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S. Succi, K. Appert and J. Vaclavik
Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
21, Av. des Bains, CH-1007 Lausanne/Switzerland

ABSTRACT

The problem of quasilinear diffusion of plasma electrons, resonantly interacting with lower-hybrid waves launched from the exterior, is examined within a self-consistent one-dimensional homogeneous model.

An analytical solution is obtained, which allows the derivation of a simple formula for the plasma current in terms of the parameters characterizing the spectral distribution of the rf power source. The major outcome is a dramatic sensitivity of the plasma current to the presence of a small, high-$N_{\parallel}$ component in the rf power source spectral distribution, in agreement with previous numerical calculations.

1. INTRODUCTION

Since the appearance of the Fisch paper (FISCH, 1978), considerable progress has been achieved in the theoretical understanding of lower-hybrid current drive experiments. In particular, most of the basic failures of the 'classical' Fisch theory (for a review, see VACLAVIK et al., 1983) have now found a rather satisfactory explanation. However, one point which still remains unresolved concerns the mechanism by which a consistent population of superthermal electrons is generated by high-phase velocity waves in a relatively cold target plasma. Several theories have been proposed in this context (for a review, see WEGROWE, 1984) among them, some numerical calculations (SUCCI et al., 1984a, 1984b) have shown that a small, high-$N_{\parallel}$ tail in the launched spectrum may suffice to account for the observed values of the plasma current.

In this note, we propose a simple analytical model which clearly confirms a dramatic sensitivity of the plasma current to the presence of a small, high-$N_{\parallel}$ component in the spectral distribution of the rf power source. A very simple formula, Eq. (30), is obtained, which can be used for the practical evaluation of the plasma current generated by the waves. This sensitivity stems from an intense cooperation between the low and high-$N_{\parallel}$ portions of the rf spectrum in rising the level of superthermal electrons.

Few different spectra have been considered, always leading to good agreement between analytical and numerical results.

2. BASIC EQUATIONS

Our investigation is based on the following model. The steady-state electron distribution functions is described by:

$$\left(\frac{\partial f}{\partial t}\right)_{LH} + \left(\frac{\partial f}{\partial t}\right)_{c} = 0 \quad , \tag{1}$$

where

$$\left(\frac{\partial f}{\partial t}\right)_{LH} = \frac{\partial}{\partial v_{\parallel}} D_{LH} \frac{\partial f}{\partial v_{\parallel}} \tag{2}$$

and

$$D_{LH}(v_{\parallel}) = 2\pi \int_{k_{\parallel}>0} \frac{d^3k}{(2\pi)^3} W(\vec{k})\, \delta(\omega_k - k_{\parallel} v_{\parallel}) \tag{3}$$

is the quasilinear diffusion term; W(k) being the wave spectrum and where

$$\left(\frac{\partial f}{\partial t}\right)_{c} = (2+Z)\,\nu_0 \frac{\partial}{\partial v_{\parallel}} \left[\frac{1}{v_{\parallel}^3}\left(v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}}\right)\right] \tag{4}$$

is the one-dimensional Fokker-Planck operator, representing the electron-electron and electron-ion collisional effects, Z denoting the ion charge state and $\nu_0 = \log\Lambda\, \omega_{pe}^3 / 4\pi n v_{te}^3$.

The steady-state wave spectral distribution is given by:

$$2\left(\gamma - \frac{\nu_0 Z}{4}\right)W + S(k,\theta) = 0 \tag{5}$$

with

$$\gamma = \frac{\pi\cos\theta}{2k^2}\left.\frac{\partial f}{\partial v_\parallel}\right|_{v_\parallel = k^{-1}}, \tag{6}$$

where the dispersion relation of the waves is assumed to be $\omega = \cos\theta$, θ being the angle between the magnetic field and the wave vector $\vec{k}$.

The term $S(k,\theta)$ represents an external source which drives the waves. For simplicity we assume:

$$S(k,\theta) = S_0 s(k)\,\delta(\cos\theta - \cos\theta_0), \tag{7}$$

where S_0 is a constant and $s(k)$ a shape function satisfying the normalization $\int s(k)k^2dk = 1$. Since we do not address the spatial problem, we identify the function $k^2s(k)$ with the distribution $p(N_\parallel)$ of the total power:

$$p(N_\parallel)dN_\parallel = k^2 s(k)\,dk, \tag{8}$$

$$N_\parallel = ck, \tag{9}$$

c being the speed of light.

