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OPEN RESONATOR FOR QUASI-OPTICAL GYROTRONS:

STRUCTURE OF THE MODES AND THEIR INFLUENCE

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ABSTRACT

The influence of the resonator geometry on the design of quasi-optical gyrotrons is presented. Basic equations and relevant properties of quasi-optical resonators are reviewed. Since confocal resonators radiate an equal amount of power from each mirror, non-confocal resonators have to be used in order to maximize the output-coupling on one side. Technical constraints on ohmic power losses and mirror diameter are discussed. Using numerical solutions of the quasi-optical resonator equations, the influence of the actual electromagnetic field profile on the electronic efficiency is calculated for several cases.

1. Introduction

The quasi-optical resonator [1] holds interesting potential advantages for high frequency, high power gyrotrons. Efficiencies up to 40% have been calculated [2]. The peak thermal loading on the mirrors at optimum values of the electric fields is below 2 kW/cm^2, a value which is within the present technical cooling capabilities. In a high power, long pulse or CW quasi-optical gyrotron, the resonator will operate at high longitudinal mode numbers: the frequency spacing between neighbouring modes is very small, thus ensuring the tunability of the device. The problem of longitudinal mode competition has been addressed by Bondeson et al. [2] who have shown that single mode operation can be achieved.

Within the framework of the gyrotron development programme for the heating of fusion plasma, the Centre de Recherches en Physique des Plasmas, in collaboration with the Laboratoire d'Electromagnétisme et d'Acoustique of the Ecole Polytechnique Fédérale de Lausanne and an industrial firm, BBC, Société Anonyme Brown, Boveri & Cie, Baden, is developing a quasi-optical gyrotron and gyroklystron operating at 120 GHz and a power level of 200 kW. The pulse length will be long enough so that the time scale required to reach the single mode, typically around 100 μs to 1 ms [2], is achieved. The design of the gyrotron and its ancilliary systems will allow a pulse length up to 100 ms.

As for all gyrotrons, the optimization of the efficiency of the interaction between the wave and the electron beam is of crucial importance. In the previous calculations [1,2], the electric field profile is assumed to be gaussian, which corresponds to a resonator with mirrors of very large radii. However, in pratice, the mirrors will have finite size and consequently the field distribution may depart from a gaussian profile. Assuming a plane wave in the interaction region may no longer be true. Another important problem is the output coupling of the electromagnetic wave energy. Coupling through holes or annular apertures [3] in the mirror of a non-confocal resonator has been implemented. Diffraction losses due to finite size mirrors can also be used provided some focusing elements are included. In both cases, the field profiles at the mirrors are modified and the power transmission coefficient as well as the field profile at the beam waist are affected by the output coupling structure.

The prerequisite for the design of the quasi-optical resonator is a thorough understanding of its properties (e.g. the one and two-way power transmission coefficient, the complex field profile in the resonator). This knowledge and the engineering considerations will lead to the selection of the parameters of the resonator and then to the computation of the efficiency η_e of the energy transfer between the wave and the electron beam. We wish to report here on these various aspects of the problems which are encountered by any designer.

The paper is organized as follows. The basic equations that are used to compute the electronic efficiency η_e of a quasi-optical gyrotron are presented in a self-consistent way. The electromagnetic field which governs the electron orbits is computed from the current density j_x due to the electron motion [2] and the formalism also takes into account the reflecting boundary conditions due to the presence of the quasi-optical resonators (Section 3). Numerical analyses of their relevant properties and design criteria will also be discussed. Finally, in section 4, we shall illustrate the effect of a real mirror geometry on the electronic efficiency.

2. Basic Equations for the Quasi-Optical Gyrotron

The quasi-optical gyrotron configuration is sketched in Fig. 1. By assuming a linear polarization for the electric field in the x-direction E_x, the electron guiding-centre motion can be described by the following complex first order differential equation:

$$\frac{d\mathcal{P}}{dz} = \frac{ik}{\gamma\beta_\parallel}\left(\frac{\Omega}{\omega} - \gamma\right)\mathcal{P} - \frac{e\gamma E_x}{2\gamma\beta_\parallel mc^2} \tag{1}$$

for the complex perpendicular momentum,

$$\mathcal{P} = \gamma\beta_\perp e^{i\theta} \tag{2}$$

where $\gamma = \sqrt{1 + (\gamma\beta_\perp)^2 + (\gamma\beta_\parallel)^2}$ is the relativistic factor, ω, k are the wave frequency and wave number of the R.F. field, Ω is the non-relativistic electron cyclotron frequency, and θ is the particle momentum space angle. The equation (1) is straightforwardly derived from orbit equations given in Ref. [1] in the case of the uniform external magnetic field (Ω/ω = const.). The parallel particle momentum $\gamma\beta_\parallel$ is thus a constant of motion. By neglecting space charge effects in the electron beam ($\vec{\nabla}\cdot\vec{E} = 0$), the electric field E_x can be calculated from Maxwell's equations:

$$(\nabla^2 + k^2) E_x = i\omega\mu_0 j_x \tag{3}$$

where the current density j_x is calculated by solving Eq. (1) for an ensemble of electrons. For a "pencil" beam, j_x is given by

$$j_x = - I_b < \frac{\beta_x}{\beta_{//}} > \delta(x)\, \delta(y) \tag{4}$$

where I_b is the beam current intensity and the bracket <...> denotes the ensemble average.

The efficiency of converting the electron beam power to R.F. power is equal to

$$\eta_e = \frac{< \gamma_{out} - \gamma_{in} >}{< \gamma_{in} - 1 >} \tag{5}$$

where γ_{in} and γ_{out} are the relativistic factors for the electrons flowing into and out of the open resonator. In principle, by solving Eq. (1) for a collection of particles coupled to the wave equation (3) with appropriate boundary conditions, we readily obtain the efficiency by performing an ensemble average over the particles. However, solving directly the partial differential equation (3), with open boundary conditions (since the resonator is open), is not an obvious task. The problem becomes more tractable by using an approximated Green-function formulation [4] which leads to a formal solution of Eq. (3):

$$E_x(\vec{r}) = -\frac{i\omega\mu_0}{4\pi} \int_V \frac{j_x(\vec{r}\,') e^{-ikR}}{R} d^3r' + \frac{ik}{2\pi} \int_S \frac{e^{-ikR}}{R} E_x(\vec{r}\,') d^2r' \tag{6}$$

where $\vec{R} = \vec{r} - \vec{r}\,'$

3. Properties of the Open Resonator

3.1 Outline of theory

The modes in an empty resonator are determined by solving the two homogeneous integral equations obtained by expressing Eq. (6) without the source term ($j_x = 0$) for the two mirrors which represent the resonator boundary surface. When the transverse cross-section of the radiation beam with respect to the resonator axis and the distance between the mirrors are much longer than the wavelength, the homogeneous equations reduce to the Huygens-Fresnel equations which can be written in the form:

$$\begin{cases} u_1(\vec{r}_1) = i\, e^{-ikd} \displaystyle\int_{S_2} K(\vec{r}_1, \vec{r}_2)\, u_2(\vec{r}_2)\, d^2 r_2 & \text{(7a)} \\[2ex] u_2(\vec{r}_2) = i\, e^{-ikd} \displaystyle\int_{S_1} K(\vec{r}_1, \vec{r}_2)\, u_1(\vec{r}_1)\, d^2 r_1 & \text{(7b)} \end{cases}$$

where $$K(\vec{r}_1, \vec{r}_2) = \frac{1}{2\pi} \exp\left[-\frac{i}{2}\left(g_1 r_1^2 + g_2 r_2^2 \right) + i\, \vec{r}_1 \cdot \vec{r}_2 \right]$$

$$g_j = 1 - \frac{d}{R_j} \qquad , \; (j = 1, 2)$$

The geometrical quantities are defined in Fig. 2. The 2-component vector $\vec{r}_j$ denotes the coordinates of mirror j, normalized to $r_o = \sqrt{d/k}$, d is the mirror separation, R_j the radius of curvature of mirror j, and S_j the surface of mirror j. In our notation, the wave number is a complex quantity $k = k_r + ik_i$; where $k_r = 2\pi/\lambda$, λ is the wavelength and k_i accounts for the diffraction loss.

Since the pioneering work of Fox and Li [5], the open resonator equations (7) have been extensively studied over the past 25 years. Various mirror geometries have been proposed and it is not our purpose to give a full account of the theory here. Comprehensive reviews and extended reference lists can be found in [6], [7] and [8].

