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ENERGY TRANSFER FROM STANDING WAVES TO GYRATING PARTICLES

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ABSTRACT

A positive-definite local power absorption density for standing waves in stable plasmas is described.

1. INTRODUCTION

An important goal of the theory of RF heating of plasmas is the prediction of power deposition profiles. The fusion engineers need these profiles as an input to their transport codes. They are then able to interpret experimental data and to extrapolate to future machines.

In toroidal machines where the heating power must be transported perpendicularly to the confining magnetic field one seeks to compute the absorbed power density as a function of the minor radius. In a situation where the wave is weakly damped by a stable plasma and yet totally absorbed in a single pass, no question arises as to how the time-averaged local energy absorption should be defined.[1] If, however, the wave is not absorbed in a single pass, as it frequently happens, and standing waves are formed in the plasma cavity or in parts of it, one might get time-averaged power deposition profiles which locally assume negative values.[2]

Should we, under such circumstances, expect local plasma cooling? Of course not! We just did not ask a precise question. We ignored the fact that the particles gyrate around the magnetic field lines. A gyrating particle interacts with the wave field at different spatial positions and we have therefore to be careful with what we mean by "local". There is no problem with a weakly damped propagating wave where a time average automatically yields a space average, too. This is not true for a standing wave where a particle with a finite Larmor radius sees a wave envelope with a non-negligible spatial variation. In order to decide whether a wave heats or cools the plasma one has to determine the energy gained by the particles.[3,4] If we identify the particles by their initial positions at a given time, say $t = 0$, we can associate the energy absorbed by these particles with their initial positions and in this way the positive-definite local power density is obtained.[3,4]

2. PHYSICAL MODEL

To be more specific, let us assume a plane plasma slab confined in $x_\ell \leq x \leq x_r$ by a magnetic field $\vec{B} = B\vec{e}_z$, $B = \text{const.}$ The particle densities $n_\alpha(x)$ and temperature $T_\alpha(x)$ are in general functions of x. For a wave field of the form $\mathrm{Re}\vec{E}(x)$ $\exp[i(k_y y + k_z z-\omega t)]$ the linear wave equation for such a plasma, correct to second order in the Larmor radius and to first order in the scale length of the equilibrium gradients (i.e. $\partial n_\alpha/\partial x$ and $\partial T_\alpha/\partial x$) has the form[5]

$$\mathrm{rotrot}\,\vec{E} = (\overleftrightarrow{\gamma} + \overleftrightarrow{\beta}\frac{d}{dx} + \frac{d}{dx}\overleftrightarrow{\beta}^{+} + \frac{d}{dx}\overleftrightarrow{\alpha}\frac{d}{dx})\,\vec{E}. \qquad (1)$$

Here the operators d/dx act on everything to their right. The particle dielectric tensors $\overleftrightarrow{\alpha}$, $\overleftrightarrow{\beta}$, $\overleftrightarrow{\beta}^{+}$ and $\overleftrightarrow{\gamma}$ satisfy certain symmetry relations. Their elements are expressible in terms of equilibrium quantities like the plasma frequencies $\omega_{p\alpha}$, the cyclotron frequencies $\omega_{c\alpha}$, the thermal velocities $v_{th\alpha}$ and the plasma dispersion functions for the cyclotron harmonics 0, ±1 and±2. Given the wave equation (1), one may derive a conservation equation of the form

$$Q^{(1)} + \frac{d}{dx}S^{(1)} = 0 \qquad (2)$$

where

$$S^{(1)} = \mathrm{Im}[\vec{E}^{*} \times \mathrm{rot}\vec{E} + \vec{E}^{*} \cdot \overleftrightarrow{\beta}^{+} \cdot \vec{E} + \vec{E}^{*} \cdot \overleftrightarrow{\alpha} \cdot \frac{d\vec{E}}{dx}]$$

and

$$Q^{(1)} = \mathrm{Im}[\vec{E}^{*} \cdot \overleftrightarrow{\gamma} \cdot \vec{E} + \vec{E}^{*} \cdot \overleftrightarrow{\beta} \cdot \frac{d\vec{E}}{dx} - \frac{d\vec{E}^{*}}{dx} \cdot \overleftrightarrow{\beta}^{+} \cdot \vec{E} - \frac{d\vec{E}^{*}}{dx} \cdot \overleftrightarrow{\alpha} \cdot \frac{d\vec{E}}{dx}]$$

