April 1985

LRP 261/85

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ABSTRACT

The ponderomotive force acting on a magnetized plasma due to an rf electric field is studied both theoretically and experimentally. The fundamental difference between the predictions of the single particle and fluid approaches is emphasized and its origin ellucidated. An experiment is described which illustrates the collective interaction of a fluid with an rf field. This collective behavior manifests itself in the perpendicular ponderomotive force by the interaction of the rf induced magnetization current with the magnetic field.

In the investigation of the nonlinear interaction of intense rf fields with a magnetized plasma, the time averaged (ponderomotive) force has attracted considerable attention.[1] Amongst the various proposed applications, the ponderomotive force has recently been invoked to interpret the observed stabilization of low frequency modes in mirror devices.[2-5] Several theoretical approaches, including single particle, fluid, stress tensor and kinetic, have been used to calculate the ponderomotive force. Each of these has found applications in its appropriate domain of validity. However, it has been noticed by numerous authors[1] that there are basic differences between the predictions derived from the various approaches. In particular, whereas in the single particle approach the ponderomotive force can be expressed as the gradient of the ponderomotive potential,[6] in the fluid approach this is, in general, not possible.[7-9] In fact, in the fluid description of the ponderomotive force perpendicular to the magnetostatic field, an extra term arises due to the interaction of the rf induced magnetization current with the magnetostatic field.

The purpose of this letter is to ellucidate the origin of the fundamental difference between the predictions of the single particle and fluid approaches. An experiment is described that demonstrates the influence of the perpendicular ponderomotive force on the motion of an ion beam. It will be shown that the interpretation of the experimental results must take into account the collective interaction of the fluid with the rf field.

We consider a fluid consisting of a single species with charge q and mass m (either ions or electrons) immersed in a uniform external magnetic field $\vec{B}_0$. The fluid is subjected to a stationary electromagnetic field having an electric field component $\vec{E}(\vec{r},t) = \mathrm{Re}[\vec{E}(\vec{r})\ e^{-i\omega t}]$. The total time averaged force acting on a fluid element can be written in the form[7-9]

$$\vec{F} = -\nabla p + \frac{q}{c} n\ \vec{v} \times \vec{B}_0 - n\ \nabla\Phi + \vec{B}_0 \times (\nabla \times \vec{M}) , \tag{1}$$

where the ponderomotive potential Φ and induced magnetization $\vec{M}$ are given by

$$\Phi = \frac{1}{16\ \pi\ n} (\delta_{\alpha\beta} - \varepsilon_{\alpha\beta})\ E^*_\alpha\ E_\beta , \tag{2}$$

$$\vec{M} = \frac{1}{16\ \pi} \frac{\partial\varepsilon_{\alpha\beta}}{\partial\vec{B}_0} E^*_\alpha\ E_\beta . \tag{3}$$

Here $\overleftrightarrow{\varepsilon}$ is the dielectric tensor for a cold, magnetized fluid. In writing Eq. (1) we have assumed that the time averaged electric field is zero. It should be noted that the expression (2) yields formally the same ponderomotive potential as is derived from a single particle approach.[6]

In the following, we adopt a cylindrical coordinate system r,θ,z with $\vec{B}_0 = B_0\ \hat{e}_z$ and assume that the amplitude of the oscillati ng field is a function of r only. For simplicity, we confine the present study to the case for which the fluid density and temperature are uniform. In a steady state, Eq. (1) then shows that the ponderomotive effects give rise to an azimuthal component of the momentum density of a fluid element:

$$m\, n\, v_\theta = \frac{1}{\Omega}\frac{d}{dr}\,(n\,\Phi - B_0\, M_z)\ , \tag{4}$$

where Ω is the cyclotron frequency. Since we are interested in a case where the induced magnetization plays a role, it is sufficient to consider $E_z = 0$. Only the azimuthal component of the oscillating electric field E_θ then remains. From Eqs. (2) and (3) we thus obtain

$$\Phi = \frac{q^2}{4\,m}\,\frac{|E_\theta|^2}{(\omega^2 - \Omega^2)} \tag{5}$$

and

$$M_z = -\,\frac{q^2 n\,\Omega^2}{2\,m\,B_0}\,\frac{|E_\theta|^2}{(\omega^2 - \Omega^2)^2}\ . \tag{6}$$

