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ABSTRACT

Differential equations for small amplitude electromagnetic perturbations in a hot nonuniformly magnetized inhomogeneous plasma are derived from the Vlasov and Maxwell equations. Assuming a slab geometry, a perturbation expansion to second order in the smallness of the Larmor radius compared to characteristic scale-lengths of the plasma and fields is used. The results are expressed in terms of an equivalent dielectric tensor operator. The latter is shown to possess appropriate conservation properties.

## 1. INTRODUCTION

The propagation and absorption of waves in nonuniform plasmas is one of the principal problems in the theory of radio-frequency heating. Until recently, the equations applied to study the problem have usually been derived using the Fourier transform of the dielectric tensor which is valid for hot uniform plasmas. This procedure, however, is not unique and often leads to equations which do not possess appropriate conservation properties.Moreover, the terms due to the gradients of equilibrium quantities are missing in the equations. Therefore the correct equations have to be derived from first principles. This problem was first addressed by Berk and Dominguez [1] who have deviced a variational method to derive the differential equations in question. However, the explicit form of the equations was given only in a number of limiting cases pertinent to the ion-cyclotron range of frequency [2-5]. The purpose of this paper is to provide a formulation of the dielectric tensor operator that can straightforwardly be implemented in a numerical code covering all frequency ranges.

The operator will be derived from the Vlasov and Maxwell equations under the following assumptions :

1) The equilibrium quantities vary in the x direction which is perpendicular to that of a given magnetic field.
2) The magnitude of the wave field is small so that the Vlasov equation may be linearized.
3) The system possesses a small parameter $\delta = \rho/L$, where $\rho$ is the Larmor radius and L is a characteristic scale-length of the variation of the equilibrium and wave field quantities in the directions perpendicular to the static magnetic field.
4) A perturbation method will be used to obtain the solution of the Vlasov equations valid up to $\delta^2$.

In Section 2, the solution of the linearized Vlasov equation is obtained to the desired order. The dielectric tensor operator is then evaluated in Section 3 assuming a specific form of the equilibrium distribution function. The resulting operator is shown to possess appropriate conservation properties .

## 2. SOLUTION OF THE LINEARIZED VLASOV EQUATION

The distribution function $\hat{f}$ for a species with charge q and mass m, of a plasma interacting with an electromagnetic field $\vec{E}$, $\vec{B}$ obeys the Vlasov equation

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\hat{f} + \frac{q}{m}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\vec{B})\right]\cdot\frac{\partial \hat{f}}{\partial \vec{v}} = 0 \, . \qquad (1)$$

Assuming a small perturbation around an equilibrium we may split $\hat{f}$ into an equilibrium part F and a fluctuating part $\tilde{f}$. The same is true for the magnetic field. Thus, the linearized Vlasov equation reads

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\tilde{f} + \frac{q}{mc}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial \tilde{f}}{\partial \vec{v}} = -\frac{q}{m}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\tilde{\vec{B}})\right]\cdot\frac{\partial F}{\partial \vec{v}} \, , \qquad (2)$$

where $\vec{B}_0 = B_0(x)\vec{e}_z$.

The equilibrium distribution function satisfies

$$v_x \frac{\partial F}{\partial x} + \omega_c(x)\left(\vec{v}\times\vec{e}_z\right)\cdot\frac{\partial F}{\partial \vec{v}} = 0 \qquad , \qquad (3)$$

where $\omega_c(x) = qB_0/mc$ is the local cyclotron frequency. The solution of (3) is an arbitrary function of the integrals of motion : $v_\perp = (v_x^2+v_y^2)^{1/2}$, $v_z$ and $P = v_y + \int^x \omega_c(x')dx'$. For simplicity, we shall consider only $F=F(v,P)$, where $v=(v_\perp^2+v_z^2)^{1/2}$.

