# CONSISTENT AND NON-CONSISTENT QUASILINEAR MODELS OF LOWER-HYBRID CURRENT DRIVE

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## Abstract

Some theoretical aspects related to the problem of current generation by lower-hybrid waves are discussed. In particular, the influence of the spectral distribution of the rf power source and the effects of the anomalous Doppler interaction on the current generation are investigated.

## Introduction

Although several experiments have demonstrated the viability of lower-hybrid current drive [1] there are virtually no theoretical models that would satisfactorily explain most of the observed features [2]. The reason for this situation may be the fact that existing models are limited by many simplifying assumptions. It is therefore desirable to reconsider some theoretical aspects of the problem which were previously ignored or thought to be unimportant.

In this paper we shall discuss the two following questions: 1) How does the rf-generated current depend on the shape of the power source spectrum ? 2) What are the microscopic stability properties of the current carrying electron distribution function ? The first question may be related to the experimental finding [1] that the injected lower-hybrid waves have seemingly too high phase velocities in order to interact with a significant number of

electrons, and yet the current generated is rather high. The second question arises if one looks for an explanation of the origin of relaxation oscillations which occur under certain conditions in all lower-hybrid current- drive experiments [1]. We would like to point out that the problem posed has to be addressed within a quasilinear model that takes into account the self-consistent evolution of the wave spectrum due to the influence of the power source, the Landau damping, the collisional damping and possibly some instability mechanisms.

## Basic Equations

We consider a magnetized, homogeneous plasma interacting with lower-hybrid waves (magnetized plasma waves) that have parallel phase velocities higher than electron thermal velocity and perpendicular wavelengths much larger than the electron Larmor radius. The evolution of the electron distribution function can then be described by the following equation

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{Cl} + \left(\frac{\partial f}{\partial t}\right)_{Cr} + \left(\frac{\partial f}{\partial t}\right)_{Dp} + \left(\frac{\partial f}{\partial t}\right)_{Ls} , \tag{1}$$

where the term

$$\left(\frac{\partial f}{\partial t}\right)_{Cl} = \frac{\nu}{2}\left\{\left(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}v_\perp^2 + \frac{\partial}{\partial v_\parallel}v_\parallel\right)\frac{1}{v^3}\left[2+\left(\frac{2}{v^2}-1-Z\right)\left(v_\perp\frac{\partial}{\partial v_\perp}\right.\right.\right.$$
$$\left.\left.+ v_\parallel\frac{\partial}{\partial v_\parallel}\right)\right] + (1+Z)\left(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\frac{v_\perp^2}{v}\frac{\partial}{\partial v_\perp} + \frac{\partial}{\partial v_\parallel}\frac{1}{v}\frac{\partial}{\partial v_\parallel}\right)\right\}f , \tag{2}$$

$$\nu = \frac{\ln\Lambda\,\omega_{pe}^3}{4\widetilde{\chi}\,n\,v_{te}^3} \tag{3}$$

represents the electron-electron and electron-ion collisional effects and Z denotes the ion charge state. We note that all the quantities in equations (1)-(3) and throughout the paper are normalized according to $k \to k\lambda_D$, $v \to v v_{te}$, $t \to t/\omega_{pe}$, $f \to fn/v_{te}^3$, and $W \to W4\pi nT_e\lambda_D^3$. The resonant Čerenkov interaction between the electrons and waves results in parallel velocity diffusion which is described by the quasilinear term .

$$\left(\frac{\partial f}{\partial t}\right)_{cr} = \frac{\partial}{\partial v_{\parallel}} D_0 \frac{\partial f}{\partial v_{\parallel}} , \tag{4}$$

$$D_0 = \pi \int \frac{d^3 k}{(2\pi)^3} \left(\frac{k_{\parallel}}{k}\right)^2 W_k \, \delta\left(\frac{k_{\parallel}}{k} - k_{\parallel} v_{\parallel}\right), \tag{5}$$

where $W_k$ is the spectral distribution of the waves whose dispersion relation is assumed to be $\omega = k_{\parallel}/k = \cos\theta$, $\theta$ being the angle between the magnetic field and the wave vector $\vec{k}$.