For a circular mirror of radius a_{jm}, the Fresnel number is defined as

$$N_j = \frac{a_{jm}^2}{\lambda d} \qquad , (j = 1, 2) \tag{8a}$$

The resonator Fresnel number is the geometrical mean of N_1 and N_2:

$$N_m = \sqrt{N_1 N_2} = \frac{a_{1m}\, a_{2m}}{\lambda d} \tag{8b}$$

The kernel in Eqs. (7) does not depend explicitely on kd. For a given resonator geometry, the eigenmodes are obtained by reducing the Eqs. (7) to an eigenvalue problem where the eigenvalues $\gamma^2 = (-ie^{ikd})^2$ are associated with the eigenfunctions (u_1, u_2). We impose the normalization condition

$$\int_{S_1} |u_1|^2 \, d^2 r_1 + \int_{S_2} |u_2|^2 \, d^2 r_2 = 1 \tag{9}$$

The eigenfunctions depend on two indices, l,p (mode TEM_{lp}). The field profile u(r) inside the resonator (for instance at the electron beam) can be calculated by using again the Huygens principle. For a given mode the electric field of the standing wave can be written as

$$E_x(\vec{r}) = E_o \, u(\vec{r}) = E_o \left[u_+(\vec{r}) + u_-(\vec{r}) \right] \tag{10}$$

where u and u denote the waves coming from mirror 1 and 2 respectively. From the argument of γ^2 we get the resonant wavelengths

$$\frac{4\pi d}{\lambda} = 2 k_r d = \mathrm{Arg}(\gamma^2) + \pi(2q+1) \tag{11}$$

where q is the longitudinal mode number. The full notation for a mode is thus TEM_{lpq}.

In the remaining part of this section we assume that only one mode is excited in the cavity so that we can easily relate E_0 and $|\gamma|^2$ to measurable quantities.

We define the transmission through mirror j by

$$T_j = 1 - \frac{\int_{S_1} |u_j|^2 \, d^2 r_j}{\int_{\mathbb{R}^2} |u_j|^2 \, d^2 r_j} \quad , (j = 1,2) \tag{12}$$

The conservation of energy requires that

$$\begin{cases} |\gamma|^2 \int_{\mathbb{R}^2} |u_1|^2 d^2r_1 = \int_{S_2} |u_2|^2 d^2r_2 \\ |\gamma|^2 \int_{\mathbb{R}^2} |u_2|^2 d^2r_2 = \int_{S_1} |u_1|^2 d^2r_1 \end{cases} \tag{13}$$

Therefore,

$$\begin{cases} T_1 = 1 - |\gamma|^2 \cdot I \\ T_2 = 1 - |\gamma|^2 / I \end{cases} \tag{14}$$

where $I = \left[\int_{S_1} |u_1|^2 d^2r_1 \right] \Big/ \left[\int_{S_2} |u_2|^2 d^2r_2 \right.$

T_1 and T_2 are called one-way losses. The two-way loss T is the fractional loss during a full round trip of the wave.

$$T = T_1 + (1 - T_1) T_2 = 1 - |\gamma|^4 \tag{15}$$

When losses are small, T becomes equal to the total transmission $T_1 + T_2$.

The power radiated around mirror j is given by

$$P_j = \int_{\mathbb{R}^2 - S_j} \frac{1}{2\mu_0} \left| Re(\vec{E} \times \vec{B}^*) \right| d^2r_j \quad , (j = 1,2) \tag{16}$$

where B = E/c.

With our normalization, the total power radiated outside the resonator can readily be expressed as

$$P = P_1 + P_2 = \frac{r_o^2}{2Z_o} |E_o|^2 \left(\frac{1}{|\gamma|^2} - 1 \right) = \frac{r_o^2}{2Z_o} |E_o|^2 \frac{T}{|\gamma|^2 (1 + |\gamma|^2)} \qquad (17)$$

($Z_o = \mu_o c = 377\ \Omega$).

For small losses, $|\gamma|^2$ is close to 1 and P tends to $Tr_o^2 |E_o|^2/(4Z_o)$. The losses due to ohmic dissipation in the mirrors are given by

$$L_j = \int_{S_j} \frac{2}{Z_o^2 \sigma \delta} |E(r_j)|^2 d^2 r_j \qquad (18)$$

where σ is the conductivity ($5.80 \cdot 10^7\ \Omega^{-1} m^{-1}$ for copper) and δ is the skin depth. ($\delta = (\omega\mu_o\sigma/2)^{-1/2}$, $\omega/2\pi$ is the frequency). With our normalization, the total ohmic losses are

$$L = L_1 + L_2 = \frac{2 r_o^2}{Z_o^2 \sigma \delta} |E_o|^2 \qquad (19)$$

As the field spills around the mirrors, it is generally impossible to recuperate all the power diffracted. We define the power collected P_{OUT} and the output-coupling efficiency ε by

$$P_{OUT} = \int_{S_h} \frac{1}{2\mu_o} \left| Re\left(\vec{E} \times \vec{B}^* \right) \right| d^2 r_j \qquad (20)$$

$$\varepsilon = \frac{P_{OUT}}{P_1 + P_2} = \frac{P_{OUT}}{P}$$

where S_h is the surface through which the energy is coupled out of the resonators.

In steady-state operation of the gyrotron, the power radiated or dissipated must balance the power brought in by the electron beam.

$$\eta_e I_b V = P + L \tag{21}$$

where I_b is the intensity of the electron beam, V the accelerating voltage and η_e the efficiency of the power transfer from the beam to the R.F. field.

From Eqs. (17) and (19) it is seen that the ratio of the ohmic losses to the power radiated are independant of r_0 and E_0.

$$\frac{L}{P} = \frac{4}{Z_0 \sigma \delta} \frac{|\gamma|^2(1+|\gamma|^2)}{T} = 4\sqrt{\frac{\pi}{Z_0 \lambda \sigma}} \frac{|\gamma|^2(1+|\gamma|^2)}{T} \tag{22}$$

If the two-way loss T is small ($|\gamma|^2$ close to 1), the ohmic losses are proportional to the ratio P_{OUT}/T. Consequently, a quasi-optical gyrotron has the same ohmic losses if P_{OUT} = 200 kW and T = 2%, or P_{OUT} = 1 MW and T = 10%.

An important experimental quantity is the quality factor Q of the resonator which is defined usually by

$$Q = \omega \; \frac{\text{Energy stored inside the resonator}}{\text{total power loss}} \tag{23}$$

The energy stored in the open resonator can be written as the product of the circulating power multiplied by the transit time d/c.

$$W = \frac{d}{c}\left[\int_{S_1} \frac{1}{2\mu_0}\left|Re(\vec{E}\times\vec{B}^*)\right| d^2r_1 + \int_{S_2} \frac{1}{2\mu_0}\left|Re(\vec{E}\times\vec{B}^*)\right| d^2r_2\right] \tag{24}$$

$$W = \frac{d}{c}\,\frac{r_0^2}{2Z_0}\,|E_0|^2 \quad = \frac{\varepsilon_0}{2}\,|E_0|^2\, r_0^2\, d \tag{25}$$

Then, using (17) and (19)

$$Q^{-1} = \frac{P+L}{\omega W} = \frac{c}{d\omega}\left[\frac{T}{|\gamma|^2(1+|\gamma|^2)} + \frac{4}{Z_0\sigma\delta}\right] \tag{26}$$

In the limit of small T and no ohmic loss, we recover the well-known formula [2]

$$Q = \frac{2d\omega}{cT} = \frac{4\pi d}{T\lambda} \tag{27}$$

One must not forget that Eqs. (12) through (27) are no longer valid if more than one mode is present in the resonator.

We now specialize some of the above formulae to the TEM_{00} mode of a resonator with circular mirrors (without a hole). When the mirror radius is large compared to the spot size, the TEM_{00} loss is small and the field is nearly gaussian with azimuthal symmetry.

$$u_j(r_j) = \frac{1}{\sqrt{\pi}} \frac{r_o}{w_j} \exp\left[-\left(\frac{r_o}{w_j} r_j\right)^2\right] \qquad , (j = 1, 2) \tag{28}$$

$$E_x(r) = E_o u(r) = E_o \frac{2}{\sqrt{\pi}} \frac{r_o}{w_o} \exp\left[-\left(\frac{r_o}{w_o} r\right)^2\right] \tag{29}$$

where $r_o = \sqrt{d/k}$, w_j is the spot size on mirror j, w_o the beam waist and the radial coordinate r is normalized to r_o. From [6] we derive

$$w_o^2 = 2 r_o^2 \frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{|g_1 + g_2 - 2 g_1 g_2|} \tag{30}$$

$$w_1^2 = 2 r_o^2 \sqrt{\frac{g_2}{g_1} \frac{1}{1 - g_1 g_2}} \tag{31a}$$

$$w_2^2 = 2 r_o^2 \sqrt{\frac{g_1}{g_2} \frac{1}{1 - g_1 g_2}} \tag{31b}$$

The electric field on axis at the beam waist is

$$E_c = E_x(o) = E_o \frac{2}{\sqrt{\pi}} \frac{r_o}{w_o} \tag{32}$$

In terms of E_c we have for the power diffracted (17), the ohmic losses (19) and the energy stored in the resonator (25):

$$P = \frac{\pi}{8 Z_0} w_0^2 |E_c|^2 \frac{T}{|\gamma|^2 (1 + |\gamma|^2)} \tag{33}$$

$$L = \frac{\pi}{2 Z_0^2 \sigma \delta} w_0^2 |E_c|^2 \tag{34}$$

$$W = \frac{\pi}{8} \varepsilon_0 |E_c|^2 w_0^2 d \tag{35}$$

3.2 Numerical calculations

We have used different methods to solve numerically the resonator equations (7). In the iterative method [5] the field distributions on the mirrors are alternatively computed until convergence is achieved. The mode with the lowest diffraction loss, usually TEM_{00}, can easily be calculated. For axisymmetric resonators the Eqs. (7) can be reduced after integration over the azimuthal angle, to an eigenvalue problem by either expanding the kernel [9] or discretizing the integrals. The TEM_{00} and higher order modes are obtained simultaneously by matrix diagonalization. The kernel expansion method is limited to Fresnel numbers less than two due to round-off problems. This limitation does not exist in the integral discretization method, where the Gauss-Legendre integration rule turns out to be the fastest [10].