3. RESULTS

In the case of a propagating wave, eq. (2) can be interpreted as the energy conservation law: $Q^{(1)}$ is the power density and $S^{(1)}$ is the total (Poynting + kinetic) energy flux. The case of a standing

Take a standing, low-frequency ($\omega \ll \omega_{ci}$), fast magnetosonic wave in a homogeneous plasma

$$E_x = 0,\ E_y = \sin kx,\ E_z = -k_z k\rho^{*2} \frac{\omega_{ci}}{\omega} \cos kx,$$

$$k^2 = \frac{\omega^2}{C_A^2} - k_z^2,\ k_y = 0,\ C_A^2 = \frac{B^2}{4\pi n_i m_i},\quad \rho^{*2} = \frac{T_e}{m_i \omega_{ci}^2}$$

and compute $Q^{(1)}$. One finds

$$-Q^{(1)} = \left| \mathrm{Im}\ Z\left(\frac{\omega}{k_z v_{the}}\right) \right| \frac{\omega_{pi}^2}{\omega^2} (k\rho^*)^2\ (3\cos^2 kx - 2\sin^2 kx).$$

Here Z is the plasma dispersion function as defined by Shafranov.[6] The integral of $-Q^{(1)}$ over the whole plasma volume is positive, as it should be for a stable plasma, but the local values can have either sign. We have found[4] that the local power absorption related to the initial locations of the absorbing particles takes the form

$$Q^{(2)} = Q^{(1)} + \frac{d}{dx} (S^{(1)} - S^{(2)}). \tag{3}$$

It can be shown that $Q(2)$ is positive-definite as long as the finite Larmor radius expansion holds. Most important, the difference between

$Q^{(1)}$ and $Q^{(2)}$ can only be brought into form (3), if equilibrium gradients are included in the dielectric tensor.

Specifically in the Alfvén Wave Range of Frequency (AWRF) we have found that the term[5]

$$\frac{\omega}{c^2\omega_{ce}k_z} \frac{d}{dx} [\omega_{pe}^2 \{1-Z(\frac{\omega}{|k_z|v_{the}})\}]$$

has to be included in γ_{yz}. A typical result for Alfvén Wave Heating is shown in Fig. 1. In this frequency range the flux in Eq. (3) takes the form[4,5]

$$S^{(1)} - S^{(2)} = \mathrm{Im}[2\beta_{zy}^{+} \mathrm{Re}(E_y E_z^*) + \alpha_{zz} \frac{d}{dx} |E_z|^2 - \frac{1}{2} \frac{d\alpha_{zz}}{dx} |E_z|^2].$$

In cases where the equilibrium temperature is small at the plasma boundary, $S^{(1)}-S^{(2)}$ is exponentially small there and we should have

$$\int_{x_\ell}^{x_r} Q^{(1)}dx = \int_{x_\ell}^{x_r} Q^{(2)}dx$$

This is perfectly the case in Fig. 1.