Let f be the Fourier transform of $\tilde{f}$ with respect to the variables t, y, z. On introducing cylindrical coordinates in velocity space ($v_\perp$, $\alpha$, $v_z$) and eliminating $\tilde{\vec{B}}$ via Faraday's law equation (2) is transformed into

$$\omega_c \frac{\partial f}{\partial \alpha} + i(\omega - k_z v_z) f - \frac{v_\perp}{2}\left(L^+ e^{i\alpha} + L^- e^{-i\alpha}\right) f = A \quad (4)$$

where $L^\pm = \partial/\partial x \pm k_y$ and

$$A = \frac{q}{m}\left\{\vec{E} + \frac{1}{i\omega}\left[\vec{v} \times (\nabla \times \vec{E})\right]\right\} \cdot \frac{\partial F}{\partial \vec{v}} . \quad (5)$$

To respect causality, $\omega$ is assumed to have a small, positive, imaginary part. Since f must be periodic in $\alpha$ we can write it as a Fourier series

$$f = \sum_{n=-\infty}^{+\infty} f_n e^{in\alpha} . \quad (6)$$

The same is true for A.

According to our assumptions we now expand F up to $O(\delta^2)$

$$F = F(v, \xi) + v_y F' + \frac{1}{2} v_y^2 F'' , \quad (7)$$

where $F' \equiv \partial F/\partial \xi$ and $\xi(x) = \int^x \omega_c(x')dx'$. Inserting (7) into (5) we can separate order by order and perform the Fourier series decomposition to obtain

$$A_m^{(0)} = \frac{q}{m}\frac{G}{v}\left\{\frac{v_\perp}{2}\Big[E_x\left(\delta_{m,1}+\delta_{m,-1}\right)+iE_Y\left(\delta_{m,-1}-\delta_{m,1}\right)\Big] + v_z E_z\,\delta_{m,0}\right\} , \tag{8}$$

$$A_m^{(1)} = \frac{q}{m}\left\{\frac{G' v_\perp}{v}\left(\frac{v_\perp}{4}\Big[iE_x\left(\delta_{m,-2}-\delta_{m,2}\right)+E_Y\left(2\delta_{m,0}-\delta_{m,2}-\delta_{n,-2}\right)\Big] + \frac{i}{2}v_z E_z\left(\delta_{m,-1}-\delta_{m,1}\right)\right) + F' E_Y\left(1-\frac{k_z v_z}{\omega}\right)\delta_{m,0}\right\} , \tag{9}$$

$$A_m^{(2)} = \frac{q}{m}\left\{\frac{G'' v_\perp^2}{2v}\left(\frac{v_\perp}{8}\Big[E_x\left(\delta_{m,1}+\delta_{m,-1}\right)+i3E_Y\left(\delta_{m,-1}-\delta_{m,1}\right)\Big] + \frac{v_z}{2}E_z\,\delta_{m,0}\right) + i\,\frac{v_\perp}{2}F'' E_Y\left(1-\frac{k_z v_z}{\omega}\right)\left(\delta_{m,-1}-\delta_{m,1}\right) + \frac{F'}{\omega}\Big[v_z k_Y E_z\,\delta_{m,0} + \frac{v_\perp}{2}\left(\delta_{m,1}+\delta_{m,-1}\right)\left(k_Y E_x + i\frac{dE_Y}{dx}\right)\Big]\right\} , \tag{10}$$

where $\delta_{n,n'}$ is Kronecker's delta and $G \equiv \partial F/\partial v$. In the expression (10) we have omitted contributions of the harmonics that are not needed in subsequent calculations.

Finally, substituting (6) into (4) and separating different orders we obtain a recursion relation

$$f_n^{(l)} = \frac{1}{i\Omega_{-n}}\left[\frac{v_\perp}{2}\left(L^+ f_{n-1}^{(l-1)} + L^- f_{n+1}^{(l-1)}\right) + A_n^{(l)}\right], \tag{11}$$

where $\Omega_n \equiv \omega - n\omega_c - k_z v_z$.

