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SELF-CONSISTENT CALCULATION OF THE PONDEROMOTIVE FORCE IN A MAGNETIZED PLASMA COLUMN

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ABSTRACT

The ponderomotive force exerted by a circularly polarized electromagnetic wavefield in a uniform, magnetized plasma column is calculated. The effect of the self-consistent nature of the wavefields in the plasma is illustrated for the case of long wavelength excitation. The contribution to the total ponderomotive force of the interaction between the induced magnetization and the magnetic field is determined. It is shown that self-consistency plays a dominant role in determining the ponderomotive potential and induced magnetization of each plasma species if the wave frequency is in the vicinity of either the cyclotron frequency or an eigenfrequency of the plasma column.

1. INTRODUCTION

There has recently been a large interest in the investigation of the ponderomotive force exerted by an electromagnetic wavefield in a magnetized plasma. Various approaches, including single particle, stress tensor, fluid and kinetic, have been used in the past to calculate the ponderomotive force. (A survey of this work is given by STATHAM and TER HAAR (1983).) Of particular interest has been the behaviour of the ponderomotive force for a wave frequency in the vicinity of the cyclotron frequency of one of the plasma species.

A number of authors have derived expressions for the ponderomotive force which exhibit singularities at the cyclotron frequencies. The apparently large value of the ponderomotive force in the vicinity of the cyclotron frequencies has led to the proposition of various applications; for example, radio-frequency plugging of open-ended devices (WATARI et al., 1978; FADER et al., 1981), isotope separation (WEIBEL, 1980) and low frequency mode stabilization (FERRON et al., 1983; YASAKA and ITATANI, 1984).

It has been pointed out (CONSOLI and HALL 1963; EUBANK, 1969; LICHTENBERG and BERK, 1975) that the apparent singularity in the single particle ponderomotive force arises due to the assumption that the motion of the particle is adiabatic. If, however, the wave frequency is in the vicinity of the cyclotron frequency, the particle may exhibit non-adiabatic motion. A correct treatment of the particle motion, including the effect of non-adiabaticity, yields a ponderomotive force which is small, and not resonant, at the cyclotron frequency (LAMB et al., 1984).

Numerous authors have used the expression for the single particle ponderomotive force in an attempt to describe the nonlinear behaviour of a plasma (see, for example, WEIBEL, 1980; AKIMUNE et al., 1981; DIMONTE et al., 1983; FERRON et al., 1983; PARKS and BAKER, 1984). However, such a non self-consistent treatment requires a great deal of caution. The self-consistency of the wavefields in a plasma is made manifest through the ponderomotive force by two important effects. Firstly, it is well known that a plasma may significantly modify an imposed oscillating electric field. In particular, if the oscillation frequency is in the vicinity of the cyclotron frequency of one of the plasma species and the density of this species is sufficiently high, the generation of self-consistent plasma currents may prevent the electric field from penetrating into the plasma. Conversely, if an eigenmode of the plasma is excited, enhanced electric fields may exist in the plasma. Secondly, it has been shown (KLIMA, 1968; KARPMAN and SHAGALOV, 1982) that the total ponderomotive force exerted on a magnetized plasma includes the interaction of the nonlinear magnetization induced by the wavefield with the external magnetic field. This interaction does not exist for the motion of a single particle in a magnetic field under the influence of an oscillating electric field (MOTZ and WATSON, 1967). Due to these two effects, to analyze correctly the ponderomotive force exerted on a magnetized plasma a self-consistent treatment is therefore required.

In the present paper we consider a self-consistent treatment of the ponderomotive force exerted by an externally excited electromagnetic wavefield on a magnetized plasma column. Both right and left circularly polarized wavefields within the frequency range of the ion cyclotron frequency are considered. While the present analysis shall be restricted to long parallel wavelengths, it will be seen that the essential features of self-consistency are retained : the significant contribution to the perpendicular ponderomotive force of the induced magnetization, and the important influence of both exclusion and enhancement of the wavefields in the plasma in determining the ponderomotive force.

2. FORMULATION

In this paper we shall consider the nonlinear interaction of an electromagnetic wavefield in a cylindrical, magnetized plasma column. The wavefield is excited by an antenna that is located at the boundary of the plasma column. The ponderomotive force exerted by the wavefield on the plasma is obtained from the time average of the solution, to second order in wave amplitude, of the appropriate fluid equations.

We shall consider a cold, collisionless, multi-species plasma having, in the absence of the wavefield, a uniform density $n_{\sigma 0}$ for each species σ, and a uniform axial magnetic field, $\underline{B}_0 = B_0\, \hat{\underline{z}}$. The equilibrium values of electric field and fluid velocity are assumed to be zero , that is $\underline{E}_0 = \underline{u}_{\sigma 0} = 0$. The plasma column is infinite in length and has a radius r_0.

The antenna excites at the plasma boundary $r = r_0$ an electric field that can be written, in complex notation, as

$$\underline{E}_1(r_0) = E_{ext}\left[m\,,\, i\,,\, 0\right] \cos k_{\parallel} z \;\exp\left\{i(m\theta - \omega t)\right\}, \quad (1)$$

where E_{ext} is a constant.

