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ABSTRACT

The ponderomotive forces in a magnetized plasma calculated from single particle and fluid theory are fundamentally different. We show that this apparent paradox, which has an analogue in classical plasma theory, can be resolved by appropriate ensemble averaging of the single particle equations.

Some 40-50 years ago, at the advent of theoretical plasma physics, much confusion had arisen in the interpretation of results obtained using single particle and fluid approaches. The application of the fluid equations to a magnetized plasma was found to lead to apparent contradictions with what might have been expected from a cursory examination of individual particle motion in a magnetic field using guiding center, or drift, theory. Thus, for example, the equation of motion for a volume element of fluid contains a term related to the pressure gradient which has no counterpart in the drift motion of a single particle. Conversely, the equation of motion for a guiding center contains terms related to the gradient and curvature of the magnetic field which have no counterparts in the motion of a fluid element.

Some of these paradoxes have been discussed by Spitzer,[1] Schlüter,[2] and Braginski[3] a long time ago. They concluded that the discrepancies stem from the difference between the macroscopic (fluid) flux density of particles and the flux density of guiding centers. The flux density of guiding centers, $n\vec{v}_c$, is defined as the mean flux of the guiding centers in a volume element. On the other hand, the fluid flux density, $n\vec{v}$, is defined as the mean flux of all the particles which are located in a volume element, regardless of where their guiding centers are located. It was shown by the above authors that

$$n\vec{v} = n\vec{v}_c + \frac{c}{q}\nabla \times \vec{M} \tag{1}$$

and

$$n\frac{dv_{\parallel}}{dt} = n\frac{dv_{c\parallel}}{dt} - \frac{1}{m}\nabla_{\parallel} p \,, \tag{2}$$

where $\vec{M}$ is the magnetization of the fluid element and p is the pressure. The symbol $\parallel$ denotes the component parallel to the magnetic field.

From macroscopic electrodynamics it is known that the total current density is the sum of conduction and magnetization current densities. Thus, from Eq. (1) it can be seen that the conduction current arises from the flux density of guiding centers. It was shown [1,3] that if relations (1) and (2) are taken into account, the single particle and fluid pictures can be reconciled.

It should be borne in mind that by the above-mentioned mean flux, an ensemble average is implicitly assumed. The appropriate ensemble average yields, as shown in Eq. (1), the magnetization current density. Of course, the procedure of ensemble averaging is meaningful only for a system of interacting particles that exhibit collective behavior (the Debye length being much smaller than the characteristic dimension of the system).

In the foregoing discussion it was tacitly assumed that the temporal variations of electric and magnetic fields are slow in comparison with the particle gyro-period and that the amplitudes of these varying fields are sufficiently small. It is now well established that if these conditions are relaxed (e.g., in the presence of intense radio-frequency fields) ponderomotive (time averaged) effects may arise.[4,5] Consequently, to describe accurately ponderomotive effects in a magnetized plasma both single particle and fluid theory must be appropriately generalized.[6]

Let us consider, to be specific, a dissipationless conducting fluid consisting of a single species immersed in a uniform external magnetic field $\vec{B}_0$. The fluid is subjected to a stationary electromagnetic field having an electric field component $\vec{E}(\vec{r},t) = \mathrm{Re}[\vec{E}(\vec{r})e^{i\omega t}]$. The total time averaged force acting on a fluid element can be written in the form [7-10]

$$\vec{F} = -\nabla p + \frac{q}{c} m\vec{v}\times\vec{B}_0 - m\nabla\Phi + \vec{B}_0\times(\nabla\times\vec{\mathcal{M}}), \tag{3}$$

where

$$\Phi = \frac{1}{16\pi m}\left(\delta_{\alpha\beta} - \varepsilon_{\alpha\beta}\right)E_\alpha^* E_\beta , \tag{4}$$

$$\vec{\mathcal{M}} = \frac{1}{16\pi}\,\frac{\partial\varepsilon_{\alpha\beta}}{\partial\vec{B}_0}E_\alpha^* E_\beta . \tag{5}$$

Here $\overleftrightarrow{\varepsilon}$ is the dielectric tensor for a cold magnetized fluid. In writing Eq. (3) we have assumed that the time averaged electric field is zero.

