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RACETRACKS IN A TOKAMAK

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ABSTRACT

Calculational methods and results are presented for racetrack-shaped, highly-elongated plasmas in a tokamak. Such equilibria may have high critical  $\beta$  and long confinement times, even at the large aspect ratios  $\sim 4$  required for reactors such as INTOR, if scaling laws determined at elongations  $K < 1.8$  extrapolate to higher  $K$ . The MHD equilibria are calculated with a free boundary code which requires shaping coil currents as input. To estimate these currents, the shape of the outermost flux surface is specified in a separate computation to be a  $K=4/1$  racetrack, and a guess at the final plasma current profile is added. A matrix equation relating uniformly-spaced  $\phi$  values on the desired flux surface to the shaping currents plus plasma current is inverted to obtain the shaping currents. The shaping coils are in vertical columns at  $R_0 \pm d$ , where  $d/a_{\text{plasma}}=1.67$ . To obtain stationary equilibria with  $K > 2$  it was found necessary to superimpose a feedback controlled quadrupole field which maintains the elongation of the outermost flux surface at a fixed value (e.g.  $K=4$ ). Without this feedback, plasmas with  $K > 2$  either shrink or diverge into two halves. The MHD input parameters were specified as  $p'=C_1\phi^L$  and  $T'T'=C_2\phi^M$ . In a circular plasma with  $B_T=2.2$  T and  $I_p=0.33 \times 10^6$  Amps,  $L=M=0.5$  gives a ratio of edge to central safety factors of  $q_e/q_c=1.8$ . With input values of  $L=M=0.5$ ,  $R_0=0.68$ ,  $a=0.18$ ,  $B_0=0.8T$ ,

a free boundary  $K=4/1$  racetrack equilibrium is obtained with  $I_p=1.00 \times 10^6$  Amps at  $q_e=2.0$ . When compared with the circular equilibrium at the same edge  $q$ , the increase in  $I/B_0$  indicates a  $(1+K^2)/2$  scaling. The central flux-surface is more elongated than the ones at the edge, and has a  $q$  of 2.1. Equilibria of this type have been computed for a wide range of  $\beta_{pol}$ .

Racetrack equilibria with  $K=4$  have been obtained with  $L=M=1.0$ , but the central flux surface elongation is considerably reduced, and  $q_e/q_c=5$  at low  $\beta_{pol}$ . With more peaked profiles,  $L=M=1.5$ , racetracks with  $K=4/1$  have not been obtained. Instead, the flux function develops a separatrix resulting in equilibria with an expanded-boundary divertor.

## 1. INTRODUCTION

In this report, highly-elongated tokamak equilibria are investigated which may possess the high beta and long confinement time properties required for a fusion reactor. A formula for the upper limit on toroidal beta has been proposed by Troyon et al. [1], based on a best fit to stability calculations by ERATO [2] of JET and INTOR equilibria, with elongations of 1.6/1. The formula for the maximum of the pressure ratio  $\beta = 2\mu_0 \int p dV / \int B^2 dV$  is  $(\beta A)_{max} = 2.2 I_N$ ,  $I_N = \mu_0 I A^2 / T_S$ ,  $A = R/a$ ,  $T_S = R B_T$ . This formula, which can be simply rewritten as

$$\beta_{max} = 2.2 \mu_0 I / a B_{T0} \quad (1)$$

(plasma current  $I$ , minor radius  $a$ , axial toroidal field  $B_T$ ), provides a good description of  $\beta$  limits observed in experiments [3], including the maximum  $\beta_T = 4.5 \pm 0.5 \%$  obtained in Doublet III [4], which had an elongation  $K = b/a = 1.38$ , aspect ratio  $A = 3.8$ , and safety factor  $q_\psi = 1.7$ . The elongation scaling of plasma current was (for elongations up to 1.8/1)

$$I_p = \left[ \frac{2\pi a^2 B_{T0}}{\mu_0 R q_\psi} \right] \left[ \sqrt{K} \left( \frac{1+K^2}{2} \right) \right] \quad (2)$$

The validity of (1) for 4/1 racetracks is being investigated separately with the ERATO code. The validity of (2) for 4/1 racetracks is studied in this paper. If these formulas for  $I_p$  and  $\beta$  continue to be valid for  $K > 2$ , then extremely high values of current and  $\beta$  can be obtained. The energy confinement time  $\tau_E$  is also proportional to current in certain plasma regimes, for example in auxiliary-heated discharges [5].