We shall restrict our analysis to the case $|m| = 1$, that is, to excitation by either a pure right ($m = +1$) or left ($m = -1$) circularly polarized wavefield. The wavefield is assumed to be standing in the axial direction with an axial wavenumber $k_\parallel$ and frequency ω determined by the external antenna.

3. LINEAR SOLUTION

In the plasma region we shall consider fields of the form :

$$A_1(r,\theta,z,t) = A_1(r,z)\exp\{i(m\theta - \omega t)\} . \tag{2}$$

These fields must satisfy the following set of linearized multi-fluid equations :

$$m_\sigma \frac{\partial \underline{u}_{\sigma 1}}{\partial t} = q_\sigma \left(\underline{E}_1 + \underline{u}_{\sigma 1} \times \underline{B}_0 \right) , \tag{3}$$

$$\frac{\partial n_{\sigma 1}}{\partial t} + n_{\sigma 0} \nabla \cdot \underline{u}_{\sigma 1} = 0 , \tag{4}$$

$$\nabla \times \underline{B}_1 = \mu_0 \sum_\sigma n_{\sigma 0}\, q_\sigma\, \underline{u}_{\sigma 1} , \tag{5}$$

$$\nabla \times \underline{E}_1 = - \frac{\partial \underline{B}_1}{\partial t} . \tag{6}$$

Here, a subscript 1 is used to denote a quantity that is first order in the wave amplitude. The displacement current has been neglected from equation (5) since the scale lengths of interest are much less than the free space wavelength.

Since we are interested in the present analysis in frequencies much less than the electron cyclotron frequency, we shall treat the electrons as a massless fluid, with $m_e = 0$. (Although electron inertia has been shown to have a dominant influence, for low plasma density, on the ion cyclotron wave (HOSEA and SINCLAIR, 1969), it will be seen that this mode plays no role for the particular case examined in the present analysis). The equation of motion for the electron fluid then becomes

$$\underline{E}_1 + \underline{u}_{e1} \times \underline{B}_o = 0$$

and, as a consequence, the electric wavefield component parallel to $\underline{B}_o$ is zero.

Under the above assumptions, a well-known solution of the set of equations (3) - (6) has electric field components whose Fourier amplitudes may be written in the following form (WOODS, 1962):

$$E_{1r}(r,z) = -\frac{\omega}{k_{\parallel}} \left[m\, k_{\parallel} A_m \frac{J_m(k_{\perp} r)}{k_{\perp} r} + C_m J_m'(k_{\perp} r) \right] \cos k_{\parallel} z$$

$$E_{1\theta}(r,z) = -i\frac{\omega}{k_{\parallel}} \left[m\, C_m \frac{J_m(k_{\perp} r)}{k_{\perp} r} + k_{\parallel} A_m J_m'(k_{\perp} r) \right] \cos k_{\parallel} z$$

$$E_{1z}(r,z) = 0 \quad , \qquad (7)$$

where J_m is a Bessel function of the first kind and $J'_m(\xi) \equiv dJ_m/d\xi$. The constants A_m and C_m are related by

$$\frac{k_\parallel A_m}{C_m} = \frac{k_\parallel^2 - \mathcal{S}}{\mathcal{D}} = \frac{\mathcal{D}}{k_\parallel^2 + k_\perp^2 - \mathcal{S}} , \tag{8}$$

where

$$\mathcal{S} = -\frac{\omega^2}{c^2} \sum_{\sigma \neq e} \frac{\omega_{p\sigma}^2}{(\omega^2 - \Omega_\sigma^2)} ,$$

$$\mathcal{D} = \frac{\mu_0 \omega n_{eo} e}{B_0} + \frac{\omega^2}{c^2} \sum_{\sigma \neq e} \frac{\omega_{p\sigma}^2 \Omega_\sigma}{\omega(\omega^2 - \Omega_\sigma^2)} , \tag{9}$$

and $\omega_{p\sigma}^2 = n_{\sigma o} q_\sigma^2 / m_\sigma \varepsilon_o$ and $\Omega_\sigma = q_\sigma B_o / m_\sigma$.

If the wave frequency is in the vicinity of the cyclotron frequency of an ion species $\sigma = i$, we may approximate $\mathcal{S}$ and $\mathcal{D}$, defined in equation (9), by

$$\mathcal{S} \simeq -\frac{\omega^2}{c^2} \frac{\omega_{pi}^2}{(\omega^2 - \Omega_i^2)} \quad ; \quad \mathcal{D} \simeq \frac{\omega^2}{c^2} \frac{\omega_{pi}^2 \Omega_i}{\omega(\omega^2 - \Omega_i^2)} .$$

We shall, for the sake of clarity, restrict the present analysis to the case for which the axial wavenumber imposed by the external antenna is small, that is,

$$k_\parallel^2 r_o^2 \ll \zeta_i^2 \quad \text{where} \quad \zeta_i = \frac{\omega_{pi} r_o}{c} . \tag{10}$$

We may then show that

$$\frac{k_{\parallel} A_m}{C_m} \simeq \frac{\omega}{\Omega_i} \quad . \tag{11}$$

The perpendicular wavenumber is then imaginary, with $|k_{\perp}|^2 \gg k_{\parallel}^2$ and $k_{\perp} r_o \simeq i\zeta_i$.

