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TOKAMAK PLASMAS

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ABSTRACT

It is demonstrated numerically that a tokamak plasma can be evolved continuously from a near-circular cross-section shape to a 4/1 vertically elongated racetrack. All intermediate stages and the final state are stable to axisymmetric MHD modes. The stabilization is provided by the vacuum vessel walls on the ideal time scale and by an orthogonal active feedback system on the resistive time scale.

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We demonstrate here the feasibility of constructing a tokamak experiment to explore  $\beta_T$  limits in high-current high-elongation plasmas. A limit on  $\beta_T$  (ratio of plasma to toroidal field pressure) in tokamaks scaling as  $I_p/aB_T$ , has previously been proposed and verified<sup>1,2</sup> in ideal-MHD calculations, for plasmas with moderate elongations up to  $\kappa=1.6$ . Here  $I_p$  is the plasma current,  $a$  is the horizontal minor radius,  $B_T$  is the toroidal field. At fixed limiter safety factor  $q_{\psi}$ ,  $I_p$  increases with increasing  $\kappa$  at fixed  $a$  and  $B_T$ , which experimentally has allowed an increase in  $\beta_T$  in quasi-steady state discharges<sup>3</sup>. Highly elongated belt-pinches<sup>4,5,6,7</sup> ( $\kappa \sim 10$ ) have obtained high  $\beta_T$  (10's of %), but are transient (10's to 100's of microseconds). Ideal and resistive MHD studies<sup>8</sup> of an infinite 1-D plasma have indicated that high-elongation configurations may have high  $\beta_T$  limits. In this letter, two key areas are specifically addressed : (1) it is demonstrated that controlled formation and maintenance of highly-elongated tokamak plasmas in a realistic geometry is possible; (2) the ideal and resistive-MHD axisymmetric stability of these plasmas is verified.

The geometry utilized in these studies is shown in Fig. 1. A rectangular vacuum vessel is placed inside two vertical stacks of field shaping coils which form a periodic structure vertically. Limiter points are indicated with an x. The plasma evolution proceeds as follows : a nearly-circular plasma with  $q_{\psi}=2$  is formed near the top of the vessel, with  $R_0=0.80$  m,  $a=0.18$  m,  $B_T=1.5$  T, and  $T_{e0} = 10^3$  eV. The MHD equilibrium and resistive simulations are begun at this point. The plasma current is then increased at the same time as the vertical elongation is increased, to 4/1 on the  $10^{-1}$  sec time-

scale. The currents are programmed so as to maintain  $q_{\Psi} \approx 2$ , in order to keep the current profile as broad as possible.

The ideal MHD equilibrium code TCVMHD<sup>9</sup> is used to calculate the preprogrammed coil currents for input to the time-dependent STARTUP<sup>10</sup> code. TCVMHD computes axisymmetric, free-boundary tokamak equilibria with a predetermined plasma shape, and the coil currents necessary to produce that shape. In order to generate input data for the STARTUP code, we computed several racetrack equilibria for Table I with elongations ranging from  $\kappa = 1.6$  to  $\kappa = 4$ . It was found that, for fixed  $q_{\Psi} = 2$  at the limiter, the plasma current scales with elongation approximately as

$$I_p \approx \frac{1 + \kappa^2}{2}$$

In the control system modeled in the STARTUP code, active stabilization and control is provided by 4 independent feedback systems for radial field (RF), quadrupole field (QF), octopole field (OF), and plasma current (OH). These systems feedback on sums and differences of the flux measurements on the flux loops 1-7 described in Table II.

A given flux measurement is in general an interpolated signal from 2 stationary flux loops. Thus, for example, if  $t_1 < t < t_2$ , then flux measurement number 1 would correspond to

$$\Psi_1(t) = \alpha\Psi(0.62,0.43,t) + (1-\alpha)\Psi(0.62,0.36,t) \quad (1)$$

where  $\alpha = (t_2-t)/(t_2-t_1)$  and  $\Psi(R,Z,t)$  is the poloidal magnetic flux at position  $(R,Z)$  at time  $t$ .