In the following sections, MHD equilibria with high vertical elongation are computed to determine the scaling of plasma current at constant  $q_\psi$  at the limiter. These equilibria can then be used in stability calculations. The calculations concentrate on plasmas with aspect ratios of  $A \approx 4$ , which will be required for fusion reactors, such as INTOR. Achieving high  $\beta_{TOR}$  is more difficult at large aspect ratio.

Plasmas with very high elongation have been produced, for example in belt pinches [6,7,8]. These plasmas tend to be transient because the high density and low electron temperature result in current peaking and shrinkage of the current channel [9]. On a longer timescale, Doublet plasmas with 3/1 elongation [10] have been produced, but due to peaked current profiles, the two magnetic axes were nearly circular, and were linked by a relatively small (<25 %) fraction of common flux. On the other hand, with flat current profiles, plasmas have been formed in the upper half of Doublet III, with a single magnetic axis, with elongations up to 2.1/1 [11,12]. With hollow profiles, the central surface has an even higher elongation, 2.4/1, leading to a central  $q$  value higher than that at the edge. These plasmas lasted 10's of milliseconds, gradually shrinking as the current increased, since the initial value of the safety factor at the edge was  $q = 10$ .

Extensive studies of vertical stability of steady-state plasmas with  $K < 1.8/1$  and peaked current profiles have been made in Doublet III [13]. These studies indicate that flattening the current profile allows higher elongations, especially with feedback control.

In the following sections, we shall first describe a "specified boundary code" developed to determine shaping coil currents used as input to the free boundary code. Then we shall discuss detailed MHD equilibrium calculations for a tokamak of similar physical dimensions as the present TCA tokamak at Lausanne. The equilibrium code used for this analysis is the Free Boundary Tokamak Code (FBT), developed at CRPP [14,15].

## 2. RACETRACK, A SPECIFIED-BOUNDARY CODE

### A. Introduction and Assumptions

The goal of this research is to compute racetrack-shaped tokamak equilibria with high elongation. This could be done with a fixed boundary code but in that case one obtains no information on axisymmetric modes and feedback requirements. Here, a free boundary code is used to determine whether or not such equilibria can be set up experimentally with actual coils. If one wants to achieve a predetermined plasma shape in a free-boundary calculation, however, the shaping coil currents must be computed in a separate calculation. For this purpose, a new code was written, named RACETRACK.

The code is based on the assumption that if the desired shape of the external flux surface at the plasma boundary is specified, and if an appropriate guess is made of the internal flux surface shapes and current profiles, then the currents in the outer shaping coils can be accurately calculated. This assumption is based on the work of Lao et al. [16], where in order to calculate the plasma boundary, only the first few moments of the current profile are needed.

### B. Matrix Equation

The program RACETRACK solves, by inversion, the matrix equation

$$A_{ij} \left( \frac{I_j}{\psi_0} \right) = \frac{\psi_i}{\psi_0} = [1], \quad (3)$$

where  $\psi_i$  is the poloidal flux function on the specified boundary of

the plasma (which has the constant value  $\Psi_0$ ),  $I_j$  are currents in the shaping coils (plus one lumped current representing the plasma), and  $A_{ij}$  is the coupling coefficient. Note that  $\Psi = RA_\theta$ , where  $A_\theta$  is the toroidal component of the magnetic vector potential, and the poloidal flux  $\Phi_i = 2\pi\Psi_i = 2\pi A_{ij}I_j$ , and therefore  $2\pi A_{ij}$  is the mutual inductance between a circle with polar co-ordinates  $R_i, Z_i$ , and a circular coil carrying current  $I_j$  at  $R_j, Z_j$ , where

$$A_{ij} = \frac{\mu_0 \mathcal{K}}{2\pi} \left[ \frac{R_i R_j}{2} \right]^{1/2} F(\mathcal{K}^2), \text{ and } (4)$$

$$F(\mathcal{K}^2) = \lim_{N \rightarrow \infty} \sum_{m=1}^N \frac{(\Delta\alpha) \cos(\alpha_m)}{[1 - \mathcal{K}^2 \cos(\alpha_m)]^{1/2}}, \alpha_m = (m - \frac{1}{2})\Delta\alpha,$$

$$\mathcal{K}^2 = \frac{2 R_i R_j}{R_i^2 + R_j^2 + (Z_i - Z_j)^2}, \Delta\alpha = \frac{\pi}{N}.$$