All quantities throughout Eqs. (1-9) are normalized according to $k \to k/\lambda_D$, $t \to t/\omega_{pe}$, $v \to v\,v_{te}$, $f \to f\,n/v_{te}^3$, $W \to W\,4\pi nT_e\lambda_D^3$.

Once a solution is found, the current is evaluated

$$\dot{j} = 2\pi \int_0^\infty \int_{-\infty}^\infty f(v_\parallel, v_\perp)\, v_\parallel v_\perp \, dv_\parallel dv_\perp \quad . \tag{10}$$

In convenient units

$$I(kA) = 6.7\times 10^4 \, n[10^{19} m^{-3}]\, T_e^{1/2}[keV]\, a^2[m]\, \dot{j} \; , \tag{11}$$

$$P(kW) = 2.9\times 10^{11} \, R[m]\, a^2[m]\, n^{3/2}[10^{19} m^{-3}]\, T_e[keV]\, S_0 \, , \tag{12}$$

P being the power needed to sustain the current I.

3. WEAK AND STRONG SOURCE REGIMES

On using the procedure indicated in MUSCHIETTI et al. (1982) Eqs. (1-9) can be reduced to the following equations:

$$W(k,\theta) = \frac{S_0\, \delta(\cos\theta - \cos\theta_0)\, s(k)}{\frac{\nu_0 Z}{2} - \pi \cos\theta\, k^{-2} F'\big|_{v=k^{-1}}} \quad , \tag{13}$$

$$vF + F'\left[1 + \frac{S_0 \cos\theta_0\, s(k=v^{-1})}{2\pi\nu_0(2+Z)\left(\frac{\nu_0 Z}{2} - \pi\cos\theta_0 v^2 F'\big|_{v=k^{-1}}\right)}\right] = 0 \, , \tag{14}$$

where F stands for $2\pi\int f(v_\parallel, v_\perp) v_\perp dv_\perp$, $v \equiv v_\parallel$ and $F' \equiv dF/dv$.

A complete knowledge of the functions W and F would, of course, require the exact solution of Eq. (14); this is a complicated first order, non-linear equation, and an exact analytical solution seems to be out of our tools. Nevertheless, we believe that much of the information we need, can be extracted by doing appropriate approximations. Again following MUSCHIETTI et al. (1982) we identify two distinct regimes where Eq. (14) can be linearized:

i) Weak source regime

If the condition $2\gamma \equiv \pi \cos\theta_0 v^2 F' \gg \frac{Z\gamma_0}{2}$ (15)

holds, Eq. (14) reduces to

$$v F + F' = \frac{\tilde{S}\, s(v)}{v^2} \tag{16}$$

with $\tilde{S} = \dfrac{S_0}{2\pi^2 \gamma_0 (2+Z)}$, $s(v) \equiv s(k = v^{-1})$. (17)

Equation (16) is readily integrated to:

$$F_W(v) = \exp\left(-\frac{v^2}{2}\right)\left(C_W + \tilde{S}\int^{v} \frac{s(z)}{z^2}\exp\left(\frac{z^2}{2}\right)dz\right). \tag{18}$$

For the inequality (15) to be fulfilled, we require that

$$\tilde{S} \ll S_c(v) \equiv \frac{v^3 F(v)}{s(v)} \tag{19}$$

or conversely

$$F(v) \gg F_c(v) \equiv \frac{\tilde{S}\, s(v)}{v^3} \quad . \tag{20}$$

(We assumed $Z\nu_0/2\pi\cos\Theta_0 << \tilde{S}\, s(v)$).