The state vector for the flux loop measurements is denoted by

$$\Psi(t) = (\Psi_1(t), \Psi_2(t), \Psi_3(t), \dots, \Psi_7(t)). \quad (2)$$

The orthogonal vectors corresponding to the 4 control systems are

$$\begin{aligned} \Psi_{RF} &= (0, +1, +1, 0, -1, -1, 0) \\ \Psi_{OH} &= (-1, -1, +1, +1, +1, -1, 0) \\ \Psi_{QF} &= (-2, +1, +1, -2, +1, +1, 0) \\ \Psi_{OF} &= (-1, -1, -1, -1, -1, -1, 6) \end{aligned} \quad (3)$$

Corresponding to each of the orthogonal flux measurement vectors of equation (2) is a vector of current amplitudes  $I(t)$

$$I(t) = (I_1(t), I_2(t), I_3(t), I_4(t), I_5(t), I_6(t), I_7(t)) \quad (4)$$

The current vectors for each of the control systems are determined by selecting coils or groups of coils near the control flux loops at each of the 8 reference times. The inductance matrix  $M(t)$  between these coil groups and the control flux loops is inverted to obtain the control current vectors at each of the 8 times

$$\begin{aligned} I_{RF}(t) &= M_{RF}^{-1}(t) \cdot \Psi_{RF}(t) \\ I_{VF}(t) &= M_{VF}^{-1}(t) \cdot \Psi_{VF}(t) \\ I_{QF}(t) &= M_{QF}^{-1}(t) \cdot \Psi_{QF}(t) \\ I_{OF}(t) &= M_{OF}^{-1}(t) \cdot \Psi_{OF}(t) \end{aligned} \quad (5)$$

Control current vectors at intermediate times are again defined by linear interpolation. For the plasma current control, a "perfect" OH system is modeled in which the poloidal flux everywhere on the computational boundary is increased at the same rate.

In the absence of plasma or additional conductors, the control systems described above are independent in the sense that the flux vector from one current vector is orthogonal to the other flux vectors. This orthogonality property is approximately preserved in the presence of conductors and plasma due to the symmetrical placement of the coils and flux loops. Thus the feedback control voltage for each of the feedback current groups is chosen proportional to the inner product of its flux vector and the flux state vector. For example, the voltage driving the radial field currents is

$$V_{RF}(t) = \gamma \Psi_{RF} \cdot \Psi(t) \quad (6)$$

where  $\gamma$  is a proportionality constant.

The STARTUP code<sup>10</sup> is a free boundary axisymmetric simulation code which models the resistive time-scale evolution of a toroidal plasma, including its interaction with the poloidal field coils and other nearby conductors. Circuit equations for the poloidal field systems are solved simultaneously with the plasma equations, allowing realistic modeling of passive and active feedback systems. The plasma is modeled as a distributed current resistive fluid whose shape and size change dynamically during the evolution to remain in near MHD equilibrium, with a single point of contact with the limiter or magnetic divertor. Flux surface averaged transport equations evolve the two-fluid adiabatic variables, the rotational transform, together with the entropy and number density for each species. The computation is carried out numerically on a background Eulerian grid on which grid points may be one of three types : conductor, vacuum, or plasma. The vacuum is modeled as a cold (here 2.5 eV) pressure-less plasma. The

plasma flux surfaces, including the moving plasma/vacuum interface, are continuously contoured to compute the surface averaged metric quantities. The computational boundary is magnetically transparent, with the plasma contribution to the boundary flux being updated using a multipole expansion of the distributed plasma current.

The disparate time scales in the equations are handled numerically by artificially enhancing the ion mass and viscosity to slow down and damp Alfvén waves, and by substepping the flux-diffusion and fast Alfvén terms. The Alfvén velocity has been reduced by a factor of  $\mu = 4500$  in the runs reported here, and a normalized viscosity coefficient of  $\nu=8.0$  was used. We have verified that these artificially large parameters do not affect the motion of the plasma on the  $L/R$  time scale of the passive coils when realistic values of resistivity and inductance are used.

The vacuum vessel is modeled as a discrete set of 72 conductors spaced 0.05 m apart. The time constant of each conductor is 12 ms, corresponding to a vacuum vessel of thickness 0.025 m with a resistivity of  $10^{-7}$  ohm-m. Independent induced currents develop in each of the 72 conductors, however a constraint is imposed that the sum of the 72 currents always equals zero. This constraint models a gap in the vacuum vessel to allow more rapid penetration of the OH flux. The time constant for response of the feedback systems is also 12 ms.

If the plasma becomes ideal MHD unstable during its evolution, this instability will grow on the modified Alfvén time, which is an order of magnitude faster than the resistive decay time of the conduc-

tors or the time scale over which the preprogrammed or feedback currents are changing. This instability motion is thus readily distinguished from the stable resistive evolution. As a check, we rerun a stable evolution sequence twice, once with the Alfvén velocity reduction factor  $\mu$  set to 3000, and once with the artificial viscosity parameter  $\nu = 12.0$ . These should give results that are indistinguishable from the original run, verifying that plasma inertia is unimportant and thus ideal MHD instabilities are absent.

The computation is initialized to the equilibrium configuration shown in Fig. 2a. The plasma is shown to evolve in a stable manner through the states shown in Figs. 2b through 2d. Examination of the currents required in the feedback systems shows that a maximum feedback current of 20 kA was required to maintain positional control of the plasma during the evolution. The plasma was thus evolved up to the maximum 4/1 elongation with axisymmetric stability found at all times.