Substitution of Eqs. (5) and (6) into Eq. (4) yields

$$m\, n\, v_\theta = \frac{q^2\, n}{4\, m\, \Omega}\,\frac{(\omega^2 + \Omega^2)}{(\omega^2 - \Omega^2)^2}\,\frac{d}{dr}\,|E_\theta|^2\ . \tag{7}$$

As can be seen from Eqs. (5) and (6) both the ponderomotive potential and induced magnetization are large if the frequency is in the vicinity of the cyclotron frequency. However, Φ changes sign at $\omega = \Omega$ whereas M_z does not. Moreover, for $\omega \simeq \Omega$ the contribution of M_z to the azimuthal momentum density dominates that of Φ, as can be seen from Eq. (7).

An experimental investigation of the effect of the perpendicular ponderomotive force was performed by injecting a beam of ions through an oscillating electromagnetic field structure, as shown schematically in Fig. 1. An ion source (Spectra-Mat model STD 250) was used to produce a low density ($n \lesssim 10^6$ cm^{-3}) beam of essentially monoenergetic sodium ions. The beam, which was uniform across its 1.0 cm diameter, was directed along a uniform axial magnetic field (B_0 = 3 kG). For the experiment described here, the beam had a parallel energy, measured by a time-of-flight technique, of $E \simeq 2$ eV with a perpendicular energy spread of approximately 0.1 eV. The electromagnetic field was produced by passing an oscillating current through a helical antenna consisting of 24 turns wrapped along a length of 67.5 cm with a radius of 6 cm. This created an azimuthally symmetric electric field directed almost entirely in the azimuthal direction. A Faraday shield was used to avoid interaction of the beam with any electrostatic field that may have been generated by the antenna. The beam, after passing through the electromagnetic field region, was collected by a small probe (0.3 cm x 0.4 cm) situated 120 cm from the ion source and movable in the r-θ plane. Both the beam and the antenna current were pulsed to facilitate detection.

With the collector probe centred on the same magnetic field line as the ion source, it was noticed that the probe current was strongly dependent on the frequency of the applied electromagnetic field. This is illustrated in Fig. 2, which shows the measured probe current for frequencies in the vicinity of the cyclotron frequency of the sodium ions. For $\omega \simeq \Omega$, Fig. 2 shows that little or no current was detected. No such decrease in the current was measured when the probe was replaced by a plate of much larger dimension (2.7 cm x 3.5 cm). Thus the decrease in current is not explained by a decrease in the transmission of the beam through the electromagnetic field region (due, for example, to any parallel ponderomotive force resulting from the axial field gradients at the ends of the antenna). By moving the probe in an azimuthal direction, it could be shown that the beam acquired an azimuthal displacement due to its passage through the electromagnetic field region. This is illustrated in Fig. 3, where the temporal dependence of the current collected on the probe is shown. Plotted in this figure is the probe current for two consecutive beam pulses, the electromagnetic field being applied only during the passage of the second pulse through the antenna region. The curve shown in Fig. 3(a) was obtained with the collector probe positioned to measure the maximum current without an applied field, while for Fig. 3(b) and (c) the probe was moved to measure maximum current with the applied field. Figures 3(a) and (b) were obtained for $\omega \gtrsim \Omega$, and Fig. 3(c) for $\omega \lesssim \Omega$. As can be seen from Fig. 3(b) and (c), the beam moves in the same azimuthal direction independent of whether the frequency of the applied field is greater or less than the cyclotron frequency. Under no experimental conditions was a displacement of the beam in the opposite direction observed. This finding is in qualitative agreement with the fact that the azimuthal displacement is dominated, for $\omega \simeq \Omega$, by the induced magnetization, as shown in Eq. (7). It should be noted that this equation, while sufficient for a qualitative analysis of the present experimental results, does not allow detailed quantitative comparison. To describe accurately the effect of the ponderomotive force for $\omega \simeq \Omega$, the influence of non-adiabaticity on the rf induced magnetization, in addition to the ponderomotive potential,[10] must be taken into account. This more complete theory, together with detailed experimental corroboration, will be the subject of a future publication.