ii) Strong source regime

If $2\gamma \equiv |\pi\cos\Theta_0 v^2 F'| \ll Z\nu_0/2$ holds, Eq. (14) reduces to

$$vF + F'[1 + b(v)] = 0 \tag{21}$$

with $b(v) = S_0 \cos\Theta_0\, s(v)/\pi Z(2+Z)\nu_0^2$

whose solution is

$$F_s(v) = C_s \exp\left(-\int^{v} \frac{z}{1+b(z)}\, dz\right) . \tag{22}$$

Once more, this solution is valid if

$$\tilde{S} \gg S_c(v) \equiv \frac{v^3 F}{s(v)} \quad , \tag{23}$$

4. ANALYTICAL SOLUTION

Since the domains of validity for the weak and strong source solution do not overlap, a boundary layer may develop and a perturbative approach could be conceived to reconstruct the exact solution. The first step in this direction consists in requiring the thickness of this boundary layer be zero, hence reconstructing the full solution as a collection of "weak" and "strong" branches (F_W and F_S) continuously connected. This simple procedure will probably be inaccurate around the critical region where $S = S_C(v)$, but it is our conviction (supported by the numerical experience) that the errors introduced will not affect the values of the current very much, and the correct order of magnitude will always be obtained.

Let us now apply this idea by using the following spectrum:

$$p(N_\parallel) = \begin{cases} 0 \ , & N_\parallel < 1 \\ \text{const}\left\{\exp - \left[\left(\frac{N_\parallel - N_0}{\Delta N}\right)^2 \ln 2\right] - \exp - \left[\left(\frac{N_m - N_0}{\Delta N}\right)^2 \ln 2\right]\right\}, & 1 \leq N_\parallel < N_m \\ p_0 \ , & N_m \leq N_\parallel \leq N_1 \ , \end{cases} \tag{24}$$

where N_0, N_m, ΔN, N_1 and p_0 are free parameters. With this choice the function $s(v)$ becomes $(N_\parallel = c/v)$:

$$s(v) = \begin{cases} 0 \ , & v_\parallel > v_2 \equiv c \\ c v^2 \, p\left(N_\parallel = \frac{c}{v}\right) \ , & \frac{c}{N_m} \equiv v_m < v_\parallel < v_2 \\ c v^2 p_0 \ , & \frac{c}{N_1} \equiv v_1 < v_\parallel < v_m \ , \end{cases} \tag{25}$$

where p_0 will be in general a small number such that $s(v < v_m) \ll s(v > v_m)$. Consequently, the weak (strong) source condition will now read as:

$$\tilde{S} \lessgtr S_c(v) = \frac{vF}{c\,p_0} \qquad \text{or} \qquad F \gtrless F_c(v) = \frac{\tilde{S}\,c\,p_0}{v} \tag{26}$$

for $v_1 < v < v_m$.

Suppose now that the weak source approximation holds in the range $[v_1, v_m]$ and the strong source approximation in $[v_m, v_2]$ (we shall justify this hypothesis later on). In this case, we obtain the following solution

$$F = \begin{cases} F_W = \exp\left(-\frac{v^2}{2}\right)\left[(2\pi)^{-\frac{1}{2}} + \tilde{S} p_0 c \displaystyle\int_{v_1}^{v} \exp\left(\frac{z^2}{2}\right) dz\right] , & v_1 < v < v_m \\ \\ F_S = \exp\left(-\frac{v_m^2}{2}\right)\left[(2\pi)^{-\frac{1}{2}} + \tilde{S} p_0 c \displaystyle\int_{v_1}^{v_m} \exp\left(\frac{z^2}{2}\right) dz\right] = \text{const.} , & v_m < v < v_2 \, . \end{cases} \tag{27}$$

This expression already exhibits the basic effect we want to discuss in connection with the evaluation of the current. Since we know that for $v > v_m$, Fisch's theory is applicable, i.e. the distribution function is flat, the crucial quantity which determines the value of the current is the value of F at $v = v_m$. In absence of the high-$N_\parallel$ part of the spectrum we would have the Maxwellian value:

$F(v_m, p_o = 0) = 1/\sqrt{2\pi}\ \exp(-v_m^2/2)$, whereas for $p_o \neq 0$, eq. (27) yields

$$F(v_m, p_o \neq 0) = (2\pi)^{-\frac{1}{2}} \exp\left(-\frac{v_m^2}{2}\right)\left[1 + (2\pi)^{\frac{1}{2}} \tilde{S} p_o c \int_{v_1}^{v_m} \exp\left(\frac{z^2}{2}\right) dz\right] . \quad (28)$$