The motion of a single particle under the influence of the same fields may be expressed in terms of the position vector

$$\vec{r}(t) = \vec{r}_c + \vec{\rho}_\Omega + \vec{\rho}_\omega , \tag{6}$$

where $\vec{r}_c$ describes the time averaged motion of the oscillating center (a generalization of the guiding center). The oscillating motion at frequencies Ω (cyclotron frequency) and ω is characterized by $\vec{\rho}_\Omega$ and $\vec{\rho}_\omega$, respectively. The time averaged force acting on the oscillating center is given by[11]

$$\vec{F}_c = \frac{q}{c}\,\dot{\vec{r}}_c\times\vec{B}_0 - \nabla\Phi , \tag{7}$$

where Φ is the ponderomotive potential, which is formally the same as that defined in expression (4).

In order to compare the single particle and fluid expressions for the ponderomotive force, we must first average Eq. (7) over an ensemble of particles. We then obtain

$$n\overline{\vec{F}_c} = \frac{q}{c} n\vec{v}_c \times \vec{B}_0 - n\nabla\Phi , \qquad (8)$$

where the bar denotes the ensemble average. Comparing Eqs. (3) and (8) it can be seen that a "new" paradoxical term appears in the fluid description: that involving the quantity $\vec{\mathcal{M}}$. Although this difference has been noticed in the literature, there has appeared, to the best of our knowledge, no discussion of how Eqs. (3) and (8) can be brought into accordance. The only exception is Ref. 12 which, in our opinion, is based on misconception. It was argued that the paradoxical term can be obtained from oscillating center theory by evaluating the time average of $\dot{\vec{\rho}}_\omega$, without involving an ensemble average. However, this is in contradiction with the central assumption of oscillating center theory that fast and slow time scales can be separated. In fact, from Eq. (6) it follows that $\langle\vec{\rho}_\omega\rangle = 0$, where $\langle\cdots\rangle$ denotes the time average: but this also implies that $\langle\dot{\vec{\rho}}_\omega\rangle = 0$. That the term in question cannot be obtained from single particle theory has independently been demonstrated by solving numerically the exact equation of motion.[13]

We now show that apparent discrepancy between Eqs. (3) and (8) can be rationalized by using an appropriate generalization of relations (1) and (2). It is well known that a single charged particle possesses a magnetic moment, which can be calculated from the classical formula.[14] For a particle under the combined influence of steady magnetic and oscillating electric fields the magnetic moment has a time averaged value given by

$$\langle \vec{m} \rangle = \frac{q}{2c} \langle \vec{v} \times \dot{\vec{r}} \rangle = \frac{iq}{4c} \left(\Omega \vec{\rho}_\Omega \times \vec{\rho}_\Omega^{\,*} + \omega \vec{\rho}_\omega \times \vec{\rho}_\omega^{\,*} \right). \tag{9}$$

The first term on the right-hand side is the usual magnetic moment of guiding center theory, while the second term is associated with the motion of the particle at frequency ω. The total magnetization of the fluid element is obtained by averaging Eq. (9) over an ensemble of particles. For an approximately Maxwellian distribution we then obtain

$$\vec{M} = n \overline{\langle \vec{m} \rangle} = -\frac{p}{B_0^2} \vec{B}_0 + \vec{\mathcal{M}} \tag{10}$$

where $\vec{\mathcal{M}}$ is the rf induced magnetization given by Eq. (5). Equations (1) and (10) can now be combined to eliminate the flux $n\vec{v}_c$ from Eq. (8). This yields

$$m \overline{\vec{F}_c} = \left[\frac{q}{c} m \vec{v} + \nabla \times \left(\frac{p}{B_0^2} \vec{B}_0 - \vec{\mathcal{M}} \right) \right] \times \vec{B}_0 - m \nabla \Phi . \tag{11}$$

On using Eq. (2) it is straightforward to show that Eq. (11) is equivalent to Eq. (3).

It should be stressed that the ponderomotive forces calculated from the single particle and fluid approaches are fundamentally different. They cannot simply be related by a transformation between reference frames. The single particle theory, of course, does not account for the collective interaction of the fluid with the rf field. This collective behavior manifests itself in the perpendicular ponderomotive force by the interaction of the rf induced magnetization current with the magnetic field. However, we have shown that by appropriate ensemble averaging of the single particle equations the apparent paradox can be resolved. It has been demonstrated that the rf induced magnetization current arises in precisely the same way as the classical diamagnetic current.

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