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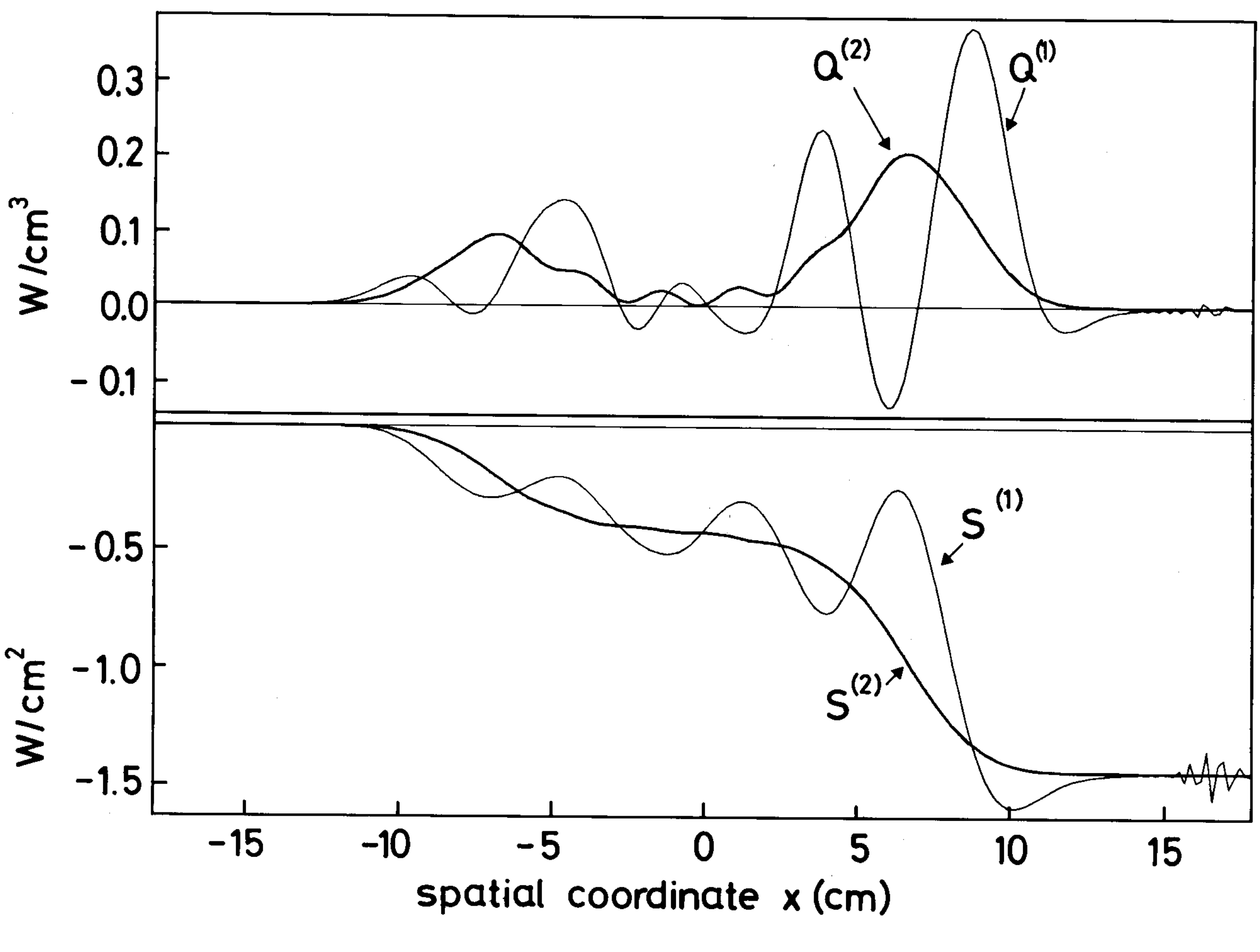

Fig. 1. First and second order power densities, $Q^{(1)}$, $Q^{(2)}$ and fluxes, $S^{(1)}$, $S^{(2)}$, as a function of position as obtained from a numerical solution[7] of the wave equation (1) in the AWRF. Shown is an Alfvén Wave Heating relevant case where a fast magnetosonic surface wave mode converts into a kinetic Alfvén wave at x = ±8.6 cm.

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GLOBAL WAVES IN THE ICRF IN JET PLASMAS

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ABSTRACT

A toroidal global wave code has been developed for the study of ICRF heating in tokamak plasmas.[1] The code takes into account the full geometry of the equilibrium and is suitable for the investigation of mode conversion scenarios. The influence of the existence of global waves is examined here.

INTRODUCTION

A global wave calculation implies the resolution of the relevant system of partial differential equations in a well-defined geometry in a bounded domain. Let us consider a toroidal plasma in an axisymmetric equilibrium surrounded by a vacuum region containing an antenna and enclosed by a conducting shell. The equations for the rf field can be written as

$$\text{rot rot } \underline{E} - \omega^2/c^2 \, \underline{\underline{\varepsilon}} \, \underline{E} = 0, \qquad (1)$$

where $\underline{\underline{\varepsilon}}$ is the dielectric tensor operator. We use the cold plasma model with vanishing electron inertia. In this approximation we write $\underline{\underline{\varepsilon}}$ in the local magnetic coordinate system $\underline{e}_{\parallel} = \underline{B}_o/B_o$, $\underline{e}_N = \nabla\Psi/|\nabla\Psi|$, $\underline{e}_{\perp} = \underline{e}_{\parallel} \times \underline{e}_N$:

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{NN} & \varepsilon_{N\perp} \\ -\varepsilon_{N\perp} & \varepsilon_{NN} \end{bmatrix} + \frac{\underline{J}_o \cdot \underline{B}_o}{B_o^2} \text{ rot}$$

$$\varepsilon_{NN} = \frac{c^2}{c_A^{\,2}} \sum_i \frac{f_i}{1-\omega^2/\omega_{ci}^2} \qquad \varepsilon_{N\perp} = i\frac{c^2}{c_A^{\,2}} \sum_i \frac{f_i\omega/\omega_{ci}}{1-\omega^2/\omega_{ci}^2}$$

$$c_A^{\,2} = B_o^2/(\mu_o \sum_j n_j m_j) \qquad f_i = \frac{n_i m_i}{\sum_j n_j m_j} \tag{2}$$