We can define a "wing gain factor" as

$$G = \frac{F(v_m, p_o)}{F(v_m, o)} .$$

From (eq. 28) we obtain $G = 1 + \sqrt{2\pi}\ \tilde{S} c p_o \int_{v_1}^{v_m} \exp(+z^2/2) dz$. For typical tokamak parameters, $\tilde{S} \sim 5 \times 10^{-3}$, $c \sim 20$, so that choosing $v_1 = 4$ and $v_m = 6$, for example, we get $G \sim 10^6 p_o$; a spectacular effect indeed.

We now come to the justification of our hypothesis that $F = F_w$ within $[v_1, v_m]$ and $F = F_s$ in $[v_m, v_2]$. As for the latter interval there is no problem since v_m will be always high enough to justify the use of Fisch formula. With regard to the lower interval we notice that with our choice (24), if the weak source condition is satisfied at $v = v_1$, it will stay so all over the range $[v_1, v_m]$. In fact, an aysmptotic expansion of $F_w(v)$, eq. (18), leads to:

$$\lim_{v \to \infty} F_w(v) = \frac{\tilde{S} c p_o}{v} \equiv F_c(v) .$$

$$N_0 = 2.2\ ,\quad N_m = 3.7\ ,\quad N_1 = 6\ ,\quad \Delta N = 0.8\ ,$$

$$n = 0.5\ ,\quad T = 1\ ,\quad \rho = 130\ ,\quad z = 1\ .$$

The quantity w in abscissa represents the percentage of power contained within N_m and N_1. The two dotted-dashed lines represent the values of the current corresponding to a plateau on the distribution function, starting from $v = c/N_1$ (upper line) and $v = c/N_m$ (lower line), whereas the circles correspond to the numerical results. One can see that the analytical formula (30) gives an excellent representation of the transition region between the two plateau levels. The deviation from the numerical results are easily interpreted. Close to the upper 'Fisch level' the condition of weak source starts to be violated ($\tilde{S} \sim S_c$ for $w \sim 2.5 \times 10^{-2}$), hence our formula cannot be applied any longer. It is pleasing to note that there where this formula becomes inadequate, one can already apply the usual Fisch formula. The deviations from the numerical results at lower w are simply due to the lack of resolution of the numerical treatment. Finally, the dashed line at the bottom is an extrapolation, since the expression (29) does not hold for $p_o \to 0$.

It should be borne in mind that the formula (30) was derived for the case of a simplified spectrum. For more complex spectra, a more elaborate collection of "weak" and "strong" pieces of the distribution function may be required. What we claim is that, when doing so, one can possibly miss factors but not orders in evaluating the current.

Finally, we would like to spend few comments on the dimensionality of the collision operator. It is our conviction, supported by the numerical and analytical results (KARNEY and FISCH, 1979; BERS et al., 1984), that the inclusion of two-dimensional effects, would once again bring only factors and not orders. The essential difference is that, in the presence of two-dimensional effects, the resonant part of the distribution function cannot be matched to an unperturbed Maxwellian any longer.

CONCLUSION

In conclusion, we have shown that a simple analytical model, derived from the one-dimensional quasilinear equations, can account for a spectacular enhancement of the current generated by LH waves once the source spectrum contains a small percentage of power at high-$N_\parallel$.

This model corroborates previous numerical results and also the idea that such a small, high-$N_\parallel$ component could be indeed responsible for the currents observed in the experiments.

We are aware that our model suffers from the limitation due to the assumed homogeneity in configuration space. However, we hardly see how such a dramatic effect could be completely cancelled once the proper spatial features would be taken into account.

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Figures Captions

Fig. 1: Dimensionless current density versus the fractional power in the wing for the following parameters: $N_0 = 2.2$, $\Delta N = 0.8$, $N_m = 3.7$, $N_1 = 6$, $S_0 = 4.4 \times 10^{-8}$.

The solid line represent the analytical result, Eq. (30), and the circles the numerical results. The dashed-dotted lines represent values corresponding to a plateau starting from $\mathcal{L}/N_1$ (upper line) and $\mathcal{L}/N_m$ (lower line).