In what follows, we restrict ourselves to circular mirrors of spherical curvature.

Confocal resonator

The mirror radii of curvature are equal to the mirror separation $R_1=R_2=d$ and thus $g_1=g_2=0$. It can be shown that the diffraction losses depend solely on the resonator Fresnel number $N_m = a_{1m} \cdot a_{2m}/\lambda d$. In References [9] and [11], the influence of a circular hole was studied. The output-coupling efficiency ε depends on the hole radius a_o and remains always less than 50% (Fig. 3). With the selected resonator dimensions, the optimum efficiency is obtained for a hole of 1.5 mm radius. For larger holes, the TEM_{10} mode becomes dominant, but its coupling efficiency is very low since the TEM_{10} profile is proportional to r at the mirror centre.

In practical situations, the thickness of the mirrors has to be taken into account. The output hole may behave as a cut-off wave guide which will modify the calculated quality factor Q and coupling efficiency. In addition, since the properties of the confocal resonator do not depend on the individual mirror radii but only on their product, one cannot increase the coupling efficiency beyond 50% by increasing the size of one mirror. Note that a coupling efficiency less than 50% is obtained because the diffraction losses are equal at both mirrors, which has been confirmed by numerical calculations with mirrors of different radii. This effect can also be proved analytically [12]. In conclusion, a quasi-optical gyrotron with a confocal resonator will always emit radiation equally from both mirrors. This may alleviate the power handling capability of the output windows, but also implies other constraints on the transmission line. For a typical output transmission T of about 1% - 2%, coupling through a central circular hole does not seem adequate, not only because the coupling efficiency is below 50% but also because of the unrealistically large microwave power flowing through it. Moreover, the presence of a central hole perturbs the TEM_{00} mode more than the TEM_{10}: transverse mode selection by diffraction losses [1] therefore no longer applies.

Non-confocal resonators

We have discretized the integrals in Eqs. (7) to compute diffraction losses of the TEM_{00} mode at both mirrors for a non-confocal axisymmetric resonator. It can be shown [13] that such a resonator (without coupling holes) depends only on three parameters: N_m, G_1, G_2 where

$$G_1 = g_1 \frac{a_{1m}}{a_{2m}} = \left(1 - \frac{d}{R_1}\right) \frac{a_{1m}}{a_{2m}}$$

$$G_2 = g_2 \frac{a_{2m}}{a_{1m}} = \left(1 - \frac{d}{R_2}\right) \frac{a_{2m}}{a_{1m}} \tag{36}$$

With no loss of generality, we can assume that mirror radii are equal: $a_{1m} = a_{2m}$. The partial transmission T_1 is plotted in Fig. 4 as a function of the parameter G_2, for several values of G_1, while the total transmission is fixed at 2%. It is seen that for low values of G_1, T_1 can become close to 2%. Therefore, it is possible to make a resonator with asymmetric diffraction losses.

For non-confocal resonators, calculations have also shown that coupling the power out through a central hole is not the best method. Although a little higher than for confocal resonators, the maximum output coupling efficiency always remains about 50%. If the hole size is increased, the TEM_{00} profile tends to decrease at the centre and to spread towards the mirror edge all the faster when the mirror radius is larger. As an alternative, output coupling through annular slots has been implemented [3]. The TEM_{00} mode is then only slightly perturbated and diffraction around the mirrors can be kept small. If we consider, for example, a 50cm long resonator with λ = 2.5 mm, with mirror radii $a_1m = a_2m$ = 6cm (N_m = 3.92), and $G_1 = G_2 = 0.5$. An annular slot 3.4 to 4.7cm in one mirror results in a coupling efficiency of 71.5% for a total transmission T = 1.82%.

3.3 Constraints on resonator dimensions

Let us consider a non-confocal resonator without output coupling holes. As mentioned in section 3.2, its properties depend only on three parameters: N_m, G_1, G_2. We can restrict the possible range of these parameters by the following considerations:

(i) The spot sizes on the mirrors must be less than a certain value W_{max}. This condition arises naturally from practical considerations concerning the mirror sizes since the resonator has to fit into a vacuum vessel of limited dimensions.

(ii) The thermal load per unit area due to ohmic losses cannot exceed a certain value P_{max} (watt/cm^2).

For small diffraction losses ($T \simeq 2\%$), we can assume that the field profile of the TEM_{OO} mode is gaussian within a good approximation (Eqs. (28) and (29)). In order to translate analytically the above conditions, we take the expression (31) for the spot size w_j. As discussed in [13], this expression is a very good approximation of the true spot size in the case of a resonator with mirrors of equal radius, as long as the diffraction losses are small.

One can obtain an approximate expression for w_1 for an arbitrary resonator by considering the equivalent resonator which has mirrors of equal radius.

	resonator 1	resonator 2
radii	a_{1_m}, a_{2_m}	a'_m
	g_1, g_2	g'_1, g'_2
spot sizes:	w_1, w_2	w'_1, w'_2

From the equivalence between the two resonators, we obtain

$$N_m = \frac{a_{1_m} \cdot a_{2_m}}{d\lambda} = \frac{(a'_m)^2}{d\lambda}$$

$$G_1 = \frac{a_{1_m}}{a_{2_m}} g_1 = g'_1 \tag{37}$$

$$G_2 = \frac{a_{2_m}}{d_{2_m}} g_2 = g'_2$$

The diffraction losses, which depend essentially on the ratio of w/a, are identical for both resonators. Therefore,

$$\frac{w_1}{a_{1m}} = \frac{w_1'}{a'_m} \qquad \text{and} \qquad \frac{w_2}{a_{2m}} = \frac{w_2'}{a'_m}$$

and hence $$w_1 = \frac{a_{1m}}{a'_m}\, w_1' = \sqrt{\frac{a_{1m}}{a_{2m}}}\, w_1' \tag{38}$$

By using (30), (37) and (38), we obtain

$$w_1^2 = \frac{a_{1m}}{a_{2m}}\, 2 r_0^2 \sqrt{\frac{G_2}{G_1} \frac{1}{1-G_1G_2}} = 2 r_0^2 \sqrt{\frac{g_2}{g_1} \frac{1}{1-g_1g_2}} \tag{39}$$

This expression is identical to (31) and in this analytical approximation the spot size of the non-confocal resonator does not depend on the mirror radii. However, it must be remembered that Eq. (39) is only valid as long as the spot size is somewhat smaller than the mirror radii, which has to be the case if the diffraction loss is small. For the confocal case, the equivalence relations and the assumption (38) lead to

$$w_1^2 = \frac{a_{1m}}{a_{2m}} \cdot 2 r_0^2 \tag{40}$$

where it is apparent that the spot size depends explicitly on the mirror radii.

For a gaussian beam, the maximal thermal load per unit area (at the centre of the mirror) is related to the total dissipation L_j by

$$L_j = \frac{\pi}{2} w_j^2 P_{max} \tag{41}$$

The above conditions, therefore, can be expressed for mirror 1 by

$$\frac{2 r_0^2}{W_{max}^2} \leq \sqrt{\frac{g_1}{g_2}(1 - g_1 g_2)} \leq \frac{P_{max}}{L_1} \pi r_0^2 \tag{42}$$

with a similar expression for mirror 2 obtained by exchanging g_1 and g_2. To illustrate the allowed domain in the (g_1, g_2) plane, the boundary curves corresponding to (42) are plotted in Fig. 5, using reasonable values for W_{max} and P_{max}. The resonator stability conditions

$$0 \leq g_1 g_2 (1 - g_1 g_2) \leq 1 \tag{43}$$

must also be satisfied otherwise the diffraction losses would be too high to obtain an operating point for the gyrotron. As is apparent from (42) and (43), we obtain another possible domain for (g_1, g_2) by reversing the sign of both g_1 and g_2.

Another criterion which is of importance for the design of a resonator is to have a plane electromagnetic wave in the interaction region. In other words, the beam waist is at a prescribed distance d_1 from mirror 1.

$$\frac{d_1}{d} = \frac{g_2(1-g_1)}{g_1+g_2-2g_1g_2} \qquad (44)$$

Curves corresponding to various ratios d_1/d are plotted in Fig. 6 as well as curves corresponding to a constant spot size. It is seen that as the beam waist gets closer to one mirror, the spot size on this mirror decreases, hence the thermal load increases.