The antenna is modelled by an infinitely thin sheet on which the rf current is prescribed. Before solving eq. (1) we first write it in a variational form in toroidal magnetic coordinates. We use appropriate boundary conditions at the plasma-vacuum interface, at the antenna and on the shell. The variational form is then discretized on a general irregular mesh and solved using finite hybrid elements. For more details, see Ref. [1]. A great advantage of the finite element method is that no WKB approximation is made. Moreover, we can try to get as close as possible to the exact solution by making convergence studies and thus measure the accuracy of the result.

The operator defined by eqs. (1) and (2) is non-compact due to the presence of singularities at the Alfvén and ion-ion hybrid resonances. The relation defining the resonances is $\varepsilon_{NN} - k_{\parallel}^{\,2} = 0$, where, in toroidal geometry, $k_{\parallel}^2$ is a differential operator. A consequence is that the resonances coincide with magnetic surfaces.[1,2] To turn around the singularities we add an imaginary part to the dielectric tensor, either directly to ε_{NN} ($\varepsilon_{NN} + i\delta$) or by replacing ω by $\omega(1 + i\nu)$.

RESULTS

In the traditional theoretical picture of ICRF heating it is often argued that it is necessary to use a scenario in which the single-pass absorption is large. In this paper we intend to moderate this belief by showing a striking counter-example.

Let us consider a JET plasma (aspect ratio 2.4, $q_0 = 1$, $q_a = 2.2$, elongation 1.68, $B_0 = 3.4$ T, $R_0 = 3$m, $n_e = 3.242\ 10^{19} m^{-3}$) containing a mixture of deuterium and hydrogen with $n_H/n_e = 30\%$. The basic theories predict for this case that a wave incident from the Low Field Side (LFS) is almost 100% reflected, but a wave coming from the High Field Side (HFS) is absorbed at the ion-ion hybrid resonance with almost 100% single-pass absorption. An approach like ray-tracing[3] would clearly discard the LFS antenna in favour of the HFS. Using the global approach, our conclusion will be opposite.

Since global modes can be excited as soon as the absorption is less than 100%, a LFS antenna can excite them but not a HFS. The results of our code confirm this fact. The total power as a function of the frequency for one toroidal wavenumber ($n = -15$) is shown in Fig. 1. For the LFS antenna we find sharp peaks that we identify as global eigenmodes of the fast wave. Notice that the main peaks are surrounded by smaller "satellite" peaks. These peaks are a result of the coupling due to the toroidal geometry: the modes have the same radial wave structure as the main peaks but a different poloidal structure. If we perform the same calculation in a 1-D slab model we find only the main peaks.[4] Thus in toroidal geometry the global modes are more densely packed than in the 1-D model and the average coupling of the LFS antenna is in this case as good as for any other scenario having a good single-pass absorption.

In Fig. 2 we show a contour plot of the amplitude of the wave left-hand polarization, $|E_+|$, for a frequency of 42.85 MHz corresponding to a main peak of Fig. 1. As expected strong reflection occurs and the wave field is confined between the LFS edge and the middle of the plasma.

The results for the HFS antenna look quite different. First of all, as expected no global mode is excited. Secondly, the coupling is very low as compared with the LFS antenna (see dashed line in Fig. 1). This surprising result is an effect of the toroidal geometry. The fast wave is evanescent near the plasma boundary due to the low density in these regions. However, it is more evanescent on the HFS than on the LFS. If we analyse the dispersion relation of the fast wave and approximate the parallel wavenumber by n/r, which is 2.4 times larger on the HFS than on the LFS, we see that the region of evanescence extends over 23 cm on the HFS and 1 cm on the LFS. This strong evanescence can be visualized on a plot of the wave left-hand polarization (Fig. 3). The amplitude of the wave is reduced by a factor of about 6 through the evanescence and thus the total power by a factor of about 36. Since there is no reflection but all the incoming power is single-pass absorbed at the ion-ion hybrid resonance, there is no possibility for the wave to build up an eigenmode and hence the coupling is weak.