Applying the boundary condition of continuity of the tangential electric field E_θ at $r = r_o$, we obtain from equation (1) the following expression:

$$C_m = - \frac{k_{\parallel}}{\omega} \frac{E_{ext}}{F_m} \quad , \tag{12}$$

where

$$F_m = m \frac{I_m(\zeta_i)}{\zeta_i} + \frac{\omega}{\Omega_i} I_m'(\zeta_i) \tag{13}$$

and I_m is a modified Bessel function.

F_m possess a zero if the frequency is equal to an eigenfrequency of the plasma column. The zero of F_{-1} may be related to the excitation of a long wavelength $m = -1$ fast wave. If we consider a low density plasma ($\zeta_i \ll 1$), it may be shown that $F_{-1} = 0$ when

$$\frac{\omega}{\Omega_i} = 1 - \frac{\zeta_i^2}{4} \quad , \tag{14}$$

corresponding to the dispersion relation for the first radial eigenmode of the m = -1 fast wave near cut-off ($k_{\parallel} \to 0$) given by COLLINS et al. (1984). Note that F_{+1} is always positive : there does not exist an eigenmode of the m = +1 wave in the vicinity of $\omega = \Omega_i$.

We note that for the specific case considered in this paper (that is, excitation at the plasma boundary of fields having a long axial wavelength), we have excluded the excitation, for $\omega \simeq \Omega_i$, of any eigenmodes (APPERT et al., 1984; COLLINS et al., 1984) except the first radial eigenmode of the m = -1 fast wave.

In general, as observed from equation (7), the electric field excited in the plasma region $r < r_o$ will be elliptically polarized. However on the plasma axis, r = 0, we find

$$\underline{E}_1(0,z) = -\frac{C_m}{2}\frac{\omega}{k_{\parallel}}\left(1 + m\frac{k_{\parallel}A_m}{C_m}\right)\left[m,i,0\right]\cos k_{\parallel}z \,, \qquad (15)$$

that is, the field is circularly polarized.

For the conditions stated above, substituting equations (11) and (12) into (15) yields the following electric field on axis :

$$\underline{E}_1(0,z) = \frac{E_{ext}}{2\,F_m}\left(1 + m\frac{\omega}{\Omega_i}\right)\left[m,i,0\right]\cos k_{\parallel}z \,. \qquad (16)$$

For right circularly polarized excitation (m = +1), the electric field on the plasma axis is therefore only weakly dependent on the wave frequency in the vicinity of the ion cyclotron frequency. However, for left circularly polarized excitation (m = -1), the field is excluded from the plasma axis at $\omega = \Omega_i$, but experiences a resonance at the zero of F_{-1}, corresponding to the excitation of the $m = -1$ eigenmode.

4. NONLINEAR SOLUTION

The ponderomotive force is obtained from the time average of the equation of motion to second order in wave amplitude. For the present study of a stationary wavefield ($dE_{ext}/dt = 0$) we may write the equation of motion, after averaging in time, as

$$q_\sigma n_{\sigma 0} \left(\underline{E}_2 + \underline{u}_{\sigma 2} \times \underline{B}_0 \right) + \underline{F}_\sigma^{NL} = 0 \quad , \qquad (17)$$

where the subscript 2 is used to denote a time-averaged second order quantity. The nonlinear (ponderomotive) force acting on species σ is given by

$$\underline{F}_\sigma^{NL} = q_\sigma \left\{ n_{\sigma 1} \underline{E}_1^* + n_{\sigma 0} \underline{u}_{\sigma 1} \times \underline{B}_1^* \right\}$$

$$- n_{\sigma 0} m_\sigma \left\{ \underline{u}_{\sigma 1} \left(\nabla \cdot \underline{u}_{\sigma 1}^* \right) + \left(\underline{u}_{\sigma 1} \cdot \nabla \right) \underline{u}_{\sigma 1}^* \right\} \quad , \qquad (18)$$

where the superscript * denotes the complex conjugate. The ponderomotive force defined in equation (18) conforms to that used by previous authors (see, for example, STATHAM and TER HAAR, 1983; LEE and PARKS, 1983).