If the electron distribution function develops a sufficiently long tail in $v_{\parallel}$-space the fast electrons in the tail may interact with the lower-hybrid waves via the anomalous Doppler resonance $\omega_{ce} = k_{\parallel} v_{\parallel}$, where $\omega_{ce}$ is the electron cyclotron frequency. The resulting pitch-angle scattering of the electrons caused by the waves is described by the quasilinear term due to the anomalous Doppler interaction

$$\left(\frac{\partial f}{\partial t}\right)_{Dp} = \left(\frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}}\frac{\partial}{\partial v_{\perp}}\right) D_1 \left(\frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}}\frac{\partial}{\partial v_{\perp}}\right) f , \tag{6}$$

$$D_1 = \pi \int \frac{d^3 k}{(2\pi)^3} \left(\frac{k_{\parallel}}{k}\right)^2 \left(\frac{k_{\perp} v_{\perp}}{2\,\omega_{ce}}\right)^2 W_k \, \delta\left(\omega_{ce} - k_{\parallel} v_{\parallel}\right) . \tag{7}$$

The last term in equation (1) can represent any momentum or energy loss effects. For reasons which will become apparent later, we wish to simulate the loss of perpendicular energy resulting from the cyclotron radiation. Thus we choose

$$\left(\frac{\partial f}{\partial t}\right)_{Ls} = \nu_{eff} \frac{1}{v_{\perp}}\frac{\partial}{\partial v_{\perp}} \left(\frac{v_{\perp}^2}{2} - 1\right) f , \tag{8}$$

where $\nu_{eff}$ is an effective loss rate.

The wave spectrum evolves according to

$$\frac{\partial W_k}{\partial t} = 2\left(\gamma_0 + \gamma_1 - \frac{Z\nu}{4}\right)W_k + S_k , \tag{9}$$

where

$$\gamma_0 = \frac{\tilde{\pi}}{2} \frac{k_\parallel^2}{k^3} \int d^3v \, \frac{\partial f}{\partial v_\parallel} \delta\left(\frac{k_\parallel}{k} - k_\parallel v_\parallel\right) \tag{10}$$

and

$$\gamma_1 = \frac{\tilde{\pi}}{8} \frac{k_\parallel^2}{k^3} \int d^3v \left(\frac{k_\perp v_\perp}{\omega_{ce}}\right)^2 \left(\frac{\partial f}{\partial v_\parallel} - \frac{v_\parallel}{v_\perp} \frac{\partial f}{\partial v_\perp}\right) \delta(\omega_{ce} - k_\parallel v_\parallel) \tag{11}$$

are the damping or growth rates corresponding to the Čerenkov and anomalous Doppler interactions, respectively. The $S_k$ represents the spectrum of an external power source that drives the waves.

The equations (1) and (9) were solved numerically using the finite-element method with the additional ansatz:
$f = F(v_\parallel)\exp[-v_\perp^2/2T(v_\parallel)]/2\pi T(v_\parallel)$. The details of the procedure can be found elsewhere [3]. For a given source spectrum, the distribution function (initially Maxwellian) and the wave spectrum (initially the thermal noise) were advanced over a relevant time interval. The total power input and the electron current density were calculated according to

$$P = 2 \int \frac{d^3k}{(2\tilde{\pi})^3} S_k , \tag{12}$$

$$j = \int v_\parallel F(v_\parallel) \, dv_\parallel . \tag{13}$$

Before presenting the results of our calculations we would like to make a few comments. In previous (non-consistent) models [4] only the first two

terms in equation (1) were considered and the diffusion coefficient, $D_0$, in equation (4) was assumed to be a given quantity irrespective of the magnitude of the power source. Such an approach is justified in a regime where j saturates, i.e., becomes independent of the magnitude of $D_0$, since a complete plateau is established on the electron distribution. Then, of course, the detailed shape of $D_0$ is irrelevant. However, the power used in the present day experiments is not sufficient, from the theoretical point of view, to establish a complete plateau even in the cases where a significant current is generated. Thus, the diffusion coefficient $D_0$, i.e., the wave spectrum, should be determined self-consistently together with the distribution function for given strength and shape of the external power source. The present formulation of the problem permits us to accomplish this goal in a natural way.

If, for the reasons already stated, the anomalous Doppler interaction becomes operative then the self-consistent treatment is anyhow mandatory since the same waves may interact simultaneously with two different groups of resonant electrons. Consequently, two different diffusion coefficients have to be determined as functions of the wave spectrum which, in turn, evolves simultaneously with the electron distribution function.