CONCLUSION

We have shown that in some cases the global approach brings a new theoretical picture of ICRF heating. We have also shown that the toroidal effects play an important role.

ACKNOWLEDGEMENT

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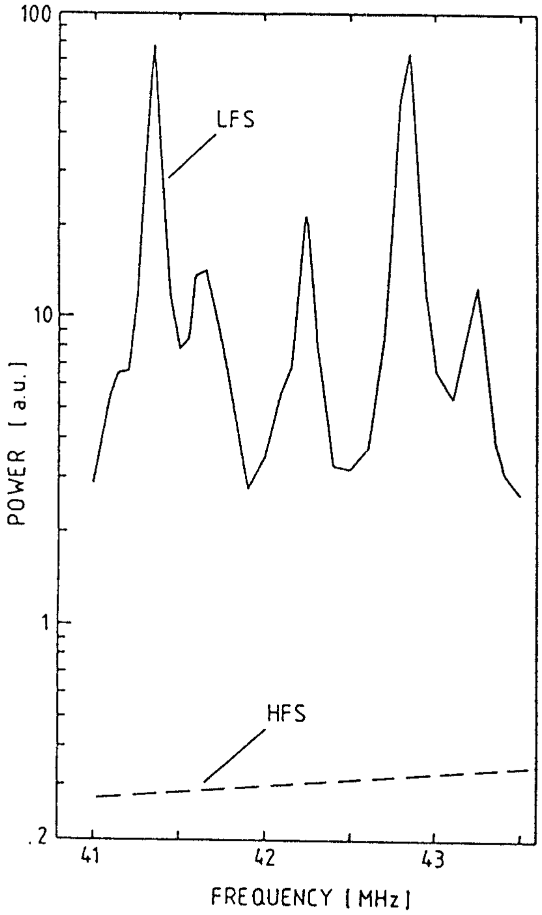