Using equations (3) to (6) to express the Fourier amplitudes of $\underline{E}_1$, $\underline{B}_1$ and $n_{\sigma 1}$ in terms of that of $\underline{u}_{\sigma 1}$, it can be shown

(KLIMA, 1968; KARPMAN and SHAGALOV, 1982; LEE and PARKS, 1983) that the ponderomotive force acting on species σ may be written as

$$\underline{F}_{\sigma}^{NL} = -n_{\sigma o} \nabla \Phi_{\sigma} + \underline{B}_{o} \times (\nabla \times \underline{M}_{\sigma}) \,, \tag{19}$$

where the ponderomotive potential and the induced magnetization are, respectively,

$$\Phi_{\sigma} = \frac{m_{\sigma}}{4} \left\{ \underline{u}_{\sigma 1} \cdot \underline{u}_{\sigma 1}^{*} + \frac{i q_{\sigma}}{\omega m_{\sigma}} \underline{B}_{o} \cdot (\underline{u}_{\sigma 1} \times \underline{u}_{\sigma 1}^{*}) \right\} \,, \tag{20}$$

$$\underline{M}_{\sigma} = i \, \frac{n_{\sigma o} q_{\sigma}}{4 \omega} \, \underline{u}_{\sigma 1} \times \underline{u}_{\sigma 1}^{*} \,. \tag{21}$$

If we specialize the above equations to the case of a circularly polarized wave, using equation (3) we may write

$$\underline{u}_{\sigma 1}(r, z) = \frac{q_{\sigma} E_{\perp}(r, z)}{m_{\sigma}(\omega + m \Omega_{\sigma})} \left[-i \,, m \,, 0 \right] \,,$$

where

$$E_{\perp} \equiv | E_r | = | E_{\theta} | \,.$$

Substituting into equations (20) and (21) yields

$$\Phi_{\sigma} = \frac{q_{\sigma}^{2} E_{\perp}^{2}}{2 m_{\sigma} \omega (\omega + m \Omega_{\sigma})} \tag{22}$$

and

$$\underline{M}_\sigma = \hat{z}\, M_{\sigma z} = \hat{z}\, m\Omega_\sigma \frac{n_{\sigma o}}{B_o} \frac{q_\sigma^2 E_\perp^2}{2 m_\sigma \omega (\omega + m\Omega_\sigma)^2} \quad . \qquad (23)$$

Note that the ponderomotive potential given in equation (22) is the same as that calculated from a single particle approach (MOTZ and WATSON, 1967). However, as seen in equation (19), in a magnetized plasma, an additional term arises due to the interaction of the induced magnetization and the external magnetic field. For a circularly polarized wavefield we may combine these two terms to obtain, after substituting equations (22) and (23) into (19), the total ponderomotive force in the form :

$$\underline{F}_\sigma^{NL} = \hat{r} \left[\frac{-n_{\sigma o} q_\sigma^2}{2 m_\sigma (\omega + m\Omega_\sigma)^2} \frac{\partial E_\perp^2}{\partial r} \right] + \hat{z} \left[\frac{-n_{\sigma o} q_\sigma^2}{2 m_\sigma \omega (\omega + m\Omega_\sigma)} \frac{\partial E_\perp^2}{\partial z} \right] \quad . \qquad (24)$$

It is interesting to note that inclusion of the contribution of the induced magnetization results in a form for the perpendicularly-directed ponderomotive force that differs from that calculated from a single particle approach. As can be seen from equation (24), whereas the parallel component changes sign as the wave frequency crosses the cyclotron frequency (for an appropriate polarization of the wavefield), the perpendicular component does not. This may be important to note in connection with attempts to explain the stabilization of low frequency modes observed in a mirror device (FERRON et al., 1983) by means of the perpendicular ponderomotive force.

For the wavefield excited by an external antenna as described in section 3, the self-consistent ponderomotive potential and induced magnetization acting on an ion species may be calculated on the plasma axis (where the wavefield is circularly polarized) and at $z = 0$ (where $|\underline{E}|$ is maximum) by substituting equation (16) into (22) and (23). We then obtain for a wave frequency in the vicinity of the cyclotron frequency of the ion species :

$$\Phi_i^{\circ} = \frac{(\omega + m\Omega_i)}{4\,\omega\,F_m^2}\;\frac{q_i^2\,E_{ext}^2}{2\,m_i\,\Omega_i^2} \tag{25}$$

and

$$M_{iz}^{\circ} = \frac{m\,\Omega_i}{4\,\omega\,F_m^2}\;\frac{n_{io}}{B_o}\;\frac{q_i^2\,E_{ext}^2}{2\,m_i\,\Omega_i^2}\;. \tag{26}$$

5. DISCUSSION

In Fig. 1 and 2 are plotted, for three values of normalized ion density ζ_i, the normalized values of Φ_i° and M_{iz}° as a function of ω/Ω_i. Figure 1 shows the dependence for a right circularly polarized wavefield ($m = +1$) and Fig. 2 for a left circularly polarized wavefield ($m = -1$).

As seen from Fig. 1(a), including the effect of self-consistency of the wavefields does not alter substantially the ponderomotive potential created by an m = +1 wavefield exerted on an ion species. Figure 1(b) shows, however, that the self-consistent treatment leads to an induced magnetization that significantly influences the perpendicular (radial) ponderomotive force. It may be noted from Fig. 1 that the ponderomotive potential and the induced magnetization remain small for a wave frequency in the vicinity of the ion cyclotron frequency. Both decrease only slightly with increasing density. These observations result from the fact that a right circularly polarized wavefield is unaffected by the ion cyclotron resonance, and for the choice of wavefield made in section 3, there does not exist an m = +1 eigenmode in the range of frequencies being considered in the present analysis.