For l=0, this relation simplifies to

$$f_n^{(0)} = \frac{A_n^{(0)}}{i\Omega_{-n}}, \tag{12}$$

and after a little algebra

$$f_n^{(1)} = -\frac{1}{\Omega_{-n}}\left[\frac{v_\perp}{2}\left(L^+ \frac{A_{n-1}^{(0)}}{\Omega_{-n+1}} + L^- \frac{A_{n+1}^{(0)}}{\Omega_{-n-1}}\right) + i A_n^{(1)}\right], \tag{13}$$

$$\begin{aligned} f_n^{(2)} = \frac{1}{\Omega_{-n}}\Bigg\{ & i\left(\frac{v_\perp}{2}\right)^2 \Bigg[L^+ \frac{1}{\Omega_{-n+1}}\left(L^+ \frac{A_{n-2}^{(0)}}{\Omega_{-n+2}} + L^- \frac{A_n^{(0)}}{\Omega_{-n}}\right) \\ & + L^- \frac{1}{\Omega_{-n-1}}\left(L^+ \frac{A_n^{(0)}}{\Omega_{-n}} + L^- \frac{A_{n+2}^{(0)}}{\Omega_{-n-2}}\right)\Bigg] \\ & - \frac{v_\perp}{2}\left(L^+ \frac{A_{n-1}^{(1)}}{\Omega_{-n+1}} + L^- \frac{A_{n+1}^{(1)}}{\Omega_{-n-1}}\right) - i A_n^{(2)} \Bigg\}. \end{aligned} \tag{14}$$

## 3. DIELECTRIC TENSOR OPERATOR

Having found the perturbed distribution function to the required order we can calculate the perturbed current density according to

$$\vec{j} = \sum_s q\pi \int_0^\infty v_\perp dv_\perp \int_{-\infty}^{+\infty} dv_z \left\{ v_\perp \left[ (f_1 + f_{-1})\vec{e}_x + \vec{e}_y i (f_1 - f_{-1}) \right] + 2 v_z f_0 \vec{e}_z \right\} , \quad (15)$$

where the symbol $\sum_s$ denotes the sum over all the plasma species. To simplify notation this symbol will be omitted in what follows. Once the current is known the dielectric tensor operator $\overleftrightarrow{\varepsilon}$ can be determined from the relation

$$\overleftrightarrow{\varepsilon} = \overleftrightarrow{I} + \frac{4\pi i}{\omega} \overleftrightarrow{\sigma} , \quad (16)$$

where the conductivity tensor operator $\overleftrightarrow{\sigma}$ is given by

$$\vec{j} = \overleftrightarrow{\sigma} \cdot \vec{E} \quad (17)$$

and $\overleftrightarrow{I}$ is the unit tensor.

In order to perform the velocity integration in equation (15) we need to specify the equilibrium distribution function. For practical purposes we choose a Maxwellian

$$F(v, \xi(x)) = N(x) \left( \frac{m}{2\pi T(x)} \right)^{3/2} \exp\left( - \frac{m v^2}{2 T(x)} \right) , \quad (18)$$

where N(x) and T(x) are the density and temperature of species, respectively. We now insert (18) into (8)-(10) and combine these with (12)-(14). On substituting the resulting expressions into (15) we can easily evaluate the integrals over $v_\perp$. In fact, only a few $v_\perp$-moments of the Maxwellian are involved. The integration over $v_z$ , however, is more complicated. First, we have to commute the operatores $L^\pm$ with the denominators $\Omega_n{}^{-1}$ in such a way that various products of the latter can be decomposed in terms of irreducible fractions. Secondly, we transform the derivatives $\partial F/\partial\xi$ into $\partial F/\partial x$. These operations are tedious and lengthy but they allow us, at the same time, to cast the expressions into more compact and symmetric form. When this is achieved, the $v_z$-integration becomes straightforward since all the integrals in question can be represented by a plasma dispersion function and its derivatives. For the sake of brevity we shall use the notation

$$\tilde{Z}_n = \frac{\omega_p^2}{\omega - n\omega_c} Z_n^s \; , \tag{19}$$

$$Z_n = \frac{T}{m} \tilde{Z}_n \; , \tag{20}$$

where

$$Z_n^s \equiv Z^s\left(\frac{\omega - n\omega_c}{|k_z|\, v_T}\right) \tag{21}$$

is the plasma dispersion function as defined by Shafranov [6], $\omega_p$ is the plasma frequency of species and $v_T = (2T/m)^{1/2}$. We can then write the final form of the dielectric tensor operator as follows :