Fig. 1: Power vs frequency for HFS and LFS antennae

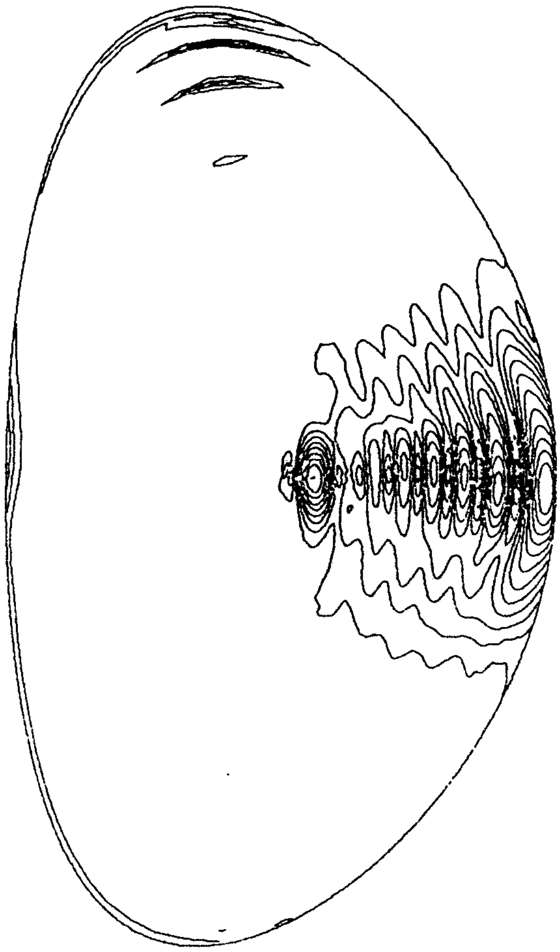

Fig. 2: Wavefield $|E_+|$ for LFS, f = 42.85 MHz

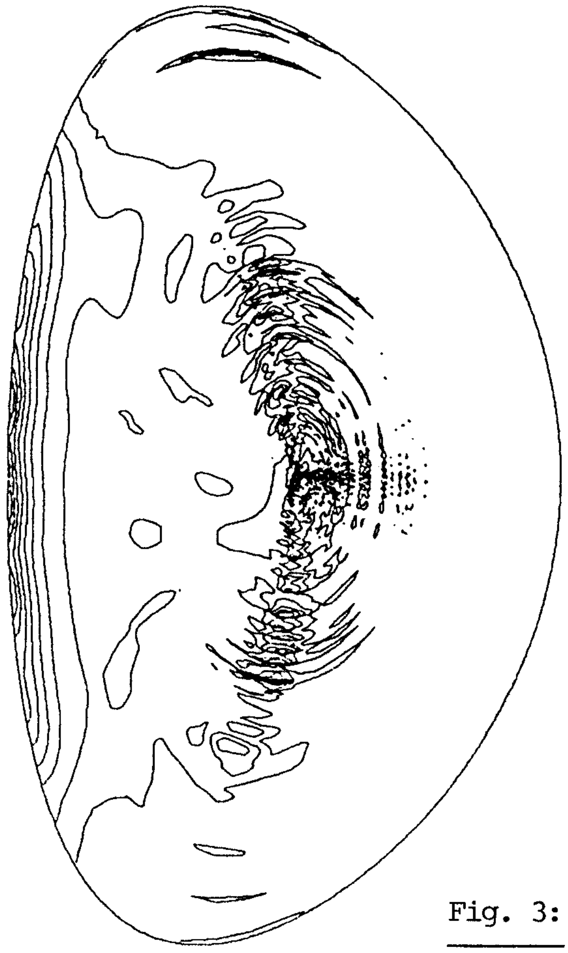

Fig. 3: Wavefield $|E_+|$ for HFS, f = 42 MHz

HFS AND LFS SCENARII FOR ICRF HEATING IN JET

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ABSTRACT

A global wave code for ICRF heating in slab geometry is used to compute the power absorbed by the plasma for high field side (HFS) and low field side (LFS) antennae. The study is made for a typical JET plasma. It is shown that LFS is at least as good as HFS due to the existence of global modes and that it should be much better in toroidal geometry.

INTRODUCTION

Performing a global wave calculation means that one resolves the equations in the plasma and vacuum with boundary conditions on the interfaces plasma-vacuum, vacuum-antenna-vacuum and vacuum-conducting shell.[1] These equations, assuming an exp(-iωt) time-dependence, taking the cold plasma model and neglecting the mass of electrons are written as follows:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{E}) - \underline{\underline{\varepsilon}}\, \underline{E} = 0 \, , \tag{1}$$

where

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_{\perp} & \varepsilon_{xy} \\ -\varepsilon_{xy} & \varepsilon_{\perp} \end{pmatrix} \, ,$$

$$\varepsilon_{\perp} = \sum_{\ell} \frac{\omega_{p\ell}^2}{c^2} \frac{\omega^2}{\omega_{c\ell}^2 - \omega^2} \, ,$$

$$\varepsilon_{xy} = -\, i \frac{\omega_{pe}^2 \, \omega}{|\omega_{ce}| c^2} + i \sum_{\ell} \frac{\omega_{p\ell}^2}{c^2} \frac{\omega \, \omega_{c\ell}}{\omega_{c\ell}^2 - \omega^2} \, .$$

The summation extends over all ion species ℓ, $\omega_{p\ell}$ and $\omega_{c\ell}$ being the plasma and ion cyclotron frequencies. The generalized equations containing the hot plasma model are discussed in reference [1]. The boundary conditions are obtained by means of Maxwell's equations in the vacuum neglecting the displacement current and assuming the prescribed current to be localized on an infinitely thin sheet. The shell is supposed to have zero resistivity.

The plasma is situated in the domain $x_{pl} \leq x \leq x_{pr}$ and is inhomogeneous along x. The conductive shell is localized at $x = x_{sl}$ and $x = x_{sr}$. The antenna lies at $x = x_a$ with $x_{pr} < x_a < x_{sr}$. The magnetic field, parallel to z-direction, varies as $1/(R_o \pm x)$ depending on whether the antenna is on the LFS or HFS respectively. The variational problem, obtained by multiplying equation (1) with a test function and integrating by part, is solved using the finite element method.[2] To get around the singularity at the ion-

ion hybrid resonance, defined by $\varepsilon_{\perp} - k_z^2 = 0$, we introduce an imaginary part in ω, $\omega(1+i\nu)$, which consequently introduces an imaginary part in $\varepsilon_{\perp}$.

RESULTS

We shall use the following typical JET parameters for the 1-D plasma: $x_{pr} = -x_{pl} = 125$ cm; $x_{sr} = -x_{sl} = 137.75$ cm; $x_a = 131.375$ cm; $R_o = 300$ cm; $B_o = 3.4 \cdot 10^4$ G; $n_e = 3.242 \cdot 10^{13}$ cm^{-3}; and a mixture of deuterium and hydrogen with $n_H/n_e = 30\%$. These parameters correspond to the ones used in reference [3].