Finally, it is desirable to minimize the mirror radii for a given thermal load. Fig. 7 is a plot of the resonator Fresnel number as a function of g in the case of equal curvatures ($|g_1| = |g_2| = g$), the total transmission being fixed to 2%. One sees that the Fresnel number increases sharply for values of g above 0.7.

For a symmetrical resonator the beam waist (30) is given by

$$w_0^2 = r_0^2 \sqrt{\frac{1+g}{1-g}} \tag{45}$$

By choosing negative values of g one reduces the beam waist and consequently the length of the interaction region. Values of g in the range -0.5 to -0.8 seem to be a good compromise.

4. Beam-resonator coupling

The usual approach [1], [2] and [14] to study the non-linear steady-state operation of the quasi-optical gyrotron is to integrate the electron orbits (Eq. 1) with an assumed profile for the R.F. field. Single mode approximation is imposed by taking into account only the lowest order empty resonator mode, i.e. a gaussian TEM_{00} mode. The electronic efficiency at the operating point is then determined by the balance between the resonator power losses and the power transferred from the electron beam to the electromagnetic field.

In order to investigate the influence of the perturbations induced by the finite size of the mirrors and the beam, we have used a self-consistent method: equation (6) is solved for the electric field E_x while the current density j_x is computed iteratively from the electron orbits. We have written a numerical code based on this idea. The main steps are as follows:

- First the resonator equations (7) are solved by the iterative method described previously (see section 3.2). Only the axisymmetric mode with lowest losses is considered.

- The obtained profile is normalized and scaled by an initial value of the electric field at a particular point in the resonator (for instance at the centre of mirror 1).

- The iterative procedure consists of:
  i) calculating the electric field at the position of the beam.
  ii) integrating the equation of motion (1) for an ensemble (typically 60) of electrons with initial momentum angles distributed uniformly in $[0,2\pi]$.
  iii) computing the current density by using Eq. (4) which corresponds to a pencil beam and recalculating the field distributions on the mirrors according to Eq. (6).

The iterative procedure is repeated until the electronic efficiency converges to a stationary value. The field frequency is adjusted slightly every 4 iterations so that the phase shift vanishes after a round trip of the wave (resonance condition). Typically 200 to 400 iterations are required to achieve convergence. The interaction region extends from $-3r_o$ to $+3r_o$ and about 100 integration steps are needed to obtain a good accuracy.

Electronic efficiencies calculated with this method are shown in Table 1 for several resonator geometries and various values of Ω/ω. For comparison, we have included the results obtained by the simpler method which assumes a gaussian plane wave in the interaction region. Self-consistent calculations show that η_e is indeed quite sensitive to the resonator shape. This can easily be understood since the radiation beam waist w_o depends strongly on the mirror curvature and the mirror diameter. Highest efficiencies η_e correspond to values of w_o smaller than r_o. A precise determination of the optimum beam waist would require plotting η_e as a function of Ω/ω for various values of w_o which would be very expensive in computer time.

The mode pattern and η_e as a function of z are shown in Figs. 8, 9 and 10. Since the resonators considered here have quality factors of the order of 120,000, no differences between empty resonator and loaded resonator field profiles are observable.

The self-consistent approach becomes quite computer-time consuming if effects such as the electron velocity spread or a realistic profile for the electron beam (annular beam) are taken into account, since a larger number of trajectories are necessary to reach a stationnary solution. For high-Q resonators, since the beam does not perturb significantly the empty resonator mode, the R.F. field needs only to be scaled at each iteration. Moreover, by using more elaborate methods to solve Eq. (21), the operation point could be found in a smaller number of iterations.

5. Conclusion

The knowledge of the basic properties of open resonators is a prerequisite for the design of a quasi-optical gyrotron. In connection with our development programme of a 120 GHz, 200 kW quasi-optical gyrotron, we have implemented various numerical codes to solve the resonator equations and to compute important quantities, such as power transmission and field profiles for given geometrical parameters. Design criteria have been discussed. It appears that ohmic losses in the mirrors impose conditions on the allowable domain of parameters (g_1,g_2). A self-consistent calculation of the electronic efficiency has also been performed, demonstrating the importance of the resonator geometry. Although not shown in this work, the coupling structure (holes, annular slots), which can perturb significantly the R.F. field profile, is also expected to influence the electronic efficiency.

Acknowledgement

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Figure captions:

Fig. 1 : Schematic diagram of a quasi-optical gyrotron.

Fig. 2 : Open resonator with circular mirrors of spherical curvature. The sign of the curvature radius is positive if the centre of curvature lies towards the inside of the resonator.

Fig. 3 : Transmission T (two-way loss) in percent for the two least lossy modes TEM_{00} and TEM_{10} and coupling efficiency of the TEM_{00} as a function of the hole radius. Resonator parameters are d = 50cm, λ = 2.5mm, $a_{1m} = a_{2m}$ = 35mm (N_m = 0.98).

Fig. 4 : Parameters (N_m, G_1, G_2) of a T = 2% transmission resonator. T_1 and T_2 are partial transmissions around mirror 1 and 2.

Fig. 5 : Domain in the (g_1, g_2) plane satisfying condition (42) for the following case: d = 50cm, λ = 2.5mm, W_{max} = 3.0cm, P_{max} = 1.5 kW/cm^2, for copper mirror L = 9.6 kW.

Fig. 6 : Solid line: curves of constant spot size, w_{co} = confocal spot size = $\sqrt{d\lambda/2\pi}$. The peak thermal load of a confocal resonator with L = 9.6 kW, d = 50 cm, λ = 2.5 mm is approx. 1.5 kW/cm^2.
Dashed line: curves of constant ratio d_1/d.

Figure captions (cont'd)

Fig. 7 : Fresnel number of a symmetrical resonator versus $g = |g_1| = |g_2|$ for a fixed transmission of T = 2%.

Fig. 8 : Thick line: electronic efficiency as a function of z/r_o ($r_o = \sqrt{d\lambda/2\pi}$).

Thin lines: amplitude and phase of the electric field at the centre of the cavity, normalized to E_c = 14.3 MV/m. The resonator is confocal with Nm = 0.7244 (entry 2 of Table 1 with Ω/ω = 1.088). The beam waist is 0.9 r_o instead of r_o, due to the finite size of the mirrors.

Fig. 9 : Same as Fig. 8 but for a non-confocal resonator with g = 0.6 (entry 7 of Table 1 with Ω/ω = 1.108). E_c = 4.38 MV/m.

Fig. 10: Same as Fig. 8 but for a non-confocal resonator with g = −0.6 (entry 8 of Table 1 with Ω/w 0 1.088). E_c = 17.0 MV/m.

TABLE 1

		Fresnel numbers		Beam waist w_o	Electronic efficiency η_e					
					Ω/ω = 1.084	1.088	1.095	1.101	1.108	1.113
	g	N_1	N_2	cm	%	%	%	%	%	%
	-	-	-	1.41[a]	0.	0.	37	24.3	14.3	10.6
nt	0.	0.7244	0.7244	1.27	0.	49.4	39.2	27.2	16.6	12.3
nt	0.3	0.8255	7.4301	1.67	0.	0.	42.1	24.6	15.7	11.3
b)	-0.3	0.8255	7.4301	1.20	0.	47.7	33.7	24.2	14.9	10.4
	0.6	0.9387	3.7548	1.97	0.	0.	0.	30.1	16.5	10.3
c)	-0.6	0.9387	3.7548	1.18	44.7	39.0	28.4	21.8	16.0	12.2
	0.6	1.0709	1.0709	2.12	0.	0.	0.	25.6	14.0	9.8
d)	-0.6	1.0709	1.0709	0.99	45.8	42.9	30.0	21.6	15.2	11.3

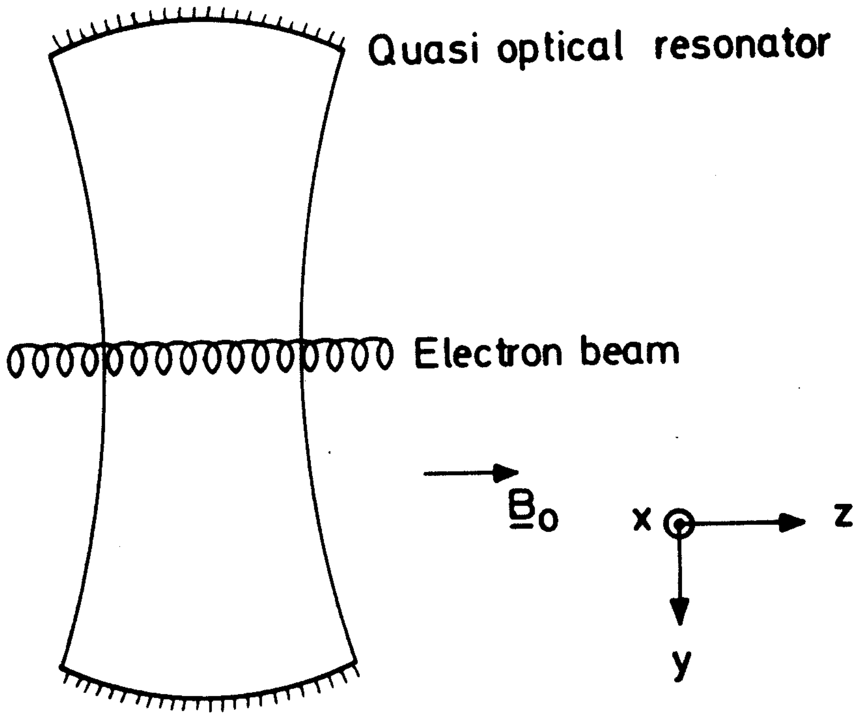
theoretical electronic efficiencies at 120 GHz for a 10A, 70kV pencil electron beam with $P_\perp/P_\parallel$
In all calculations, the resonator length is 50 cm and the total transmission T is fixed to 2%.
indicates that no operating point of the gyrotron is reached.