The zero order contribution

$$\overleftrightarrow{\varepsilon}^{(0)} = \overleftrightarrow{\gamma}^{(0)} \equiv \begin{pmatrix} \gamma_{xx}^{(0)} & i\gamma_{xy}^{(0)} & 0 \\ -i\gamma_{xy}^{(0)} & \gamma_{xx}^{(0)} & 0 \\ 0 & 0 & \gamma_{zz}^{(0)} \end{pmatrix} , \tag{22}$$

$$\gamma_{xx}^{(0)} = -\frac{1}{2\omega}\left(\tilde{Z}_1 + \tilde{Z}_{-1}\right) + 1 , \quad \gamma_{xy}^{(0)} = -\frac{1}{2\omega}\left(\tilde{Z}_1 - \tilde{Z}_{-1}\right) ,$$

$$\gamma_{zz}^{(0)} = \frac{2}{(k_z v_T)^2}\left(\omega_p^2 - \omega \tilde{Z}_0\right) + 1 ,$$

which is formally the same as in the case of a uniformly magnetized homogeneous plasma.

The first order contribution

$$\overleftrightarrow{\varepsilon}^{(1)} = \begin{pmatrix} 0 & 0 & i\beta_{xz}\frac{d}{dx} \\ 0 & 0 & -\beta_{yz}\frac{d}{dx} \\ i\frac{d}{dx}\beta_{xz} & \frac{d}{dx}\beta_{yz} & 0 \end{pmatrix} + \overleftrightarrow{\gamma}^{(1)} , \tag{23}$$

$$\beta_{xz} = \frac{1}{2\omega\omega_c k_z}\left[(\omega - \omega_c)\tilde{Z}_1 - (\omega + \omega_c)\tilde{Z}_{-1}\right] ,$$

$$\beta_{YZ} = \frac{1}{2\omega\omega_c k_z}\left[2\omega\tilde{Z}_0 - (\omega-\omega_c)\tilde{Z}_1 - (\omega+\omega_c)\tilde{Z}_{-1}\right],$$

$$\overleftrightarrow{\gamma}^{(1)} \equiv \begin{pmatrix} 0 & 0 & -i\gamma_{xz}^{(1)} \\ 0 & 0 & \gamma_{Yz}^{(1)} \\ i\gamma_{xz}^{(1)} & \gamma_{Yz}^{(1)} & 0 \end{pmatrix}, \tag{24}$$

$$\gamma_{xz}^{(1)} = -k_Y\beta_{Yz}, \quad \gamma_{Yz}^{(1)} = -k_Y\beta_{xz} + \frac{1}{\omega\omega_c k_z}\frac{d}{dx}\left(\omega_p^2 - \omega\tilde{Z}_0\right).$$

The second order contribution

$$\overleftrightarrow{\varepsilon}^{(2)} = \begin{pmatrix} \frac{d}{dx}\alpha_{xx}\frac{d}{dx} & i\left(\frac{d}{dx}\alpha_{xY}\frac{d}{dx} + \beta_{xY}\frac{d}{dx}\right) & 0 \\ i\left(-\frac{d}{dx}\alpha_{xY}\frac{d}{dx} + \frac{d}{dx}\beta_{xY}\right) & \frac{d}{dx}\alpha_{YY}\frac{d}{dx} & 0 \\ 0 & 0 & \frac{d}{dx}\alpha_{zz}\frac{d}{dx} \end{pmatrix} \tag{25}$$