It should be emphasized that the interpretation of the present experimental results necessitates the use of a fluid approach. Even though the particle density in such a beam experiment is very low, nevertheless the Debye length ($\lambda_D \lesssim 0.2$ cm) is much smaller than the beam diameter and the collective behavior of particles plays a dominant role. This behavior manifests itself in the perpendicular ponderomotive force by the interaction of the induced magnetization current with the magnetic field. We note that this interaction does not yield a contribution in the parallel direction. Consequently, an ion beam subject to the parallel ponderomotive force behaves as if the motion of individual particles was not collective.[10]

In order to comprehend more profoundly the relevance of the above results it is important to understand the origin of the difference between the single particle and fluid approaches. The motion of a single particle in a uniform magnetic field under the influence of a stationary rf field may be expressed in terms of the position vector $\vec{r}(t) = \vec{r}_c + \vec{\rho}_\Omega + \vec{\rho}_\omega$, where $\vec{r}_c$ describes the time averaged motion of the oscillating center.[6] The oscillatory motion at frequencies Ω and ω is characterized by $\vec{\rho}_\Omega$ and $\vec{\rho}_\omega$, respectively. The particle possesses a magnetic moment[11] which has a time averaged value

$$\langle\vec{m}\rangle = \frac{q}{2\,c} \langle\vec{r} \times \dot{\vec{r}}\rangle = \frac{iq}{4\,c} (\Omega\, \vec{\rho}_\Omega \times \vec{\rho}^{\,*}_\Omega + \omega\, \vec{\rho}_\omega \times \vec{\rho}^{\,*}_\omega) \,. \qquad (8)$$

The first term on the right-hand side is the usual magnetic moment of guiding center theory, while the second term is associated with the motion of the particle at frequency ω . The time averaged force acting on the oscillating center is given by[6]

$$\vec{F}_c = \frac{q}{c} \dot{\vec{r}}_c \times \vec{B}_0 - \nabla\Phi \ . \tag{9}$$

It is well known[12,13] that the particle flux in the fluid description and the oscillating center flux differ. As shown by Braginskii[13] the total current density is the sum of the drift (conduction) current and the magnetization current:

$$q\, n\, \vec{v} = q\, n\, \dot{\vec{r}}_c + c\, \nabla \times \overline{n\langle\vec{m}\rangle} \ , \tag{10}$$

where the bar denotes an average over the particle distribution function. For an approximately Maxwellian distribution, the total magnetization can be written using Eq. (8) as

$$\overline{n\langle\vec{m}\rangle} = - \frac{p}{{B_0}^2} \vec{B}_0 + \vec{M} \ , \tag{11}$$

where the rf induced magnetization $\vec{M}$ is given by Eq. (3). In addition, as shown by Spitzer,[12]

$$F_\parallel - nF_{c\parallel} = -\nabla_\parallel p. \tag{12}$$

Equations (9) to (12) can then be combined to yield the time averaged force acting on a fluid element:

$$\vec{F} = [\frac{q}{c} n\, \vec{v} + \nabla \times (\frac{p}{{B_0}^2} \vec{B}_0 - \vec{M})] \times \vec{B}_0 - n\, \nabla\Phi \ . \tag{13}$$

It is straightforward to show that Eq. (13) is equivalent to Eq. (1).

It should be stressed that the ponderomotive forces calculated from the single particle and fluid approaches are fundamentally different. The contribution of the magnetization current cannot be obtained from single particle theory (as may be concluded from Ref. 14), either using an oscillating center approximation or from the solution of the exact equation of motion.[15] Physically, the magnetization current arises through the collective motion of the particles and requires mathematically an ensemble average as shown above. A more detailed discussion of the intrinsic subtleties of this problem is deferred to a forthcoming publication.[16]

In summary, we have in the present letter analyzed in detail the ponderomotive force in a magnetized plasma. The connection between the single particle and fluid approaches has been elucidated. We have shown that even for a low density beam, the collective behavior of particles must be taken into account to describe accurately their motion under the influence of the perpendicular ponderomotive force. The results of this letter thus indicate that an interpretation of plasma ponderomotive effects based on a single particle approach may, in some cases, be insufficient.