## Influence of the Spectral Distribution of the rf Source on Current Generation

In order to investigate the dependence of the rf-generated current on the shape of the power source spectrum we use the following model

$$S_k = S(k)\, \delta(\cos\theta - \cos\theta_0),$$

$$S(k) = \begin{cases} S, & k_2 < k < k_m \\ \sigma S, & k_m < k < k_1 \\ 0, & \text{elsewhere} \end{cases} \tag{14}$$

where S and $\sigma$ are constants, and $\sigma \ll 1$. We note that due to the Čerenkov resonance condition $\omega = k_\| v_\|$ there is a one-to-one correspondence between k and $v_\|$: $(v_\|)_1 \equiv v_1 = 1/k_1$ etc.

In this section we confine ourselves to the cases where $\omega_{ce}$ is sufficiently high so that the anomalous Doppler interaction cannot be operative. Moreover, we omit also the loss term, which we believe to be

unimportant in the absence of the anomalous Doppler interaction. Thus, we ignore the last two terms in equation (1) and $\gamma_1$-term in equation (9) and search for a steady-state solution.

We have performed a series of computations with different values of the parameters S, $\sigma$, $v_1$, and Z while the value of $\cos\theta_0$ was fixed at 0.7 for convenience. (This choice is immaterial unless the waves are nearly perpendicular.) The values of S and $\sigma$ were varied in such a manner that the total power was constant. The results presented below were obtained for typical PLT (Princeton Large Torus) parameters [5]: n = $5\times10^{12}$ cm$^{-3}$, $T_e$ = 1 keV, R = 1.3 m, a = 15 cm, P = 130 kW, $v_m$ = 7.5, and $v_2$ = 16. Figure 1 shows the dimensionless current density, j, as a function of $\sigma$ for three different values of $v_1$, and Z = 1. For reference, the dashed line indicates a total current of 100 kA. One can see that a tail of the power source spectrum with $\sigma$ = 0.05 is able to generate a total current of about 200 kA for $v_1$ = 3. This value is fairly close to that observed in the experiment [5]. Moreover, we observe that the values of the current density for $v_1$ = 2 are smaller than those for $v_1$ = 3 and $v_1$ = 4. This fact indicates that, at the power level considered, Landau damping is still efficient in preventing the formation of a complete plateau on the electron distribution function. A further remarkable feature is the saturation of j as $\sigma$ is increased over a certain value, typically 0.1. This is due to the fact that an increasing fraction of the total power is transferred into the tail of the spectrum at the expense of its main component. As a result, a partial destruction of the plateau at high velocities occurs. Figure 2 shows the non-Maxwellian part of the electron distribution function, $F-F_M$, for the case $\sigma$ = 0.05, $v_1$ = 3.5, Z = 1. Curve 2 was obtained using a simplified, one-dimensional collision operator [3] whereas for curve 1 the collision operator given in Eq. (2) was retained. The one-dimensional operator can be derived from the two-dimensional one in the limit $v_\parallel^2 \gg T$. From Fig. 2 one can see that in the 2-D case the distortion of the distribution function is larger than in the 1-D case, as one would expect. Furthermore, the distribution function exhibits two plateaus separated by a smooth transition region, which demonstrates that the competition between collisional and Landau damping is not yet "over" at the power level considered.

Thus, we have shown that the inclusion of a small tail at the high-$n_\parallel$ side of a power spectrum in the modelling of lower-hybrid current-drive experiments can provide an explanation for the interaction between the launched waves and the low-velocity electrons. We note that the results obtained are

not affected by the anomalous Doppler interaction if $\omega_{ce} > 3$, which is the case in many experiments.