$$+ \overleftrightarrow{\gamma}^{(2)},$$

$$\alpha_{xx} = \frac{1}{2\omega\omega_c^2}\left(Z_2 + Z_{-2} - Z_1 - Z_{-1}\right),$$

$$\alpha_{XY} = \frac{1}{2\omega\omega_c^2}\left(Z_2 - Z_{-2} - 2Z_1 + 2Z_{-1}\right),$$

$$\alpha_{YY} = \frac{1}{2\omega\omega_c^2}\left(Z_2 + Z_{-2} - 3Z_1 - 3Z_{-1} + 4Z_0\right),$$

$$\alpha_{ZZ} = \frac{1}{2\omega\omega_c^2 k_z^2}\left[(\omega-\omega_c)^2\tilde{Z}_1 + (\omega+\omega_c)^2\tilde{Z}_{-1} - 2\omega^2\tilde{Z}_0\right],$$

$$\beta_{XY} = \frac{1}{2\omega\omega_c^2}\left[2k_Y\left(Z_1 + Z_{-1} - 2Z_0\right) + \frac{d}{dx}\left(Z_1 - Z_{-1}\right) + \frac{1}{\omega_c}\frac{d\omega_c}{dx}\left(Z_{-1} - Z_1\right)\right],$$

$$\overleftrightarrow{\gamma}^{(2)} \equiv \begin{pmatrix} \gamma_{xx}^{(2)} & i\gamma_{xy}^{(2)} & 0 \\ -i\gamma_{xy}^{(2)} & \gamma_{YY}^{(2)} & 0 \\ 0 & 0 & \gamma_{zz}^{(2)} \end{pmatrix}, \tag{26}$$

$$\gamma_{xx}^{(2)} = \frac{1}{2\omega\omega_c^2}\left\{-\frac{1}{2}\frac{d^2}{dx^2}\left(Z_1 + Z_{-1}\right) + \frac{1}{\omega_c}\frac{d\omega_c}{dx}\frac{d}{dx}\left(Z_2 + Z_{-2} - \frac{Z_1}{2} - \frac{Z_{-1}}{2}\right)\right.$$
$$+ \frac{d^2\omega_c}{dx^2}\left[\frac{1}{\omega_c}\left(Z_2 + Z_{-2} - Z_1 - Z_{-1}\right) + \frac{d}{d\omega}\left(Z_1 - Z_{-1}\right)\right]$$
$$+ \left(\frac{d\omega_c}{dx}\right)^2\left[\frac{3}{\omega_c}\frac{d}{d\omega}\left(Z_{-1} - Z_1\right) + \frac{4}{\omega_c^2}\left(Z_1 + Z_{-1} - Z_2 - Z_{-2}\right)\right]$$
$$+ k_y\left[\frac{d}{dx}\left(Z_1 - Z_{-1} - Z_2 + Z_{-2}\right) + \frac{d\omega_c}{dx}\left\{2\frac{d}{d\omega}\left(Z_1 + Z_{-1}\right)\right.\right.$$
$$\left. + \frac{1}{\omega_c}\left(4Z_2 - 4Z_{-2} + 5Z_{-1} - 5Z_1\right)\right\} + \frac{\omega_c}{\omega}\left\{\frac{d}{dx}\left(Z_1 + Z_{-1}\right)\right.$$
$$\left.\left.\left. + \frac{d\omega_c}{dx}\frac{d}{d\omega}\left(Z_1 - Z_{-1}\right)\right\}\right]\right\} - k_y^2\alpha_{YY} ,$$

$$\gamma_{XY}^{(2)} = \frac{1}{2\omega\omega_c^2}\Big\{\frac{1}{2}\frac{d^2}{dx^2}\big(Z_{-1}-Z_1\big)+\frac{1}{\omega_c}\frac{d\omega_c}{dx}\frac{d}{dx}\Big(Z_2-Z_{-2}+\frac{Z_{-1}}{2}-\frac{Z_1}{2}\Big)$$