The following sections describe the detailed coil positions, location of the  $\Psi_0$  surface, plasma current distribution, and the matrix inversion procedure to obtain  $(I_j/\Psi_0)$ . The  $\Psi_0$  and the  $I_j$  are found by specifying the desired value of plasma current.

### C. Poloidal Field System

The poloidal field system is shown in Fig. 1. The coils allow room for limiters, vacuum wall, thermal and electrical insulation, and (possibly) antennae for (RF wave) heating. We assume a plasma major radius of 0.680 meters, minor radius of 0.180 m ( $A=R/a=3.8$ ), and a plasma half-height of up to 0.720 m, giving  $K=4/1$  elongation. There are two vertical coil stacks, at  $R=0.380$  m on the inside, with 8 coils above and 8 coils below the midplane, spaced 0.100 m apart and at  $R=0.980$  m on the outside, with 4 coils above, and 4 coils below, 0.200 m apart, which allows reasonable access from the outside and excellent access top and bottom. The ratio of the distance  $d$  (from coils to plasma center) to minor radius  $a$  is 1.67, relatively large for shaping experiments.

#### D. Flux Surface Description

The outermost plasma flux surface is chosen to be a racetrack, with two semicircles of radius  $a$ , top and bottom, connected by vertical straight lines of length  $2h$ . (A circle is obtained with  $h=0$ ). Since the plasma is chosen to be up-down symmetric, the flux equation is solved only in the upper half plane.

The locations of the values  $\Psi_i$  on the flux surface are chosen to give equal spacing along the inside. For example, with eight coils, the total distance along the straight section plus half the upper semicircle is divided into eight. On the outside, there are four points corresponding to four coils. Since the lumped plasma current represents an additional current, an additional  $\Psi_i$  value is added at  $z=0$  on the inside, making 13 equations in 13 unknowns for a 4/1 race-track plasma with twelve coils above the midplane. The contributions of coil currents below the midplane to the  $\Psi_i$  values above the midplane are included.

#### E. Plasma Current Distribution

The plasma current distribution is specified with the functional form  $J/J_0=(1-\alpha^\gamma)^\delta$ , where  $\alpha$  is the co-ordinate labelling the surfaces of constant current density, and  $\gamma$  and  $\delta$  are input parameters. Note that  $\gamma=2$ ,  $\delta=1$  gives a parabola. The surfaces are assumed to be nested racetracks, all with the same height-to-width ratio. All these surfaces have the same major radius  $R_{new}$ , which may be shifted outwards from the flux surface major radius  $R_0$ , to simulate the outward Shafranov shift. In the outward direction along the midplane,  $\alpha=r/a_{minor}$ . An equally-spaced grid inside the current channel is set up. All the plasma current contributions to each  $\Psi_i$  on the plasma surface are then summed to form a single matrix element.

#### F. Matrix Inversion and Solution

The matrix is inverted by reducing it to row-echelon form, the diagonal set to 1, and the elements above then zeroed. The last column is the solution vector with the  $I_j/\Psi_0$  values. The value of  $\Psi_0$  is

calculated by specifying the plasma current, and all the  $I_j$  shaping coil currents are calculated. These currents are then used as input to the free boundary MHD equilibrium code.

### 3. RACETRACK RESULTS

As will be shown in the next section, the RACETRACK program gives a good estimate of the coil currents required to produce a specified external flux shape and current profile distribution. The code can therefore be used to rapidly study a wide range of equilibrium conditions.