On the contrary, the self-consistent nature of the wavefields modifies substantially the ponderomotive potential created by an m = -1 wavefield exerted on an ion species. In addition, a large contribution to the perpendicular ponderomotive force is obtained from the induced magnetization. Several important features may be observed from the curves shown in Fig. 2. It may first be noted that both the ponderomotive potential and the induced magnetization exhibit a resonance at a frequency just below the ion cyclotron frequency. This corresponds, as noted in section 3, to the excitation of a long wavelength m = -1 fast wave. The eigenfrequency of this mode lies always below the ion cyclotron frequency (APPERT et al., 1984; COLLINS et al., 1984), and its excitation results in a large electric field amplitude in the plasma and hence a large ponderomotive force. Figure

2(a) also shows that for $\omega = \Omega_i$ the ponderomotive potential Φ_i^o is zero for m = -1 excitation. This results from the exclusion of the electric field from the axis of the plasma column, as noted in section 3. However, as seen from Fig. 2(b), the induced magnetization M_{iz}^o is non-zero for $\omega = \Omega_i$, leading to a non-zero perpendicular ponderomotive force. For frequencies $\omega \gtrsim \Omega_i$, Fig. 2 shows a sharp reduction of Φ_i^o and M_{iz}^o for increasing plasma density. For frequencies sufficiently far from both the eigenfrequency and the ion cyclotron frequency, the effect of increasing the ion density is small : the plasma then does not significantly affect the penetration of the m = -1 wavefields.

For $\omega \gtrsim \Omega_i$ (that is, the frequency regime which does not contain a plasma eigenmode), the maximum value of the ponderomotive potential created by an m = -1 wavefield is reduced and occurs at a higher frequency if the plasma density is increased. From equation (25) the maximum of Φ_i^o for $\omega \gtrsim \Omega_i$ occurs at a frequency ω_o given by

$$\frac{2}{F_{-1}} \left. \frac{\partial F_{-1}}{\partial \omega} \right|_{\omega_o} = \frac{\Omega_i}{\omega_o(\omega_o - \Omega_i)} \quad . \tag{27}$$

For the particular wave excitation considered in this paper, substitution of F_{-1} from equation (13) yields for $\zeta_i \ll 1$,

$$\omega_o \simeq \Omega_i \left(1 + \frac{\zeta_i^2}{4} \right) \quad . \tag{28}$$

The maximum value of Φ_i^o is then given by

$$\Phi_i^o(\omega_o) \simeq \frac{1}{\xi_i^2} \frac{q_i^2 E_{ext}^2}{2 m_i \Omega_i^2} . \qquad (29)$$

From equation (28), it is therefore seen that the effects of wavefield self-consistency will dominate in the ponderomotive potential if

$$\xi_i^2 \gtrsim 4 \left(\frac{\omega}{\Omega_i} - 1 \right)$$

or

$$\omega_{pi} \gtrsim \frac{2c}{r_o} \left(\frac{\omega}{\Omega_i} - 1 \right)^{1/2} . \qquad (30)$$

The above-described behaviour of the ponderomotive potential is illustrated in Fig. 3, which shows the frequency dependence of Φ_i^o for an m = -1 wavefield in the frequency regime $\omega \gtrsim \Omega_i$.

It should be noted that in the present analysis, kinetic effects have been neglected. These effects may play an important role for frequencies close to the ion cyclotron frequency, depending on the form of the oscillating field and the ion distribution function. To avoid cyclotron damping effects, which will result in a plasma-wave interaction not accomodated by the present fluid approach, a necessary condition to be satisfied is

$$k_{\parallel} V_{i\parallel} \ll |\omega - \Omega_i| , \qquad (31)$$

where $V_{i\parallel}$ is the ion thermal velocity parallel to $\underline{B}_0$. LAMB et al. (1984) have shown that the same inequality as (31) must be satisfied for the motion of a single ion in an oscillating (electrostatic) field to be adiabatic (if $V_{i\parallel}$ is identified as the parallel ion velocity). This reflects the fact that both phenomena, cyclotron damping and ion non-adiabaticity, describe the same physical processes: the non-conservative interaction of particles with a oscillating field. The result of this interaction is that if the inequality (31) is not satisfied, the ponderomotive force exerted on the ions will be substantially reduced from that calculated assuming a conservative interaction. Thus, for wave frequencies sufficiently close to the ion cyclotron frequency, the ponderomotive force exerted on the ion species will be modified by both kinetic effects and the effect of wavefield self-consistency investigated in this paper. We note, however, that if the plasma density is sufficiently high, that is

$$\xi_i^2 > \frac{4\, k_\parallel V_{i\parallel}}{\Omega_i} \, , \qquad (32)$$

there will exist a frequency regime for which kinetic effects are negligible, but wavefield self-consistency plays a dominant role in the determination of the ponderomotive force. It should be noted that the inequality (32) may be readily satisfied experimentally (see, for example, FERRON et al., 1983).