## Role of Anomalous Doppler Interaction

For moderate values of $\omega_{ce}$ the anomalous Doppler interaction can become active in the cases studied in the preceeding section. Its influence on the evolution of the system can appear in two manners. First, the driven waves with the phase velocities close to $v_1$ can interact with the fast electrons in the tail of the distribution function via the anomalous Doppler resonance. The intensity of these waves, however, is smaller or approximately equal to the intensity of those waves that interact with the fast electrons via the Čerenkov resonance. Since the diffusion coefficient due to the anomalous Doppler interaction is by the factor $(k_{\perp}v_{\perp}/2\omega_{ce})^2$ smaller than that due to the Čerenkov interaction the effects of the anomalous Doppler interaction are negligible. Second, the electron distribution can be a flat function of $v_{\parallel}$ for $v_{\parallel} > v_1$ with electrons that have more parallel energy than perpendicular. It is well-known /6/ that this anisotropy can destabilize magnetized plasma waves that have phase velocity $\omega/k_{\parallel} = 1/k > v_1$ through the anomalous Doppler resonance. In the quasilinear stage of his instability /3/ the fast electrons are pitch-angle scattered. The associated loss of their parallel momentum could then result in a reduction of the generated current. At the same time, however, the intensity of the unstable waves with the phase velocities close to $v_1$ is raised. As a result, a part of electrons from the bulk of the distribution function is diffused via the Čerenkov effect. Thus, the generated current would tend to increase. The net result of these competing processes can be very complicated and depends, of course, on the particular values of the parameters that govern the evolution of the system. Below we shall illustrate the behaviour of the system in three qualitatively different cases.

In the first study we considered the case where, in the absence of the anomalous Doppler interaction, the distribution function develops a complete plateau. (See curve 2 in Fig. 3). Assuming $\omega_{ce} = 2$ one can show by means of equations (3) and (4) that this distribution is unstable. We followed the evolution of the system once again but with the anomalous Doppler interaction "switched on". One sees from curve 1 in Fig. 6 that the total wave energy exhibits a small burst during the quasilinear stage of the instability and then rather rapidly saturates at a relatively high value. Due to this, a considerable number of electrons are diffused from the bulk to the region of the

plateau as can be seen from curve 1 in Fig. 3. It is important to note that the current carried by this distribution is approximately the same as that carried by the distribution labelled 2. Thus, the net result of the instability is simply a redistribution of the current carriers in $v_\|$-space as evidenced by the disappearance of the high energy tail.

Next, we investigated the case where the "unperturbed" distribution function exhibits two plateaus separated by a transition region with a strongly negative slop (see curve 2 in Fig. 4). For $\omega_{ce} = 2$ this distribution is obviously less unstable than that considered above. Moreover, the waves which have their Čerenkov resonances in the transition region cannot be affected by the anomalous Doppler interaction since the corresponding resonances lie in the region of the velocity space where there are no electrons. Therefore, we assume $\omega_{ce} = 1.5$ in order to make the case more interesting. We have then two groups of unstable waves. The one which has the Čerenkov resonances in the plateau region close to the bulk and the other which has these resonances in the transition region with the negative slope. The former appears to be much more unstable than the latter. The temporal evolution of the total wave energy is represented by curve 2 in Fig. 6. We observe that the wave energy exhibits roughly two bursts. This can be interpreted tentatively as follows. As the tail of the distribution function grows due to the action of the source the first group of waves is destabilized. Since there are not yet many electrons in the tail, the resulting wave energy is rather small and the instability is rapidly quenched. Thus, the anomalous Doppler interaction is unable to stop a further growth of the tail. Once there is enough electrons in the tail to destabilize even the weakly unstable waves the wave energy starts to grow again. Since this is a slower process the second burst lasts much longer than the first one. Even so the wave energy during the burst is very high the anomalous Doppler interaction is unable to destroy the second plateau as evidenced by curve 1 in Fig. 4. The wave energy is spent to extend the transition region and to raise the level of the first plateau. Once again the currents carried by both distributions are approximately the same.

Finally, we followed the evolution of the system for the same case as above but with the energy-loss term (8) included. The effective loss rate was assumed to be $\nu_{eff} = 10^{-1}\nu$. The first stage of the evolution is not essentially changed due to this mechanism. During the second stage, however, the destabilizing effect of the loss term (increase in the energy anisotropy) becomes apparent. The wave energy exhibits rather early two relatively short

bursts and then rapidly saturates (see curve 3 in Fig. 6). As can be seen from the comparison of curves 2 in Figs. 4 and 5 the level of the wave energy is not high enough to modify the region of the first plateau. This indicates that the saturation of the instability in this case is caused by the loss term itself. Indeed, this term diminishes not only the perpendicular energy (destabilizing effect) but via the anomalous Doppler interaction also the parallel energy. Thus, at the end there is no free energy at all to support the instability as evidenced by curve 2 in Fig.5. The current carried by this distribution is smaller by about a factor of three than that in the previous case.