We plot the Z-position of the plasma centroid versus time as curve A in Figure 3. Curves B and C show the result for the same calculation repeated with the mass enhancement parameter  $\mu = 3000$  and with the artificial viscosity  $\nu = 12$ . The near identity of these three curves verifies the axisymmetric stability during the calculation. For comparison, we plot additional curves D, E, F in Figure 3 corresponding to rerunning the original calculation with the feedback systems turned off and the vacuum vessel walls removed at times  $t_1$ ,  $t_3$  and  $t_6$ . (Table I.) In these cases, axisymmetric instability on the ideal time scale is evident.



To demonstrate the separation of time scales in the simulation, we plot in Fig. 4 simulation results for  $.154 \text{ s} < t < .163 \text{ s}$  of the Z-position of the magnetic axis vs time for the standard case (feedback and walls), an ideal MHD unstable case with the conducting walls removed at  $t = .154 \text{ s}$ , and a resistive unstable case with the active feedback system shut off at  $t = .154 \text{ s}$ . The disparate time scales are evident.

In conclusion, we have found and simulated a method for obtaining highly elongated tokamak plasmas on the  $10^{-1}$  sec time scale by using a combination of preprogrammed currents, passive conductors, and an orthogonal feedback system. The equilibria all have  $q_{\psi} \approx 2$  at the limiter and broad, but not hollow, current profiles consistent with the MHD evolution equations assuming classical Spitzer resistivity. The continuously-evolved equilibria between a near-circular plasma and a 4/1 elongated racetrack plasma are all stable both to ideal and resistive axisymmetric MHD modes.

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TABLE I. Preprogrammed currents (kA) in each of the 16 EF coils, plasma current and elongation at 8 reference times. Linear interpolation is used to define currents at intermediate times.

	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$
$I_1$	162	164	39	-86	-110	-134	-133	-104
$I_2$	0	-112	-78	-80	-106	-127	-119	-96
$I_3$	-40	-66	-53	-58	-60	-71	-68	-48
$I_4$	174	218	232	224	228	199	220	240
$I_5$	123	91	79	71	96	99	96	105
$I_6$	-148	-107	-88	-93	-101	-94	-103	-108
$I_7$	-140	-254	-198	-153	-151	-167	-168	-154
$I_8$	88	80	-41	-173	-207	-174	-198	-203
$I_9$	0	0	-9	-4	-166	-251	-234	-203
$I_{10}$	0	0	65	74	90	72	-58	-154
$I_{11}$	0	0	0	0	0	0	-61	-108
$I_{12}$	0	0	0	0	0	0	132	105
$I_{13}$	0	0	0	0	0	0	133	240
$I_{14}$	0	0	0	0	0	0	-28	-48
$I_{15}$	0	0	131	54	176	72	-39	-96
$I_{16}$	0	0	49	140	-78	-131	-127.4	-104
$I_p$	332	541	584	763	1069	1174	1425	1700
$\kappa$	1.6	2.0	2.25	2.5	3.0	3.25	3.62	4.0

TABLE II. Z-position of control flux loops used at 8 reference times. At intermediate times, control flux is interpolated using loop positions in table. Flux loops 1, 2, 6 have  $R = 0.62$  m, flux loops 3, 4, 5 have  $R = 0.98$  m, while flux loop 7 has  $R = 0.8$  m,  $Z = 0.72$  m.

Loop #	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$
2 and 3	.513	.440	.480	.500	.550	.560	.560	.560
1 and 4	.430	.360	.315	.250	.180	.135	.070	.000
5 and 6	.353	.280	.150	.000	-.19	-.29	-.42	-.56

FIGURE CAPTIONS

1. Computational grid, coil, vessel and flux loop geometry.
2. Flux surface geometry for a stable evolution sequence at  
(a)  $t = t_1 = 0$ , (b)  $t = t_3 = 0.075$  s, (c)  $t = t_5 = 0.137$  s, and  
(d)  $t = t_8 = 0.210$  s.
3. Simulation results of Z-position of magnetic axis vs time for physical sequence with mass enhancement and viscosity parameters  
(a)  $\mu = 4500$ ,  $\nu = 8.0$ ; (b)  $\mu = 3000$ ,  $\nu = 8.0$ ; (c)  $\mu = 4500$ ,  
 $\nu = 12.0$ . Curves (d), (e) and (f) illustrate ideal MHD instability when passive conducting walls are removed.
4. Simulation results of Z-position of magnetic axis vs time for different conditions illustrating time scales of stable formation, resistive instability and ideal instability.