ACKNOWLEDGMENTS

The authors gratefully acknowledge B.M. Lamb, P.J. Paris and M.Q. Tran for their contribution towards obtaining the experimental results. This work was supported by the Swiss National Science Foundation.

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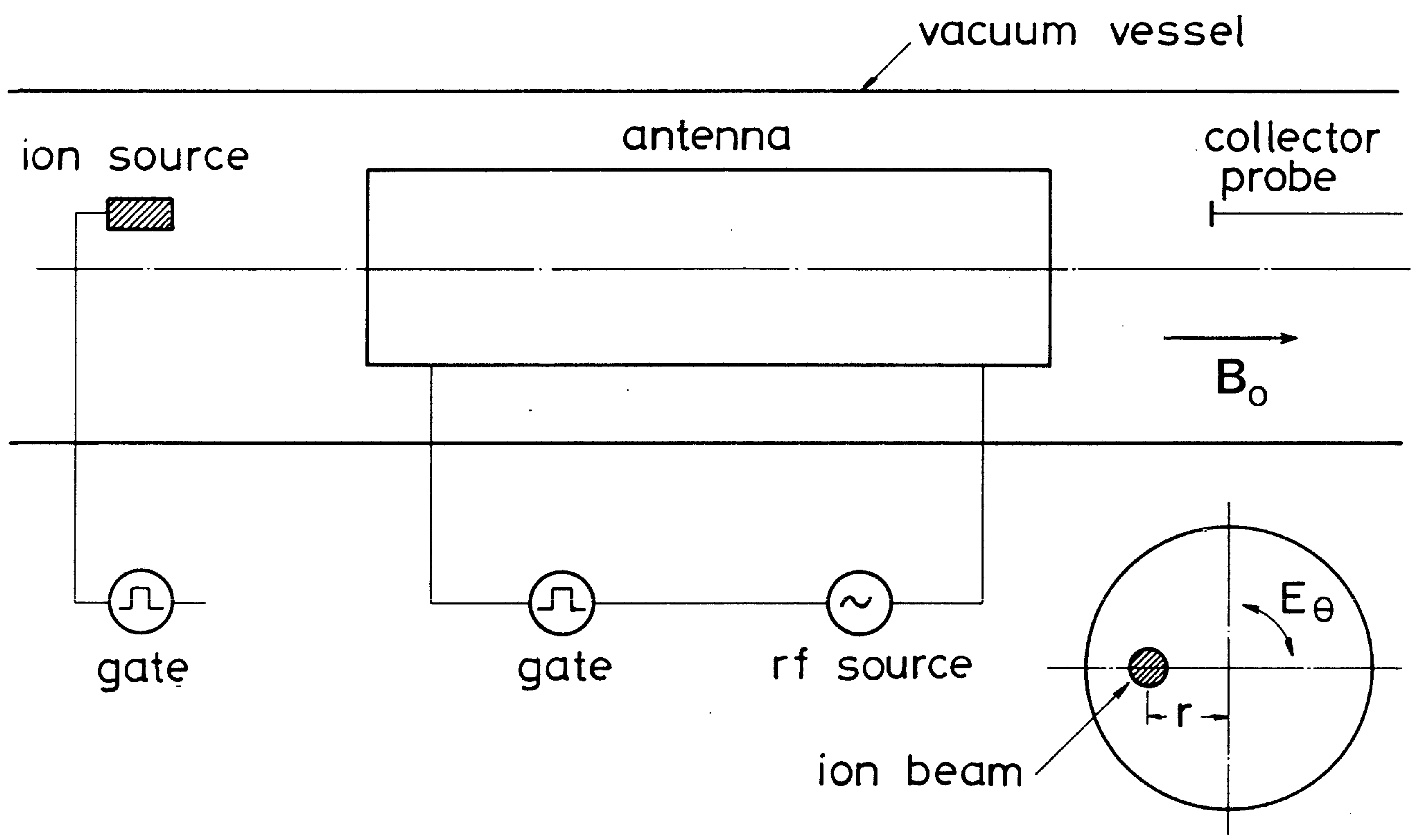
FIGURE CAPTIONS

FIG. 1. Schematic diagram of the experiment.

FIG. 2. Measured probe current as a function of the frequency of the rf field in the vicinity of the cyclotron frequency, Ω.