ds to the beam waist of a confocal resonator with large mirrors. ($w_o = r_o = \sqrt{d\lambda/2\pi}$, with
and λ = 2.5 mm)

1, $|G_2| = 0.9$; $T_1 = 1.81\%$, $T_2 = 0.19\%$

3, $|G_2| = 1.2$; $T_1 = 1.67\%$, $T_2 = 0.33\%$

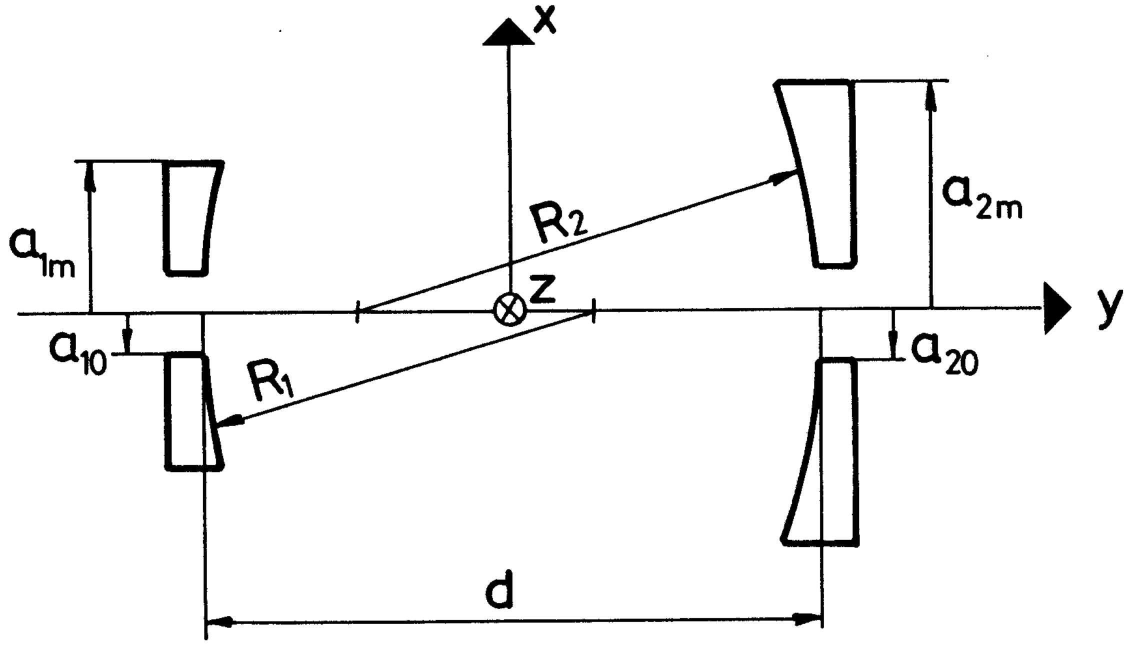
$_2| = 0.6$, $T_1 = T_2 = 1.00\%$



SCHEMATIC OF A QUASI-OPTICAL

RESONATOR

FIG. 1



Open resonator with spherical mirrors

FIG. 2

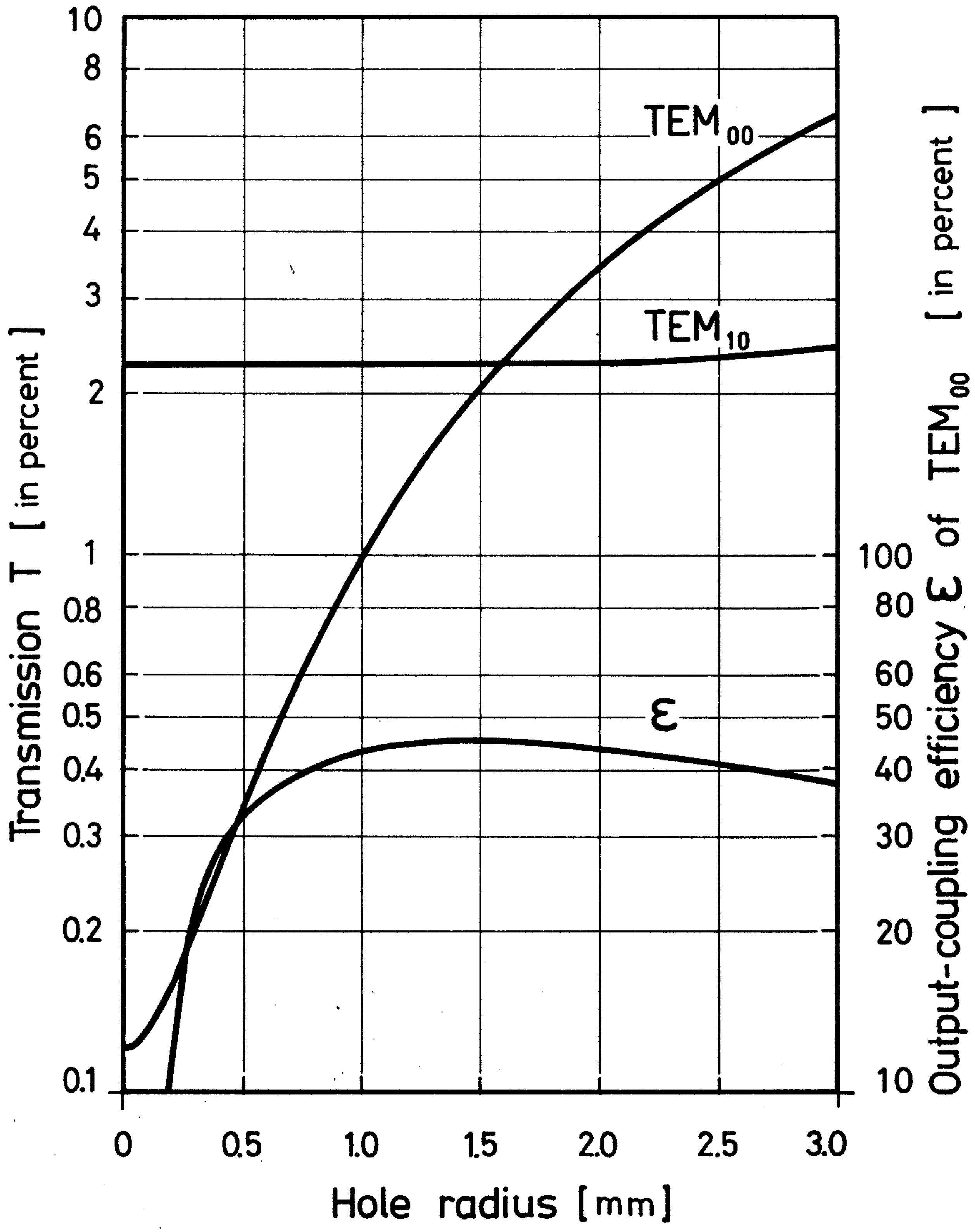