$$+\frac{d^2\omega_c}{dx^2}\Big[\frac{1}{\omega_c}\Big(Z_2-Z_{-2}+\frac{Z_{-1}}{2}-\frac{Z_1}{2}\Big)+\frac{d}{d\omega}\big(Z_1+Z_{-1}\big)\Big]$$

$$+\Big(\frac{d\omega_c}{dx}\Big)^2\Big[\frac{1}{\omega_c^2}\Big(4Z_{-2}-4Z_2+\frac{7}{2}Z_1-\frac{7}{2}Z_{-1}\Big)-\frac{3}{\omega_c}\frac{d}{d\omega}\big(Z_1+Z_{-1}\big)\Big]$$

$$+k_y\Big[\frac{d}{dx}\big(Z_1+Z_{-1}-Z_2-Z_{-2}\big)+2\frac{d\omega_c}{dx}\Big\{\frac{d}{d\omega}\big(Z_1-Z_{-1}\big)$$

$$+\frac{1}{\omega_c}\big(2Z_0-3Z_1-3Z_{-1}+2Z_2+2Z_{-2}\big)\Big\}+\frac{\omega_c}{\omega}\Big\{\frac{d}{dx}\big(Z_1-Z_{-1}\big)$$

$$+\frac{d\omega_c}{dx}\frac{d}{d\omega}\big(Z_1+Z_{-1}\big)\Big\}\Big]\Big]\Big\}-k_y^2\,\alpha_{XY}\qquad,$$

$$\gamma_{YY}^{(2)} = \frac{1}{2\omega\omega_c^2}\Big\{-\frac{3}{2}\frac{d^2}{dx^2}\big(Z_1+Z_{-1}\big)+\frac{1}{\omega_c}\frac{d\omega_c}{dx}\frac{d}{dx}\Big(Z_2+Z_{-2}+\frac{5}{2}Z_1$$

$$+\frac{5}{2}Z_{-1}-4Z_0\Big)+\frac{d^2\omega_c}{dx^2}\Big[\frac{1}{\omega_c}\big(Z_2+Z_{-2}+Z_1+Z_{-1}-4Z_0\big)$$

$$+\frac{d}{d\omega}\big(Z_1-Z_{-1}\big)\Big]+\Big(\frac{d\omega_c}{dx}\Big)^2\Big[\frac{3}{\omega_c}\frac{d}{d\omega}\big(Z_{-1}-Z_1\big)+\frac{4}{\omega_c^2}\big(2Z_0$$

$$-Z_2-Z_{-2}\big)\Big]+2\frac{\omega_c}{\omega}\frac{d}{dx}\Big[\frac{1}{\omega_c}\frac{d}{dx}\Big(\frac{T}{m}\omega_p^2\Big)\Big]+k_y\Big[\frac{d}{dx}\big(Z_{-1}-Z_1$$

$$+Z_{-2}-Z_2\big)+\frac{d\omega_c}{dx}\Big\{2\frac{d}{d\omega}\big(Z_1+Z_{-1}\big)+\frac{1}{\omega_c}\big(4Z_2-4Z_{-2}-3Z_1$$

$$+3Z_{-1}\big)\Big\}+\frac{\omega_c}{\omega}\Big\{\frac{d}{dx}\big(Z_1+Z_{-1}\big)+\frac{d\omega_c}{dx}\frac{d}{d\omega}\big(Z_1-Z_{-1}\big)\Big\}\Big]\Big]\Big\}$$