The dispersion relation of the fast wave has its cut-off on the LFS of the ion-ion hybrid resonance. Therefore it is expected that the wave will be totally reflected and no energy will be transferred from the wave to the plasma. On the contrary, for the HFS antenna the resonance occurs before the cut-off and the wave energy is totally absorbed by the plasma. We have calculated the total power versus the frequency for $k_z = 0.05$ and $k_y = 0.01$ (Fig. 1). This value of k_z corresponds to a toroidal wavenumber $n = 15$ on the magnetic axis ($R_o = 300$ cm) in toroidal geometry.[3] We see that for the HFS the coupling is about constant, corresponding to the 100% single-pass absorption mechanism. But for the LFS there are sharp peaks due to the existence of eigenmodes. The wave reflected by the cut-off can build up a standing wave and the coupling is large. The global waves cannot be excited with a HFS antenna, as can be seen in Fig. 1.

If we make the same study in toroidal geometry using the code LION[4], we see that there are more peaks for LFS due to toroidal coupling of modes having different poloidal structures and that the total power for HFS is much lower than for LFS.[3] Thus there is a much larger difference between LFS and HFS in toroidal geometry than in a slab. We shall show that this is a toroidal effect.

Let us analyse the dispersion relation of the fast wave obtained in the WKB approximation valid outside the ion-ion hybrid resonance:

$$k_F^2(x) = \frac{h(x)}{g(x)} , \qquad (2)$$

where

$$h(x) = \varepsilon_\perp - k_z^2 + \varepsilon_{xy}^2 / \varepsilon_F , \qquad g(x) = (\varepsilon_\perp - k_z^2)/\varepsilon_F ,$$

$$\varepsilon_F = \varepsilon_\perp - k_y^2 - k_z^2 .$$

Figure 2 shows the wavenumber $k_F{}^2$ versus x near the plasma boundary for ω = 43.5 MHz. We see that in slab geometry (continuous line) $k_F{}^2$ is negative within 10 cm on the HFS and 3 cm on the LFS. Thus there is a short domain of evanescence as the wave enters into the plasma. As $|k_F|$ is small the asymmetry between both sides is small and the average total power transmitted to the plasma in the frequency range considered is about the same for HFS and LFS antennae. For the former the good coupling is due to single-pass absorption while for the latter due to global waves.

In toroidal geometry we can approximate k_z by n/r, n being the toroidal wavenumber. The variation of the parallel wavenumber enhances the asymmetry of the dispersion relation between HFS and LFS. This is shown in Fig. 2 (dotted line) where we see that the evanescence region extends up to 23 cm for HFS and only 1 cm for LFS.

Thus, for the HFS the evanescence domain is about 2.5 times larger in toroidal geometry than in 1-D geometry and moreover $|k_F|$ is about 3 times larger. This yields a factor of about 6 for the amplitude of the fields. As the power is proportional to $|E_+|^2$, the total power in toroidal geometry for HFS should be about 40 times lower than for slab geometry, which is exactly the case shown in Ref. [3]. We have checked this using our 1-D code by taking $k_z = n/(R_o+x_{pl}) \simeq 0.09$ for HFS. The power drops to 0.4 at 43.5 MHz.

CONCLUSION

We have calculated, in a slab geometry for typical JET plasma parameters, the total power transmitted to the plasma from HFS and LFS antennae. We have shown that, in spite of what could be deduced from the single-pass absorption, LFS is at least as good as HFS on the average due to the existence of global modes.

Finally, by analysing the dispersion relation of the fast wave we have shown that in a tokamak the toroidal effect considered should not change the coupling for the LFS but lowers it dramatically for the HFS antenna.