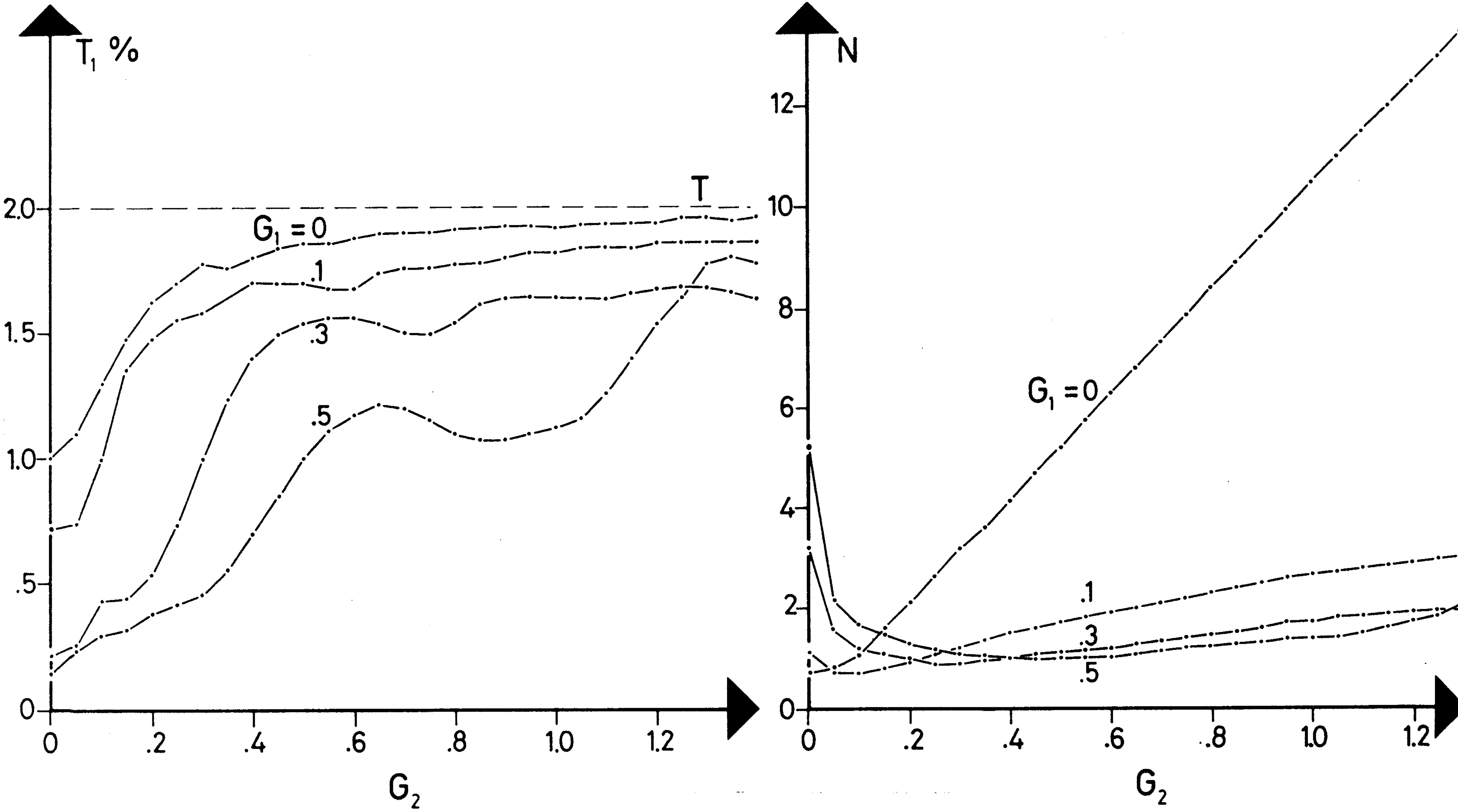
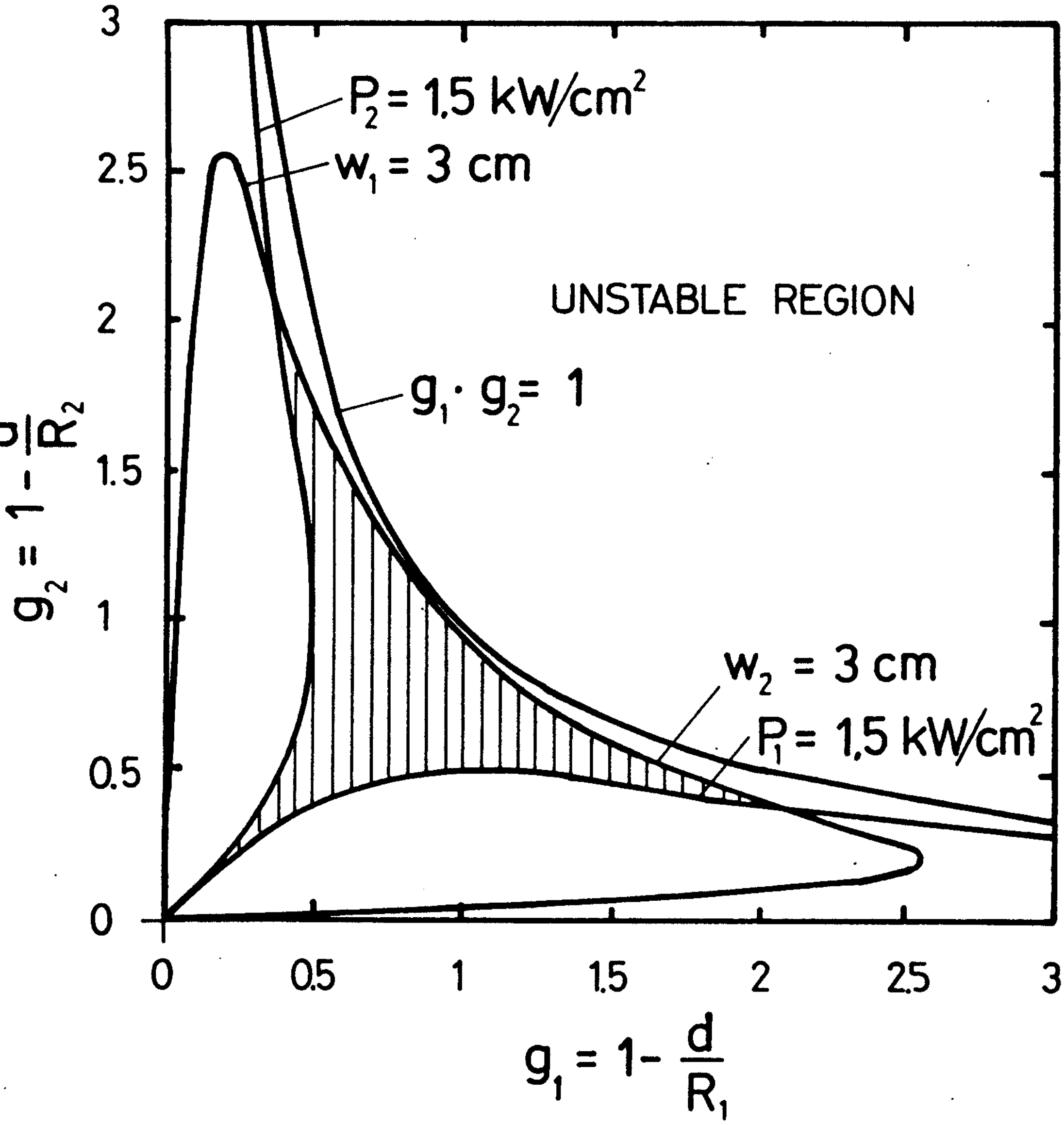

10
8
6
5
4
3
2
1
0.8
0.6
0.5
0.4
0.3
0.2
0.1
Transmission T [in percent]
TEM$_{00}$
TEM$_{10}$
ε
100
80
60
50
40
30
20
10
Output-coupling efficiency ε of TEM$_{00}$ [in percent]
0
0.5
1.0
1.5
2.0
2.5
3.0
Hole radius [mm]

FIG. 3



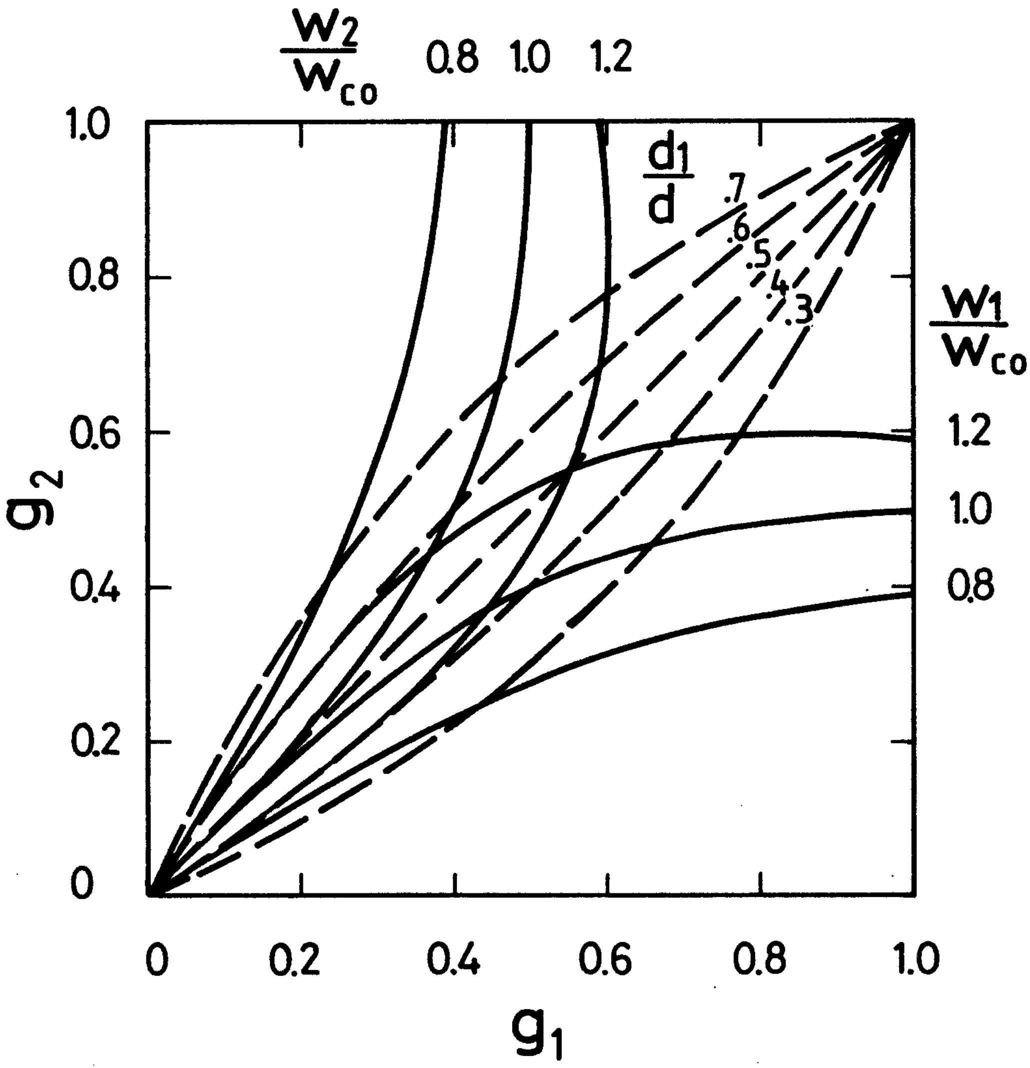
T1 %
2.0
1.5
1.0
.5
0
T
G1 = 0
.1
.3
.5
0
.2
.4
.6
.8
1.0
1.2
G2
N
12
10
8
6
4
2
0
G1 = 0
.1
.3
.5
0
.2
.4
.6
.8
1.0
1.2
G2