$$-k_y^2\,\alpha_{XX}\qquad,$$

$$\gamma_{zz}^{(2)} = \frac{1}{2\omega k_z^2}\left\{\frac{\omega}{\omega_c^2}\left[\frac{d^2}{dx^2}\left(\omega_p^2-\omega\tilde{Z}_0\right)+\frac{1}{\omega_c}\frac{d\omega_c}{dx}\frac{d}{dx}\left(\omega\tilde{Z}_0-\omega_p^2\right)\right]\right.$$
$$-k_y\left[\frac{d}{dx}\left\{\frac{1}{\omega_c^2}\left[(\omega-\omega_c)^2\tilde{Z}_1-(\omega+\omega_c)^2\tilde{Z}_{-1}\right]\right\}+2\frac{d}{dx}\left(\frac{1}{\omega_c}\right)\frac{d}{d\omega}\left(\omega^2\tilde{Z}_0\right)\right.$$
$$\left.\left.+2\frac{\omega}{\omega_c}\frac{d}{dx}\tilde{Z}_0\right]\right\}-k_y^2\alpha_{zz} .$$

In order to avoid confusion it should be noted that the operator d/dx in the expressions for $\overleftrightarrow{\beta}$, $\overleftrightarrow{\gamma}^{(1)}$ and $\overleftrightarrow{\gamma}^{(2)}$ operates only on the equilibrium quantities.

Equations (23) through (26) are the main result of this paper. As can be seen from (23)-(26) the dielectric tensor operator is of a Hermitian form, i.e., exhibits certain symmetry properties. On the other hand, it does not satisfy the Onsager reciprocity relation. For instance, $\gamma_{ik}^{(2)}(-B_0) \neq \gamma_{ki}^{(2)}(B_0)$. The reason for this symmetry breaking is the fact that the unperturbed state of the system in question, described by the distribution function F(v,P), is not a state of a thermodynamical equilibrium, but only a steady state.

In the geometry considered the time averaged Poynting theorem may be written in the form

$$\frac{dS}{dx} = -\frac{1}{2}\mathrm{Re}\left(\vec{E}^{*}\cdot\vec{j}\right) = -\frac{\omega}{8\pi}\,\mathrm{Im}\left(\vec{E}^{*}\cdot\overleftrightarrow{\varepsilon}\cdot\vec{E}\right), \tag{27}$$

where Eqs. (16) and (17) have been used. Here S is the time averaged x-component of the Poynting vector given by

$$S = \frac{c}{8\pi} \mathrm{Re}\,(\vec{E}^* \times \vec{\tilde{B}})_x = \frac{c^2}{8\pi\tilde{\omega}} \mathrm{Im}\left[E_y^*\left(\frac{dE_y}{dx} - i k_y E_x\right) + E_z^*\left(\frac{dE_z}{dx} - i k_z E_x\right)\right]. \tag{28}$$

Let us now dispense with the dissipative part of the plasma dispersion functions. On making use of Eqs. (23)-(26) we can then transform Eq. (27) into

$$\frac{d}{dx}(S + S_T) = 0, \tag{29}$$

where

$$S_T = \frac{\omega}{8\tilde{\pi}}\left\{\mathrm{Im}\left[\alpha_{xx} E_x^* \frac{dE_x}{dx} + \alpha_{yy} E_y^* \frac{dE_y}{dx} + E_z^*\left(\alpha_{zz}\frac{dE_z}{dx} + \beta_{yz} E_y\right)\right] + \mathrm{Re}\left[\alpha_{xy}\left(E_x^*\frac{dE_y}{dx} - E_y^*\frac{dE_x}{dx}\right) + \left(\beta_{xy} E_y^* + \beta_{xz} E_z^*\right) E_x\right]\right\} \tag{30}$$

may be identified as an energy flux density due to plasma thermal motion. Equation (29) implies that the total energy flux density $S+S_T$ is constant, a result which was to be expected since $\overleftrightarrow{\varepsilon}$ is Hermitian in the absence of dissipation.

## 4. CONCLUSION

We have derived the linear dielectric tensor operator for a hot nonuniformly magnetized inhomogeneous plasma in a slab geometry. The tensor is valid up to second order in $\rho/L$ and explicitly takes into account the gradients of equilibrium quantities. Its form is Hermitian and therefore suitable for implementation in a numerical code based on a variational formulation.

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