#### A. Typical Results

The input values and results of a typical example are presented here :

Input : Major Radius  $R_0 = 0.68$  m  
Minor Radius  $a = 0.18$  m  
Length Straight Section  $L = 0.54$  m  
(Gives a 4/1 elongated racetrack)  
Number of inner coils above midplane  
at ( $R = 0.38$  m) = 8  
Number of outer coils above midplane  
at ( $R = 0.98$  m) = 4  
Plasma current = 1000 Kiloamperes  
Major Radius current channel =  $R_{new} = 0.69$  m  
Current Profile  $(J/J_0) \sim (1-\alpha^{2.0})^{1.0}$

Table 1 - Location of  $\Psi_j$  points on flux surface :

<u>R(m)</u>	<u>Z(m)</u>
.5	.103
.5	.206
.5	.309
.5	.411
.5	.514
.516	.615
.583	.691
.680	.72
.777	.691
.86	.514
.86	.309
.86	.103
.5	0.



The currents in the coils are then calculated as follows :

coil #	coil current (kiloamps) (-turns)	R-coil (m)	Z-coil (m)
1	-62.2	.38	.05
2	-61.0	.38	.15
3	-56.4	.38	.25
4	-48.7	.38	.35
5	-39.4	.38	.45
6	18.8	.38	.55
7	14.9	.38	.65
8	55.5	.38	.75
9	2.6	.98	.70
10	-33.7	.98	.50
11	-99.8	.98	.30
12	-102.0	.98	.10
Plasma	1000	-	-

Note that all coils are treated as single turn coils. These currents are obviously the same below the midplane. When all 24 coil currents are added together, the ratio of their algebraic sum to the plasma current is  $-0.82/1$ . This ratio implies that the external shaping coils are acting like a conducting shell, and will reduce the volt-second requirements to the shaping coils. Also, the ratio of the sum of the absolute values of the coil currents to the plasma current is  $1.19/1$ .

#### B. Plasma Current Ramp

In order to reach full current at full elongation, there are essentially two methods of increasing the current. One method is to try to maintain the full plasma elongation during the current increase, for example as in belt-pinch or the original doublet experiments. The second method is to establish a low- $q$ , nearly circular plasma, and then increase the elongation as the current is increased. The external shaping coil program required to do this has been calcu-

lated and is displayed in Fig. 2. The current in each shaping coil is plotted as a function of elongation  $K$ . The plasma current is  $(250 \text{ kA})K$ . (Actually, the current should increase as  $\sim(1+K^2)/2$  to maintain constant  $q$ ). The maximum elongation is  $5/1$ , so there are 10 coils inside instead of 8, and 5 on the outside.

As the current and elongation are increased, the coils on the inside near the midplane start with positive current, which then becomes negative. The currents on the outside start negative and become more negative. As high elongations are reached, the currents near the midplane become negative and constant, while the positive pulling current "propagates" upwards, pulling the plasma with it. This type of current start up requires complicated power supplies, but it may be easier to control the plasma. Note that there is scatter in the calculated currents at half integer values of the elongation. This scatter illustrates how higher order moments may be produced with a non-optimal choice of location of points on the flux surface.

#### C. Comments on Plasma Shift and Current Profile Changes

The choice of position shift of the current channel can be used to simulate the shift due to high plasma  $\beta_{pol}$ . A shift of 0.01 m corresponds to low  $\beta_p \sim 0.3$  equilibria, and a shift of 0.02-0.03 m corresponds to higher  $\beta_p \sim 1$  equilibria.

For the current profiles,  $J/J_0 \sim (1-\alpha^{2.0})^{1.0}$  gives about the same current profile as  $p' \sim TT' \sim \Phi^{1.0}$  in the MHD code, and  $J/J_0 \sim (1-\alpha^{2.0})^{0.5}$  corresponds to  $p' \sim TT' \sim \Phi^{0.5}$ , where

$$\Phi = (\Psi - \Psi_{limiter}) / (\Psi_{axis} - \Psi_{limiter}).$$

#### 4. MHD FREE BOUNDARY EQUILIBRIUM CALCULATIONS

With shaping currents input from the RACETRACK program, the Free Boundary Tokamak code (FBT) solves the Grad-Shafranov equation to find the MHD equilibrium [14]. The code uses "radial position" feedback control, by calculating the value of plasma current such that the plasma outer flux surface will touch both the inner and outer limiters