FIG. 3. Temporal dependence of the probe current both without (first pulse) and with (second pulse) the applied rf field. The diagrams on the right-hand side show the position, viewed along the $\vec{B}_0$ direction, of the probe and of the beam on exit from the antenna region.



vacuum vessel
ion source
antenna
collector
probe
B_o
gate
gate
rf source
E_θ
r
ion beam

Fig. 1

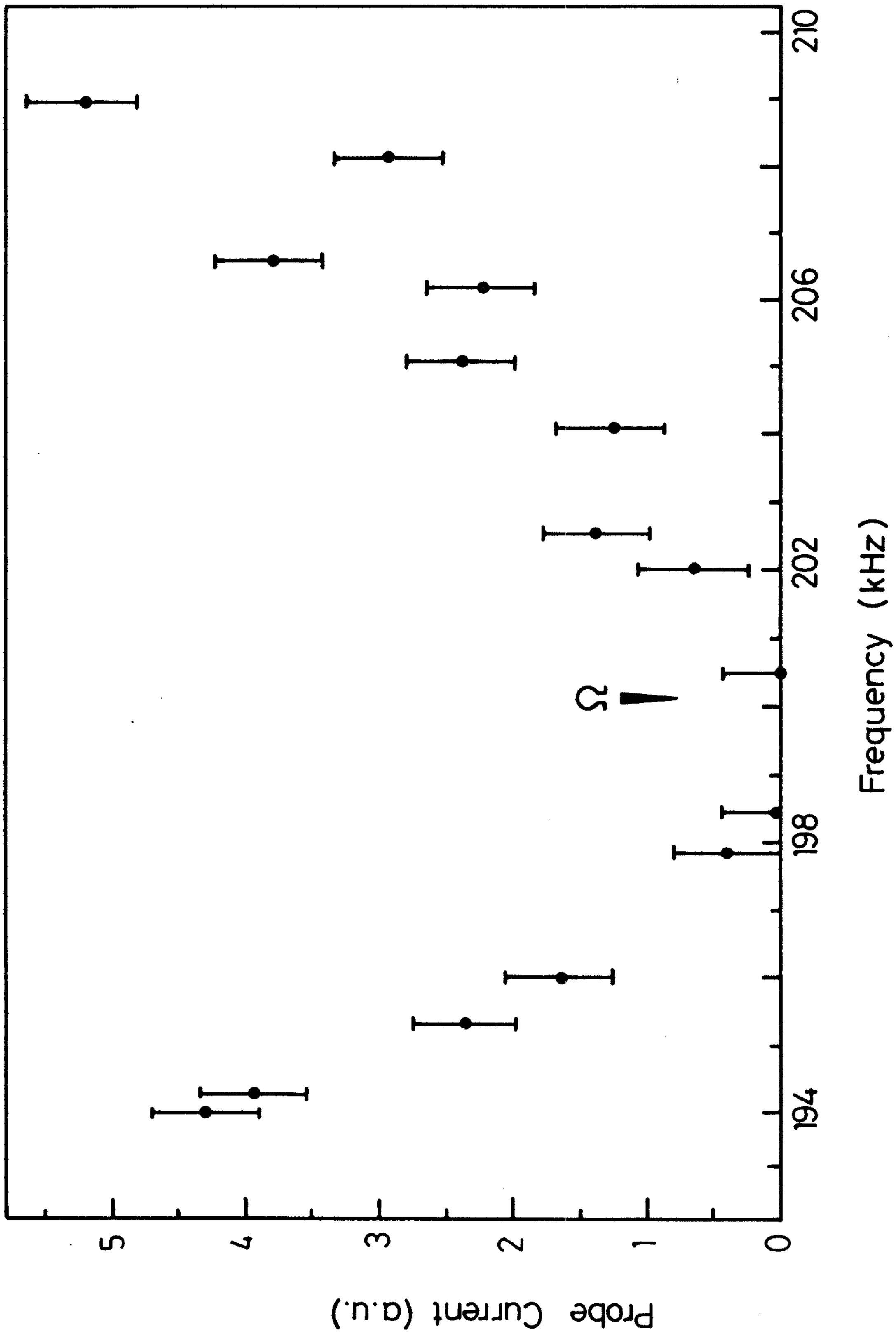


Fig. 2

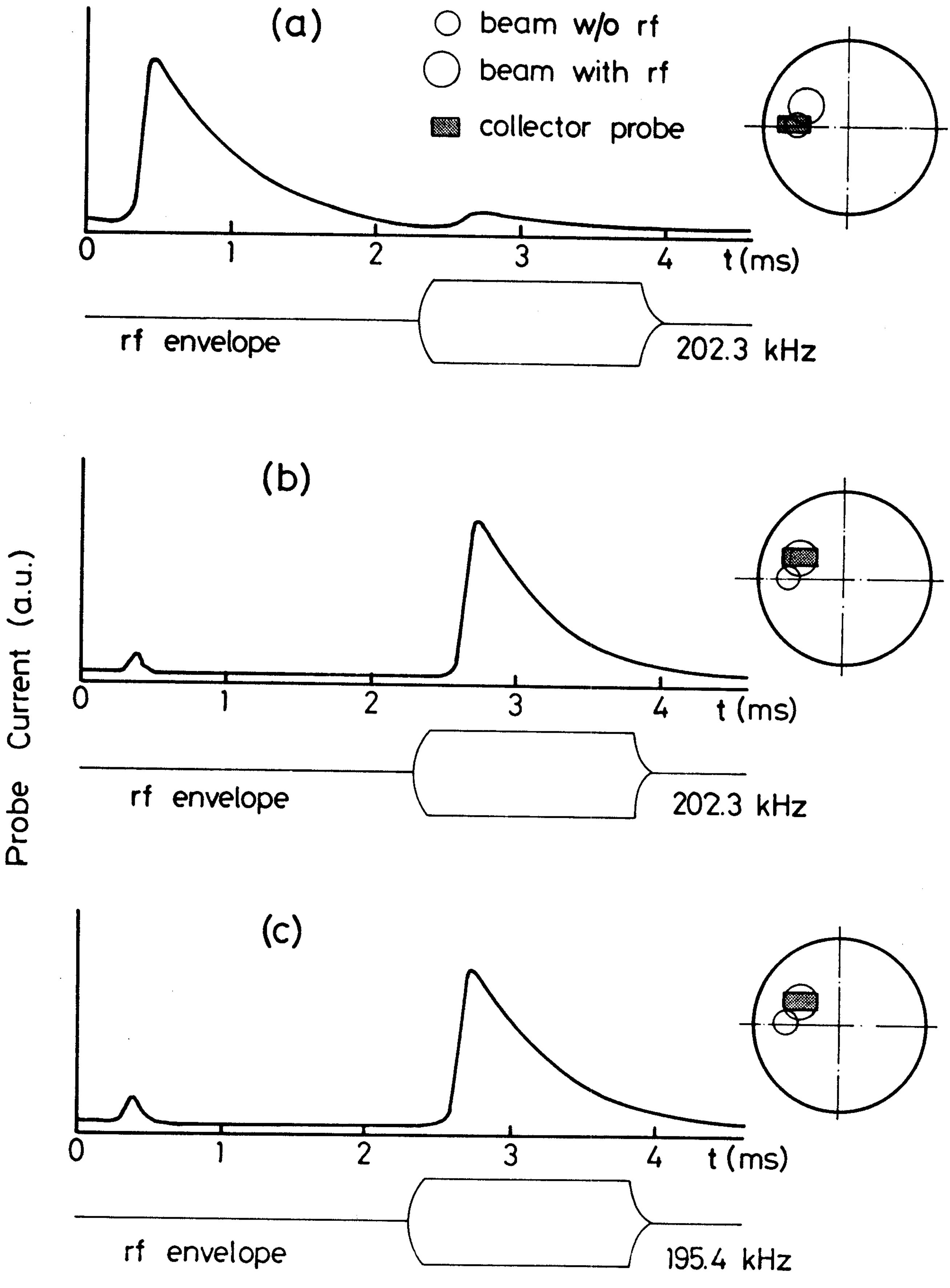
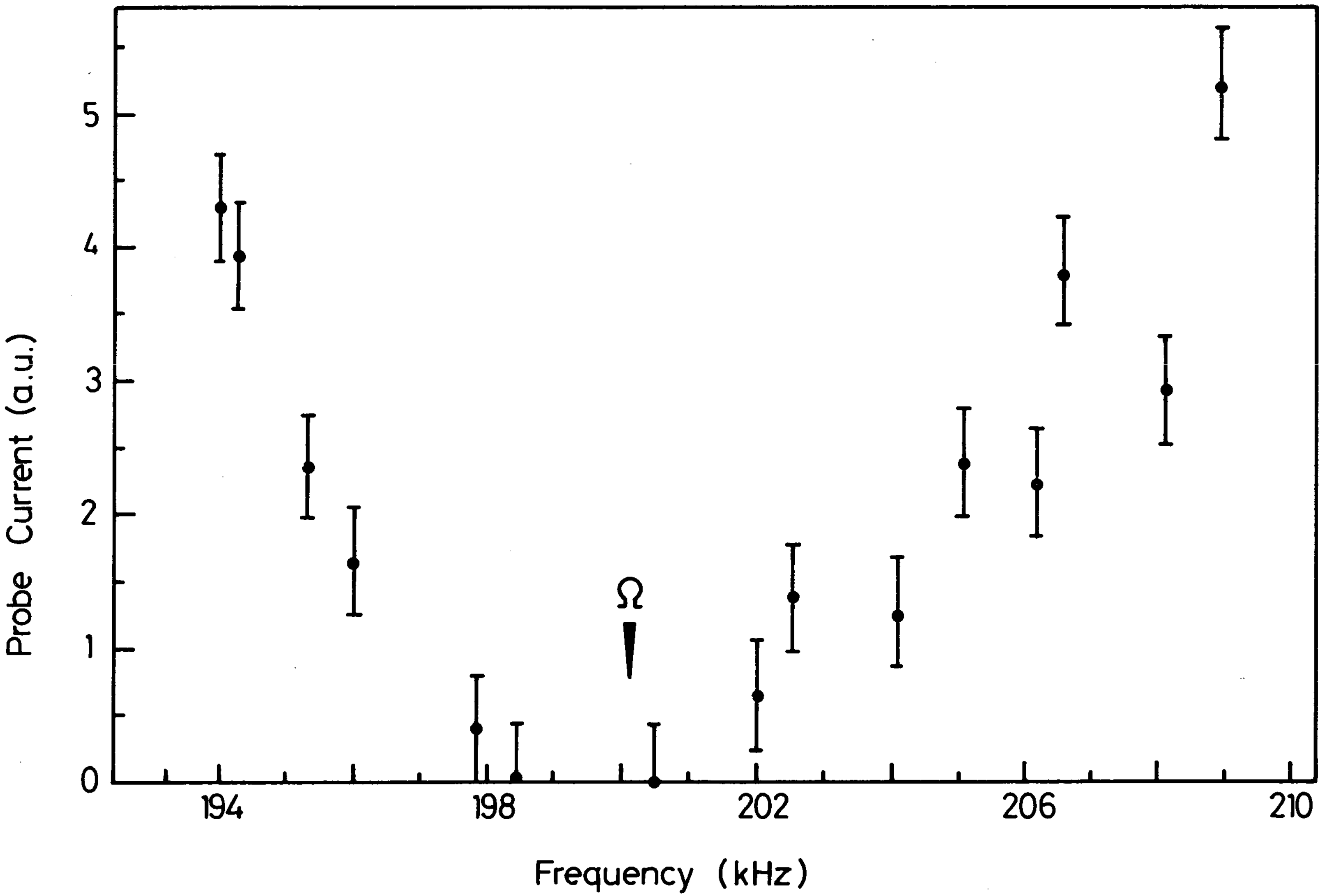

