-hand mirror to vary the resonator length \( d \) from 10 to 90 cm. A stepping motor driving a threaded rod is used to monitor small longitudinal displacements over a 25 mm range (10 \( \lambda \)) with a precision of 1 \( \mu \)m. The longitudinal mode separation is 420 MHz for \( d = 36 \) cm.

The brass mirrors were machined on a numerically controlled lathe with a precision of \( 10^{-2} \) mm (\( < \lambda/100 \)) and polished. The alignment of the mirrors is critical. We have found that typically the measured Q values decrease by a factor of two when one of the mirrors is tilted by 0.5\(^\circ\).

3. RESULTS

Before investigating the influence of the central step, we have measured the resonator Q at 120 GHz as a function of mirror spacing. This was done for a resonator without steps in order to establish the experimental method. For confocal resonators (without steps or slots) it is well known that the quality factor decreases monotonically when the mirror spacing increases (see for example [4]). In our case, we have a more complicated resonant structure and the Q depends on the cavity length in a rather complex way. We have used a computer code [5] based on Kirchoff's formulation of the Huygens-Fresnel Principle to calculate the theoretical Q of the resonator of the TEM\(_{00}\) modes. Both diffraction and ohmic losses have been taken into account. We have estimated the RF conductivity of brass at \( 0.86 \times 10^7 \) \( \Omega^{-1} \) m\(^{-1}\), i.e. 60% of the DC value. The comparison between computed and measured
Q is shown in Fig. 4.

The agreement is fairly good for d larger than 390 mm. For 
\( d = 288 \text{ mm} \), the resonator is confocal and the theoretical diffraction 
loss of the TEM\(_{00}\) is minimum (0.2%) leading to a very large Q of 
\( 2 \times 10^5 \). We believe that the reason why we did not observe such a high 
Q value was because of the imperfections of the mirror surface which 
can lower the RF conductivity and/or the coupling between the wave- 
guide and the resonator. For d < 310 mm modes other than TEM\(_{00}\) are 
also excited, thus complicating the identification on the spectrum 
analyser of TEM\(_{00}\) modes. For d < 300 mm, the measurement therefore 
becomes very difficult. Nevertheless, we think our method is adequate 
to measure Q values up to \( 6 \times 10^4 \), for d > 330 mm.

We now investigate the effect of the central step. In Figures 
5 - 10 we have plotted the resonator Q as a function of the longitudi-
nal mode number n, where 280 < n < 296. For the TEM\(_{00n}\) mode the 
resonant frequency is very well approximated by

\[
f = \frac{c}{2d} \left[ n + \frac{1}{n} \cos^{-1}(1 - \frac{d}{R}) \right]
\]

(c is the speed of light).

For \( n = 287 \), \( d = 360 \), \( f = 119.742 \text{ GHz} \). The computer code which 
takes the mirror size into account gives 119.743 GHz.
It is seen on Fig. 5 that the resonator Q increases slowly with the frequency since the resonator Fresnel number \( N = \frac{a^2}{d\lambda} \) is proportional to the frequency. Our measured Q values are typically 40% smaller than calculated ones. In Figure 5 we have also plotted a former result (for \( n = 287 \)) taken with an unstepped resonator. This value is closer to the expected one because there is no surface discontinuity due to the central step at \( r = a_e \). In Figures 6 - 10 the step height is a integer multiple of \( \lambda/2 \) in order to achieve resonance. The effect of the step height is clearly seen on Figs. 8 - 9. For example, with a step height of 5 mm (Fig. 8) the Q increases by a factor of 4 between modes 282 and 286, whereas in the absence of steps we barely have a difference of 10%. The measured Q's are only one third of the expected values. So far we have no definitive explanation for this decrease in Q although we suspect that the circular surface of the step may cause additional diffraction losses, an effect which was neglected in our calculations. In this case, not only is the Q value reduced but also the coupling efficiency, defined as the ratio of the power coupled through the slots to the total diffracted power. Another possible reason for the decrease in Q values may be tighter tolerances on the stepped mirrors and the alignment. A hint to this suggestion is the extreme sensitivity of our numerical results for Q with respect to the geometrical parameters.
4. CONCLUSION

Our measurements indicate that it is possible to manufacture Fabry-Pérot resonators for millimeter waves with mode selection properties. However, the quality factor is below that predicted by the simple quasi-optical Huygens-Presnel theory. We believe that 3-d calculations would yield a better agreement with experiment. In applications which do not require a high output coupling efficiency, the use of stepped mirrors reduces the number of natural frequencies of an open resonator.

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REFERENCES


FIGURE CAPTIONS

Fig. 1: Resonator cross-section

Fig. 2: Principle of measurement

Fig. 3: Experimental setup (photo)

Fig. 4: Quality factor of the TEM_{00} mode at 119.743 GHz as a function of the mirror spacing. The step height is 0.

Fig. 5: Quality factor as a function of the longitudinal mode number. The step height is 0.

Fig. 6: idem
Step height is 1.25 mm

Fig. 7: idem
Step height is 2.5 mm

Fig. 8: idem
Step height is 5.0 mm

Fig. 9: idem
Step height is 10.0 mm

Fig. 10: idem
Step height is -1.25 mm
Fig. 1

Fig. 2

$\Delta f$ 

$Q = \frac{f}{\Delta f_{ir}}$

$f_{ir} = f - nf_{lo}$

$f_{lo} \approx 12 \text{ GHz}$
Fig. 4
HEIGHT OF CENTRAL STEP: 0 MM

Fig. 5

HEIGHT OF CENTRAL STEP: 1.25 MM

Fig. 6
HEIGHT OF CENTRAL STEP: 10 MM

Fig. 9

HEIGHT OF CENTRAL STEP: -1.25 MM

Fig. 10