FIG. 4



$P_2 = 1{,}5\ kW/cm^2$
$w_1 = 3\ cm$
UNSTABLE REGION
$g_1 \cdot g_2 = 1$
$w_2 = 3\ cm$
$P_1 = 1{,}5\ kW/cm^2$
$g_2 = 1 - \frac{d}{R_2}$
$g_1 = 1 - \frac{d}{R_1}$
0
0.5
1
1.5
2
2.5
3

FIG. 5



$\frac{W_2}{W_{co}}$
0.8 1.0 1.2
$\frac{d_1}{d}$
.7
.6
.5
.4
.3
$\frac{W_1}{W_{co}}$
1.2
1.0
0.8
1.0
0.8
0.6
0.4
0.2
0
g_2
0 0.2 0.4 0.6 0.8 1.0
g_1

FIG. 6

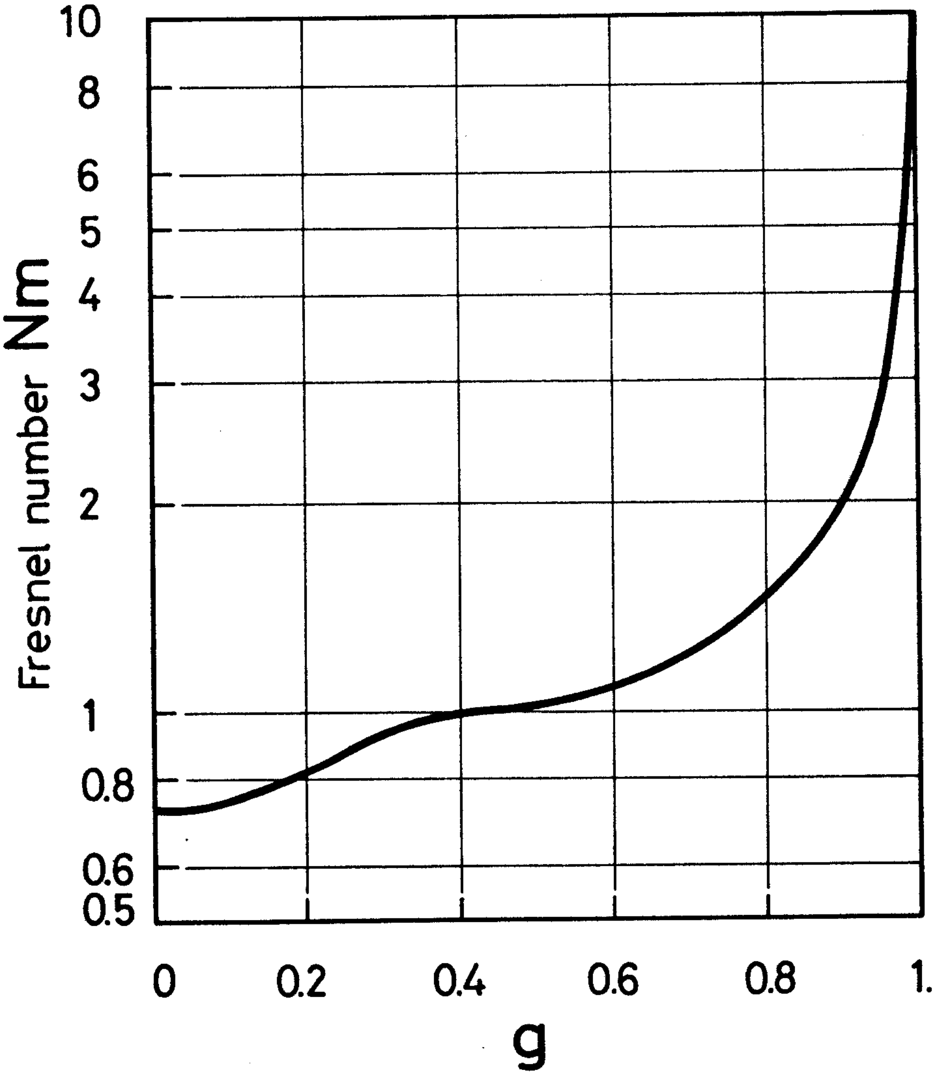


FIG. 7

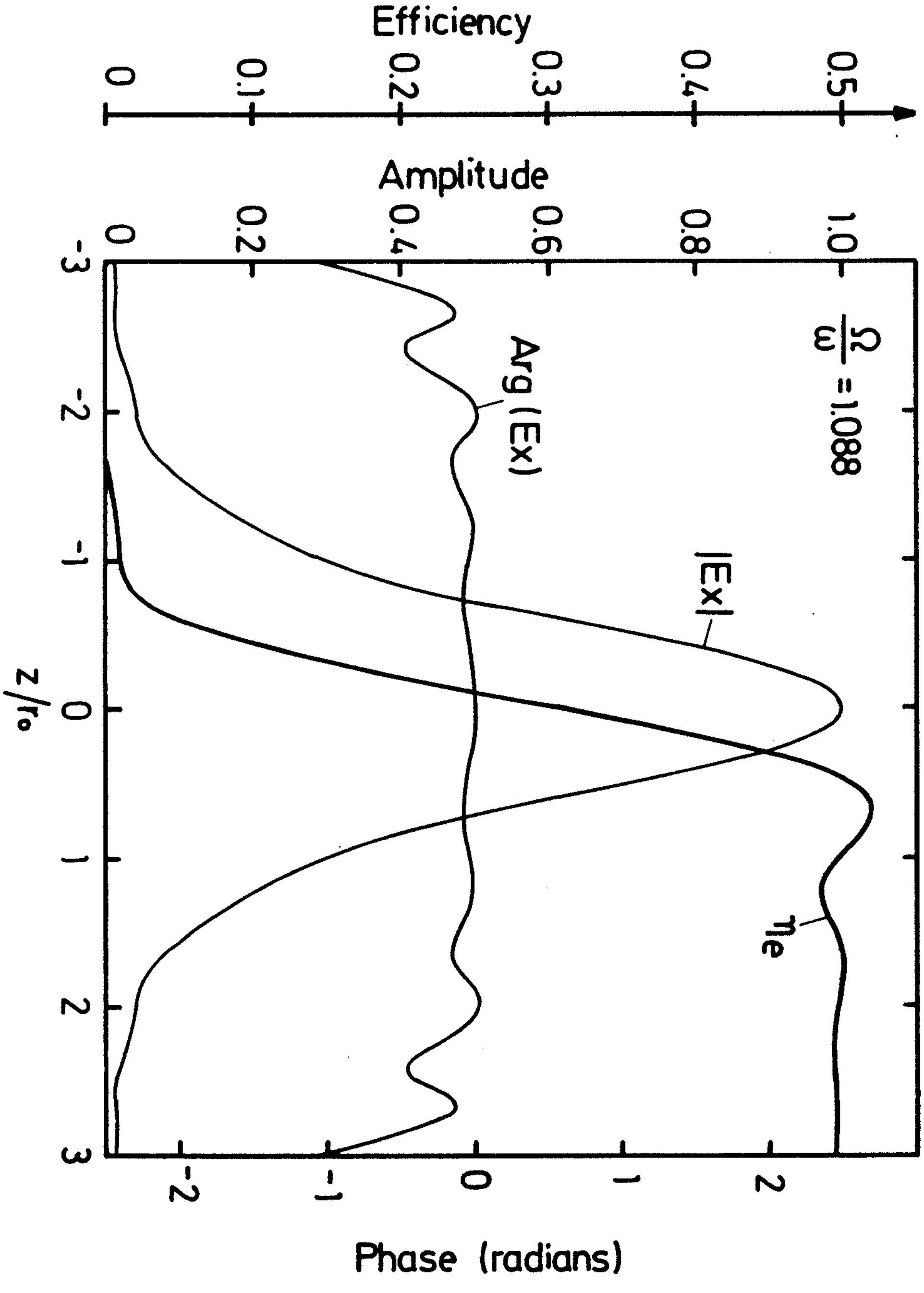
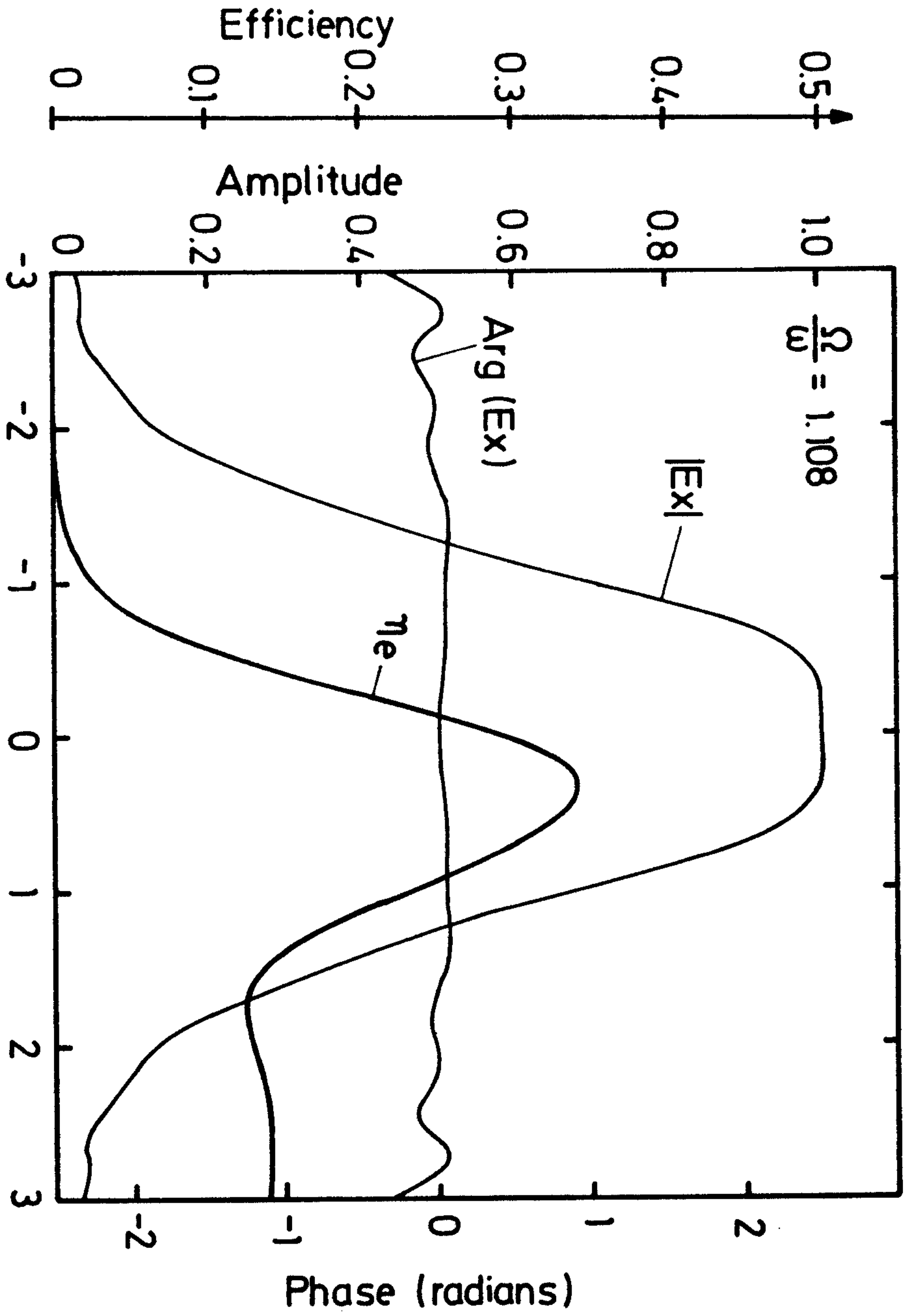