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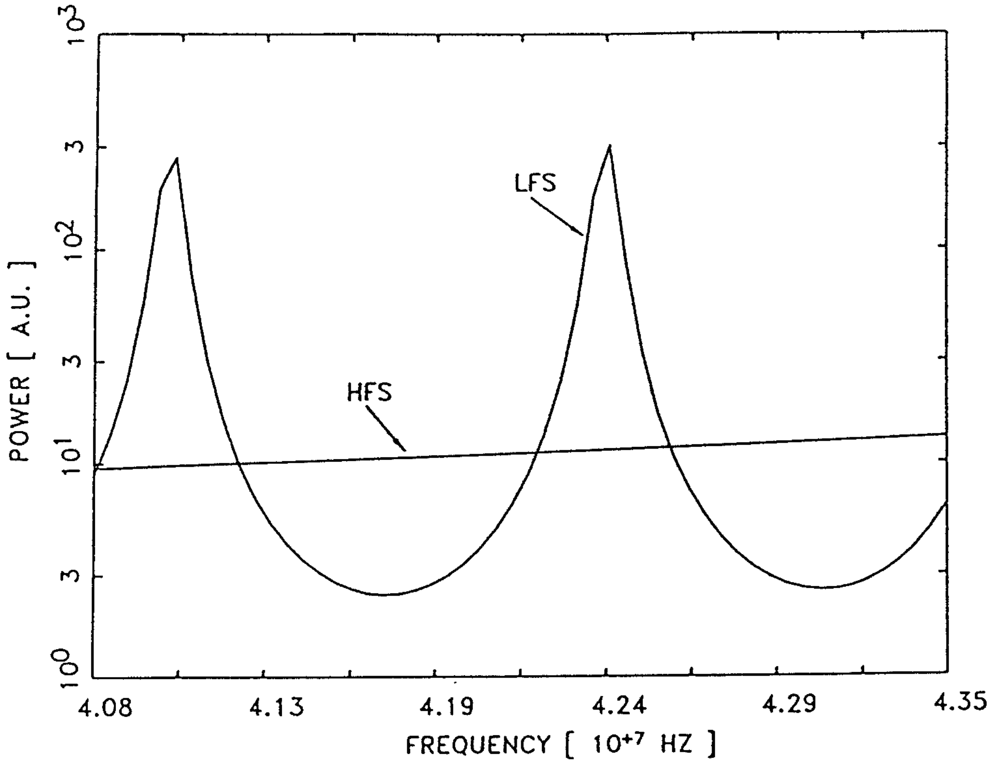

Fig. 1

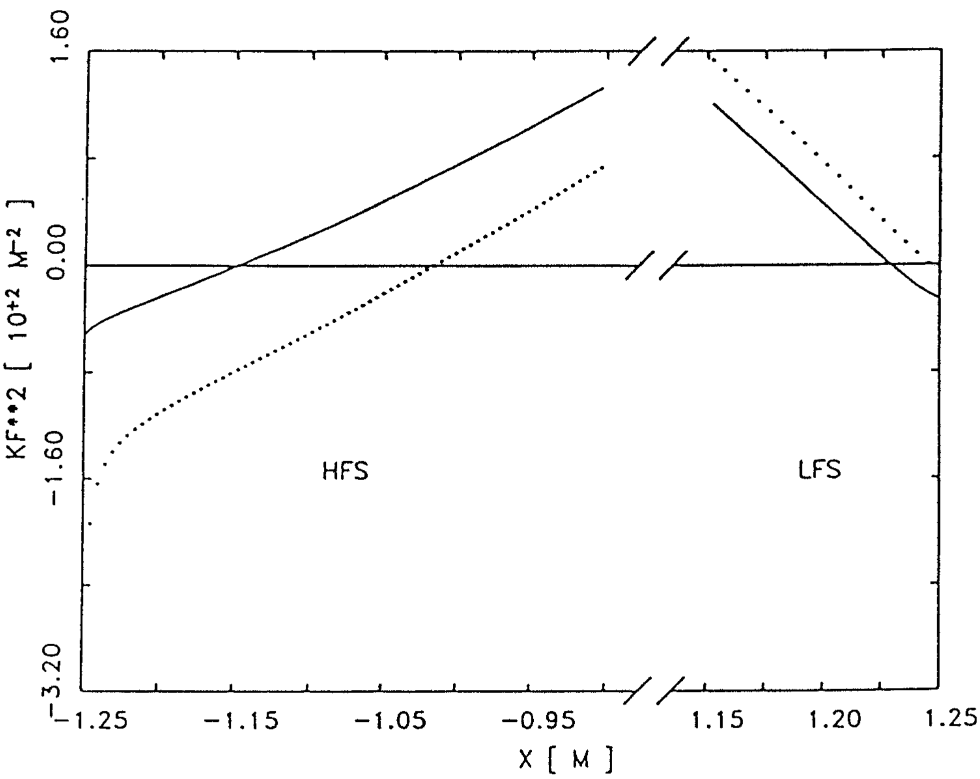

Fig. 2

Fast wave radial wavenumber $|k_F|^2$ vs x for constant k_z (continuous line) and $k_z = n/r$ (dotted line)