(a)
beam w/o rf
beam with rf
collector probe
0
1
2
3
4
t (ms)
rf envelope
202.3 kHz
(b)
Probe Current (a.u.)
0
1
2
3
4
t (ms)
rf envelope
202.3 kHz
(c)
0
1
2
3
4
t (ms)
rf envelope
195.4 kHz

Fig. 3


Probe Current (a.u.)
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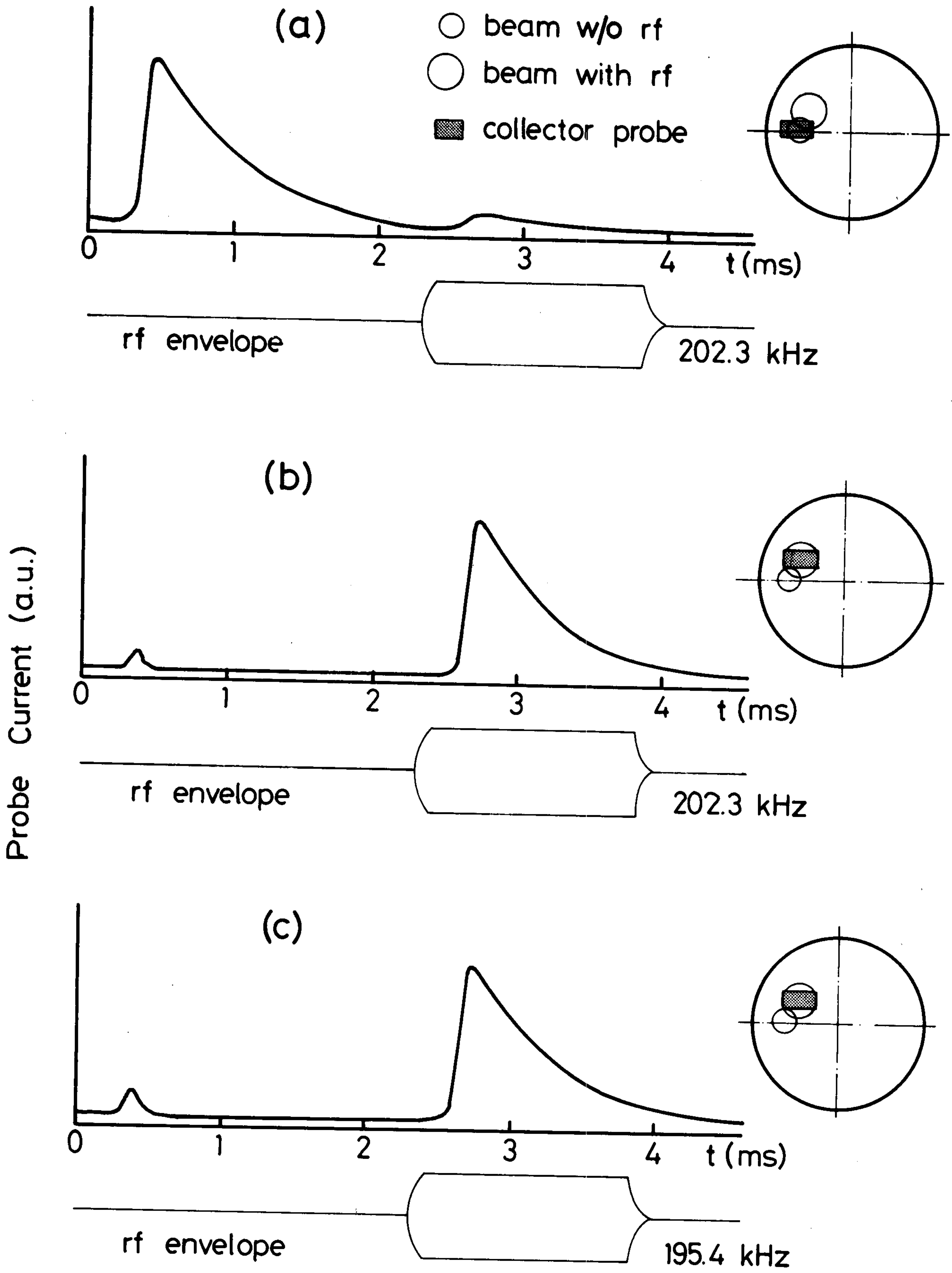
1
2
3
4
5
194
198
202
206
210
Frequency (kHz)
Ω



(a)
beam w/o rf
beam with rf
collector probe
0
1
2
3
4
t (ms)
rf envelope
202.3 kHz
(b)
Probe Current (a.u.)
0
1
2
3
4
t (ms)
rf envelope
202.3 kHz
(c)
0
1
2
3
4
t (ms)
rf envelope
195.4 kHz