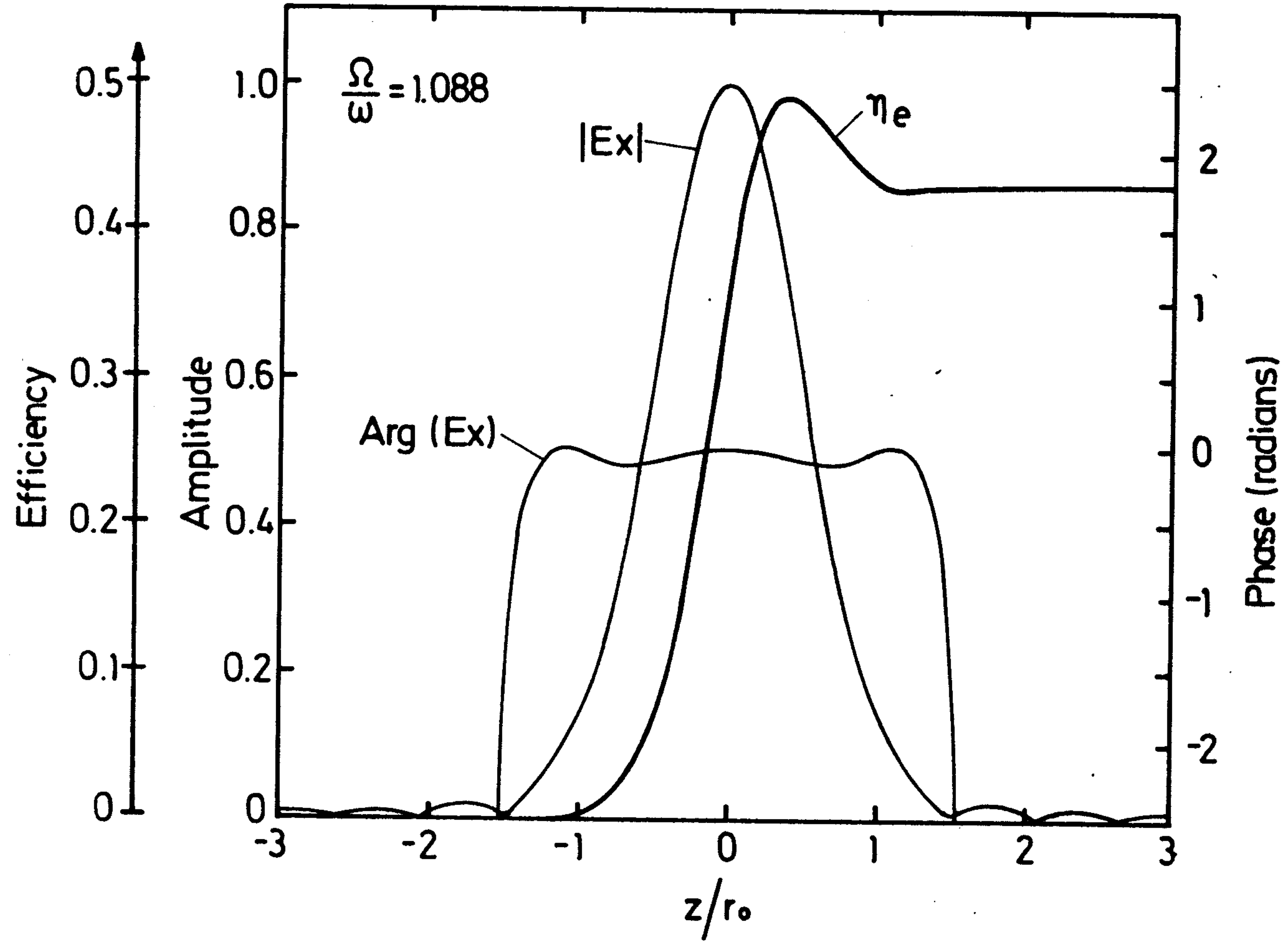

Efficiency
0
0.1
0.2
0.3
0.4
0.5
Amplitude
0
0.2
0.4
0.6
0.8
1.0
Ω/ω = 1.088
Arg (Ex)
|Ex|
ηe
-3
-2
-1
0
1
2
3
z/r₀
-2
-1
0
1
2
Phase (radians)

FIG. 8



Efficiency
Amplitude
Phase (radians)
z/r₀
|Ex|
Arg (Ex)
η_e
$\frac{\Omega}{\omega} = 1.108$

FIG. 9



Efficiency
0.5
0.4
0.3
0.2
0.1
0
Amplitude
1.0
0.8
0.6
0.4
0.2
0
$\frac{\Omega}{\omega} = 1.088$
|Ex|
η_e
Arg (Ex)
-3
-2
-1
0
1
2
3
z/r₀
Phase (radians)
2
1
0
-1
-2