The dashed line at the bottom is an extrapolation for $p_0 \rightarrow 0$.

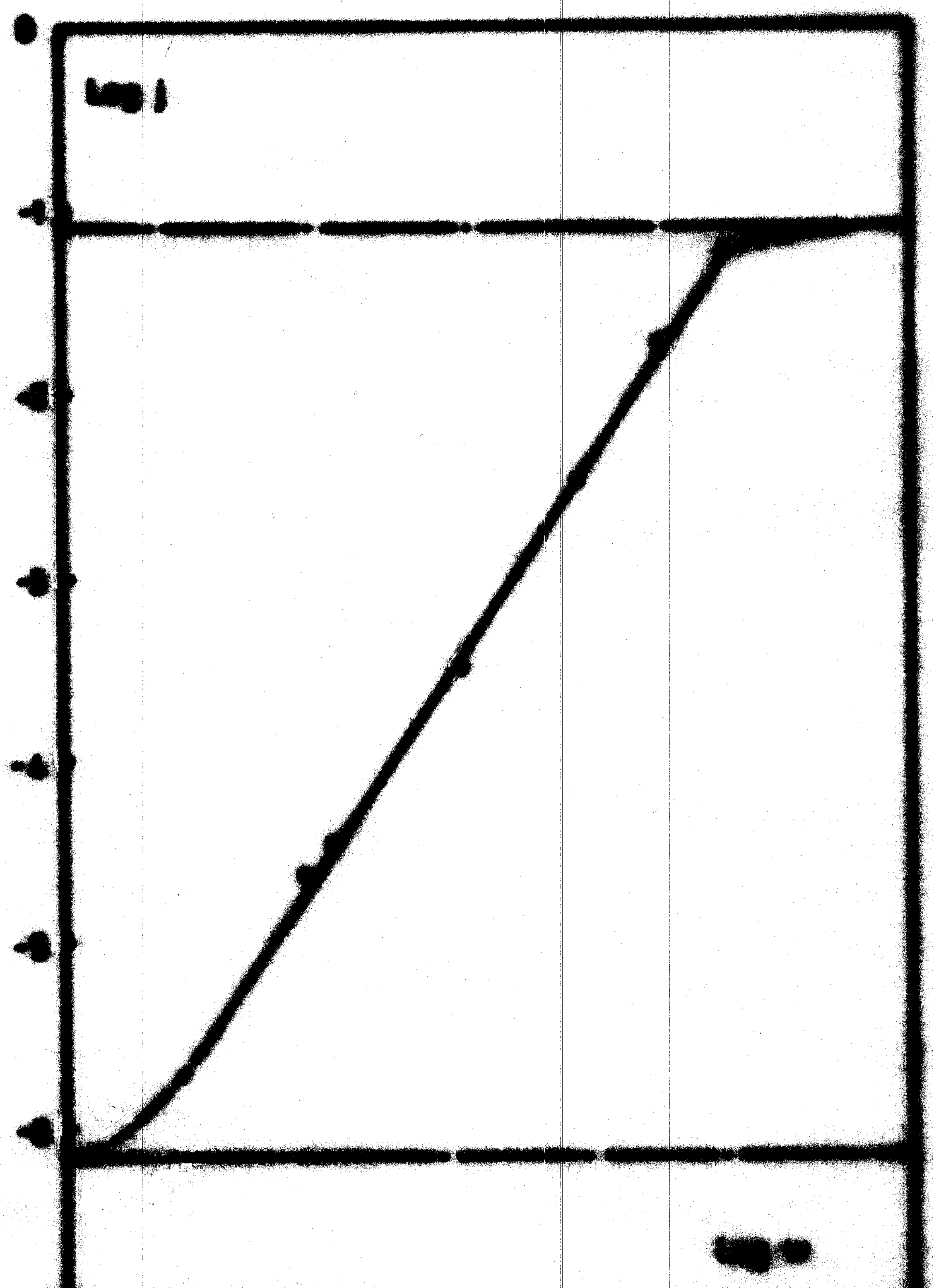

Fig. 1