We would like to point out that the three particular cases discussed above by no means exhaust the complexity of physical situations involving the interplay between the action of the external source, the anomalous Doppler interaction and possibly an energy loss mechanism. In order to gain a more complete understanding of the underlying physics a more systematic investigation is required.

## Acknowledgements

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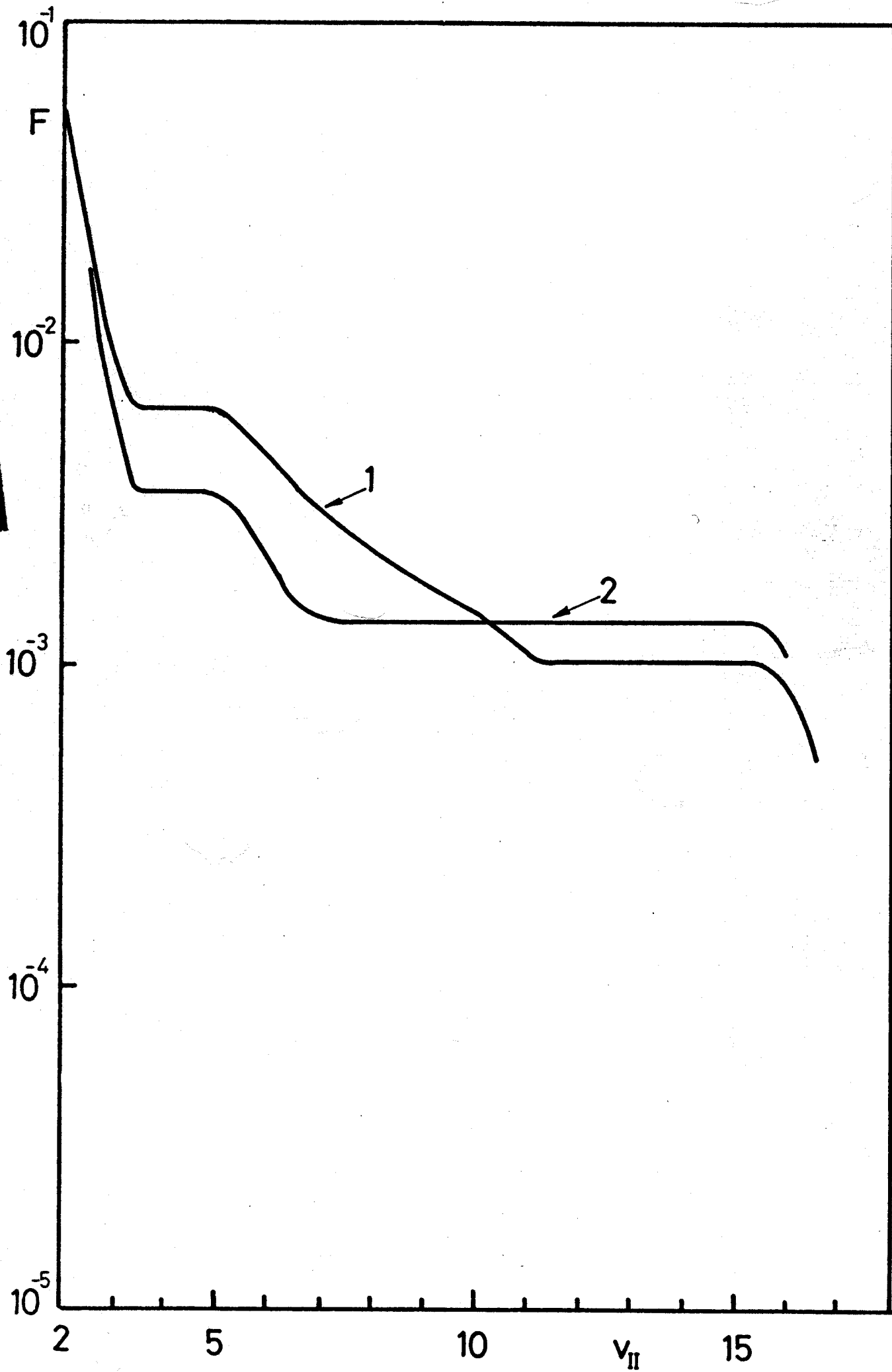/6/ KADOMTSEV, B.B., and POGUTSE, O.P., Sov. Phys. JETP, 26 (1968), 1146.
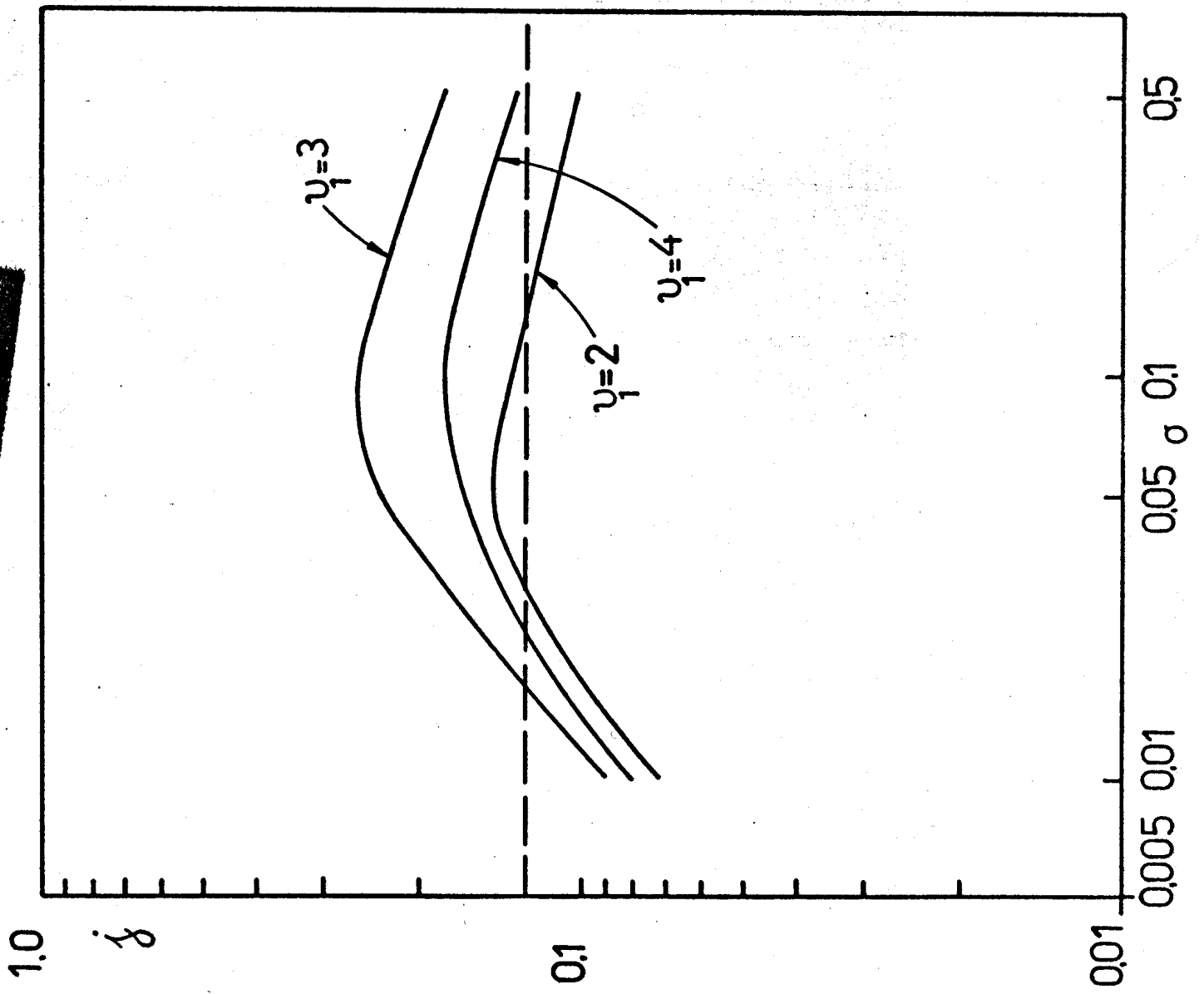
FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6