6. CONCLUSION

In the present paper we have considered a self-consistent calculation of the ponderomotive force exerted by an electromagnetic wavefield in a magnetized plasma. The influence of wavefield self-consistency is manifested by two important effects : the modification of the wavefields in the plasma, particularly at the cyclotron and eigenfrequencies, and the contribution to the perpendicular ponderomotive force from the interaction of the induced magnetization and the external magnetic field.

By examining the specific case of long wavelength excitation in a cylindrical plasma column, the modification of the ponderomotive force due to the effects of self-consistency has been illustrated. A strong modification has been calculated for left circularly polarized excitation if the frequency is in the vicinity of the ion cyclotron frequency or the eigenfrequency of the m = -1 fast wave.

It is pointed out that the specific results obtained in this paper are relevant to the particular choice of wavefield that has been considered. However, it should be stressed that the effect of accounting for the self-consistent nature of the wavefields in a plasma is of universal importance. The detailed manner in which the ponderomotive force is modified depends critically on the specific method of excitation. The frequency and wavenumber spectrum of the antenna, as well as the particular plasma parameters, must be included in any investigation of the ponderomotive force exerted on a plasma. Finally, while this paper has been concerned with frequencies in the vicinity of the ion cyclotron frequency, similar behaviour is to be expected in the range of the electron cyclotron frequency.

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FIGURE CAPTIONS

FIG. 1 : Frequency dependence, for three values of normalized ion density ζ_i, of the normalized values of (a) the ponderomotive potential Φ_i^o, and (b) the induced magnetic moment M_{iz}^o, for a right circularly polarized wavefield ($m = +1$).

FIG. 2 : Frequency dependence, for three values of normalized ion density ζ_i, of the normalized values of (a) the ponderomotive potential Φ_i^o, and (b) the induced magnetic moment M_{iz}^o, for a left circularly polarized wavefield ($m = -1$).

FIG. 3 : Frequency dependence, for $\omega \gtrsim \Omega_i$ and four values of normalized ion density ζ_i, of the normalized value of the ponderomotive potential Φ_i^o for a left circularly polarized wavefield ($m = -1$).

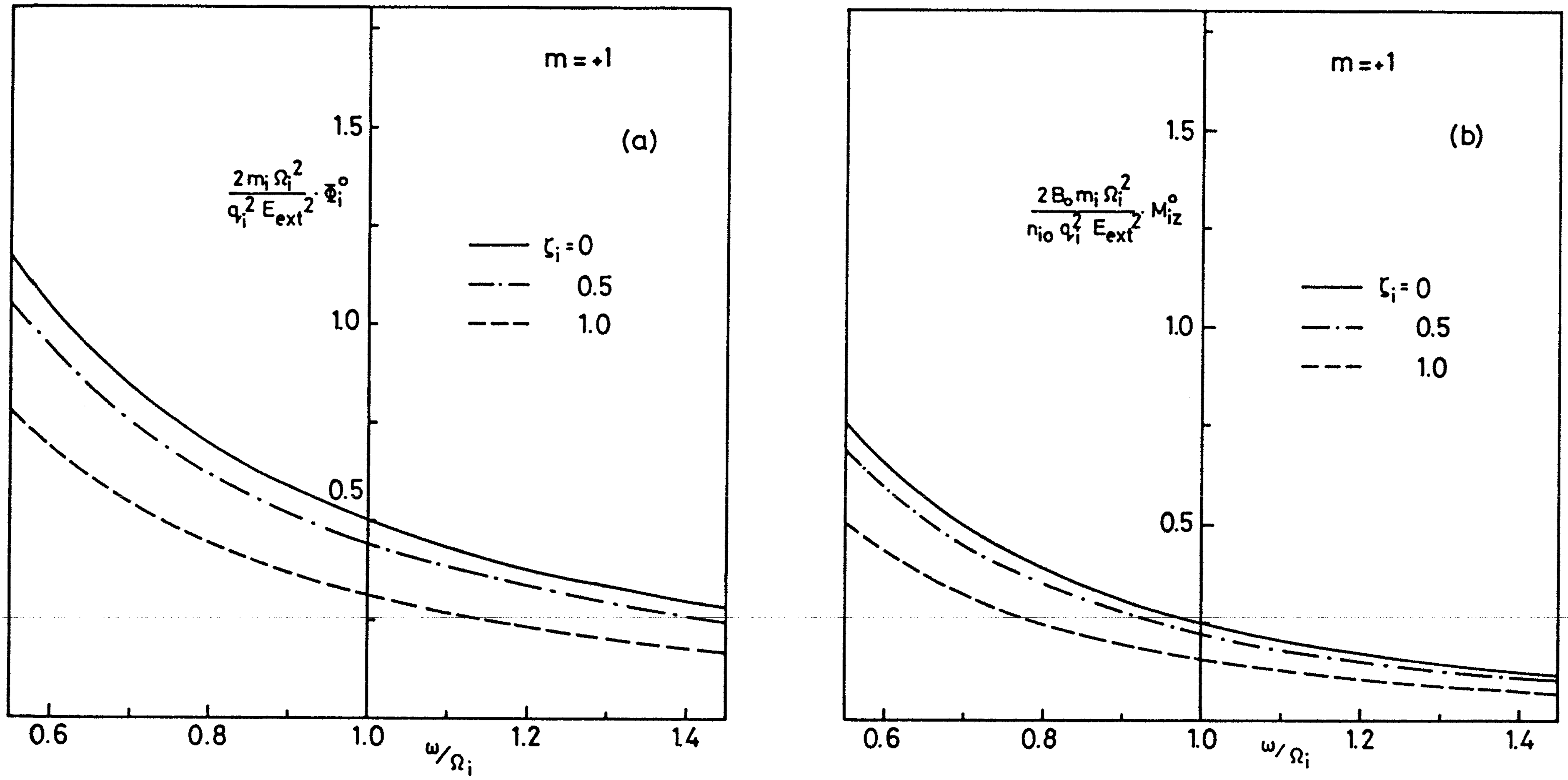
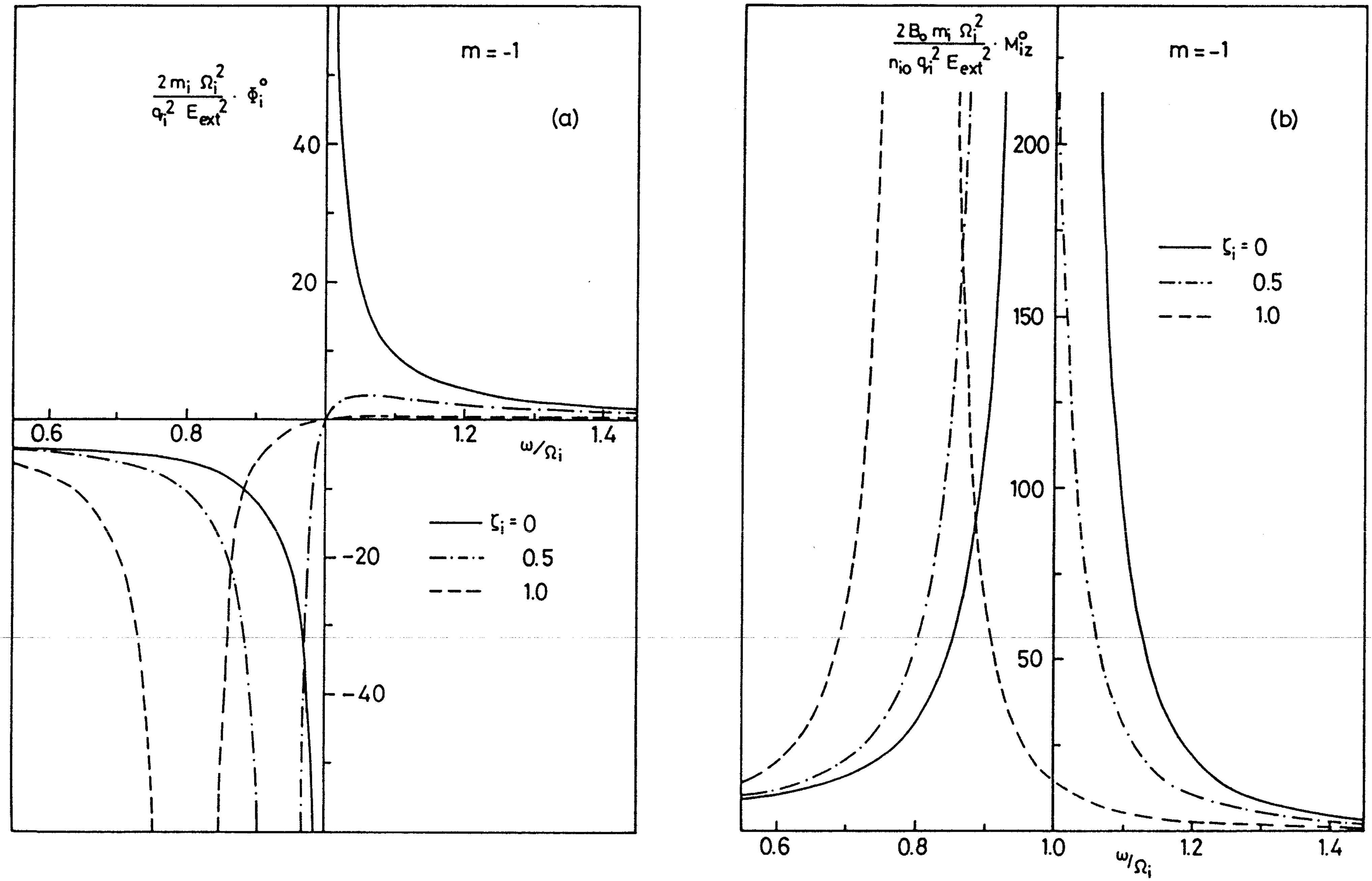
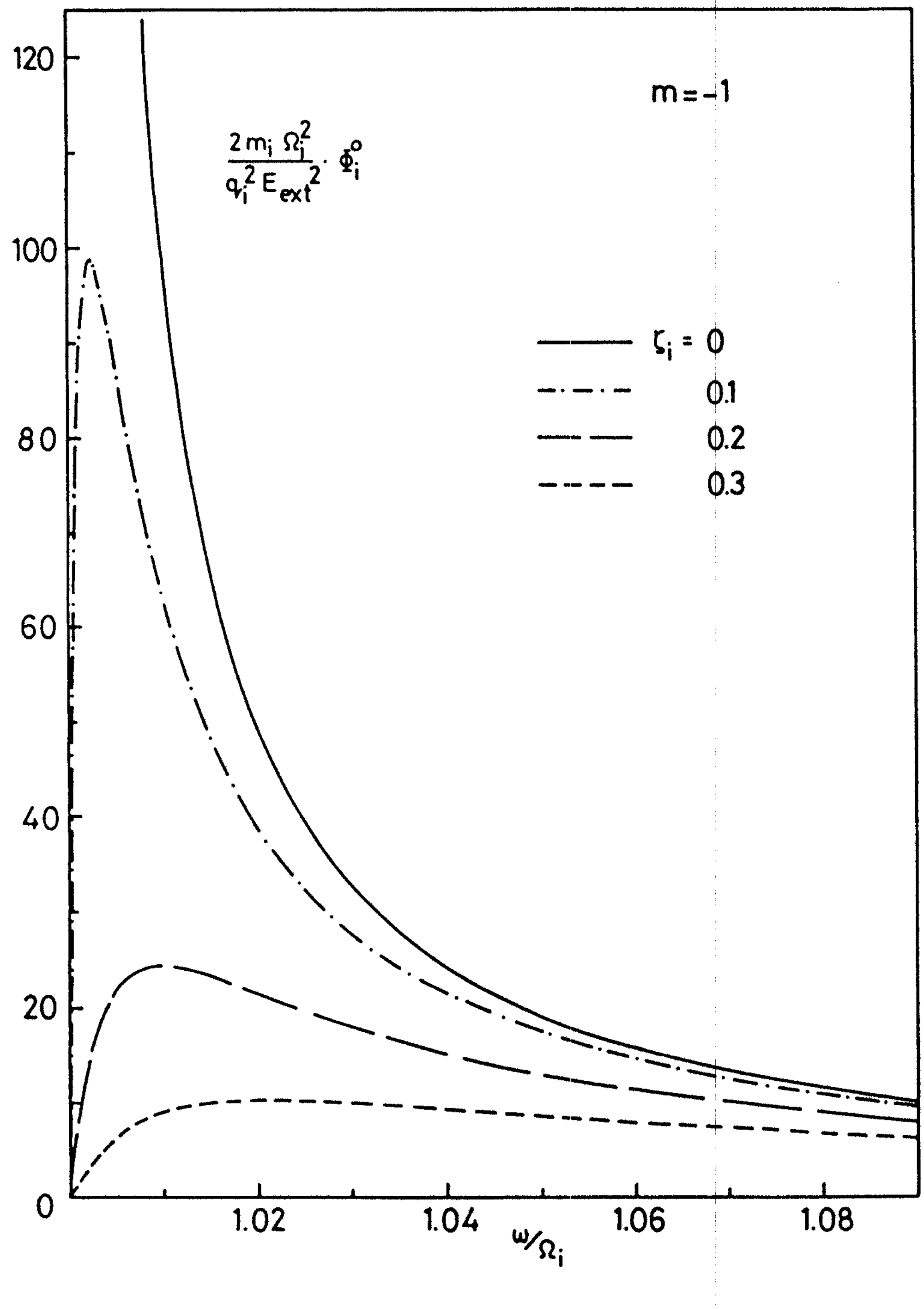

m = +1
(a)
$\frac{2 m_i \Omega_i^2}{q_i^2 E_{ext}^2} \cdot \Phi_i^o$
$\zeta_i = 0$
0.5
1.0
0.5
1.0
1.5
0.6
0.8
1.0
1.2
1.4
ω / Ω_i
m = +1
(b)
$\frac{2 B_o m_i \Omega_i^2}{n_{io} q_i^2 E_{ext}^2} \cdot M_{iz}^o$
$\zeta_i = 0$
0.5
1.0
0.5
1.0
1.5
0.6
0.8
1.0
1.2
1.4
ω / Ω_i

Figure 1



$\frac{2 m_i \Omega_i^2}{q_i^2 E_{ext}^2} \cdot \Phi_i^o$
m = -1
(a)
40
20
-20
-40
0.6
0.8
1.2
1.4
ω/Ω_i
$\zeta_i = 0$
0.5
1.0
$\frac{2 B_o m_i \Omega_i^2}{n_{io} q_i^2 E_{ext}^2} \cdot M_{iz}^o$
m = -1
(b)
200
150
100
50
0.6
0.8
1.0
1.2
1.4
ω/Ω_i
$\zeta_i = 0$
0.5
1.0

Figure 2



120
100
80
60
40
20
0
m = -1
$\frac{2 m_i \Omega_i^2}{q_i^2 E_{ext}^2} \cdot \Phi_i^o$
ζ_i = 0
0.1
0.2
0.3
1.02
1.04
1.06
1.08
ω/Ω_i

Figure 3