at  $z=0$ ,  $R=R_0+a$  and  $R=R_0-a$ . The plasma current is therefore a calculated output of the code. The FBT version used here assumes up-down symmetry, so that no dipole field feedback is required to stabilize the vertical-displacement instability. However, for highly elongated plasmas, a quadrupole feedback was found necessary. For this purpose, a point at the top of the desired racetrack is specified to have the same flux as the limiter points on the midplane and a combination of currents to form a quadrupole field is varied to obtain stability, as a perturbation to the initial guess. In many cases, with good initial guesses, the perturbation currents needed to stabilize the plasma are of the order of a few percent.

#### A. Racetrack Equilibria with flat Current Profiles

In order to achieve the maximum central plasma elongation, very flat current profiles were studied. Current profiles with  $(J/J_0) = (1-\alpha^2)^{0.5}$  at  $R_{new} = 0.70$  m were input to the Racetrack code, with  $I_p = 1000$  kAmps. The calculated shaping coil currents (1-12) are (-29.9, -23.3, -27.7, -23.0, -24.8, 35.1, 5.6, 123.9, -93.3, -97.9, -41.4, -4.7). For the MHD code, we specified  $p' = C_1 \phi^{0.5}$ ,  $\pi\pi' = C_2 \phi^{0.5}$  and  $\beta_p^{input} = 1.0$ . The input value of  $\beta_p^{input}$  is used to specify the ratio between  $p'$  and  $\pi\pi'$ , or equivalently,  $C_1$  and  $C_2$  as follows:  $\beta_p^{input} = [1 + C_2/R^2 \mu_0 C_1]^{-1}$ . The limiters are at  $z=0$ ,  $R=0.50$  m and  $R=0.86$ , giving  $R_0=0.68$  m,  $a=0.18$  m. The calculated MHD equilibrium is shown in Fig. 3.

The external flux surface is very close to an ideal racetrack. The calculated current is 1002 kA, the magnetic axis is at  $R=.703$  m, and the stabilizing currents are  $\approx 4$  % of the input currents. The central plasma pressure is 1.1 ATM. If the toroidal field is chosen to be 8.0 kG, in order to obtain  $q_\psi(a)=2.0$ , then  $q_\psi$  near the center, on the first flux surface, is  $q(nc) \approx 2.0$ . This is a consequence of the fact that the elongation is much higher near the axis (7/1) than on the edge (4/1). The central toroidal beta value then is 43 %.

By contrast, a nearly circular plasma equilibrium, shown in Fig. 4, with the same current profile, at  $I_p=330$  kA and 22 kG, gives  $q(a)=2.0$ ,  $q(nc)=1.17$ ,  $\beta_{pol}=0.3$ ,  $p(0)=0.4$  ATM, and  $\beta_T(0)=2.1$  %. For

similar edge  $q$ , these equilibria indicate that  $I_{\text{plasma}}/B_T(0)$  scales as  $(1+K^2)/2$ , in contrast to eq. (2) where there is an additional  $\sqrt{K}$  term, probably due to triangularity in Doublet III plasmas.

The effect of a more peaked current profile on elongation is shown in Fig. 5, where  $p' \sim \pi r' \sim \Phi^{1.0}$ , and  $\beta_{\text{pol input}}=1.0$ . The outer surface is still a 4/1 racetrack, but the inner surface is reduced to a 3.4/1 elongation. In this case,  $B_T=12$  kG,  $q(\text{nc})=1.0$ ,  $q(a)=5.0$ ,  $p(0)=2.1$  ATM and  $\beta_{\text{TOR}}(0)=36$  %. The plasma current is 993 Kiloamperes.

The effect of a very peaked current profile,  $p' \sim \pi r' \sim \Phi^{1.5}$ , is evident in Fig. 6. In this case, we were unable to obtain a 4/1 equilibrium. Instead, the flux function develops a separatrix very near to the plasma limiting surface. The configuration is close to that of an expanded boundary divertor. The central flux surface is nearly circular, the plasma current is 286 kA, and  $\beta_p=0.3$ . For  $B_T=20$  kG, we find  $q(\text{nc})=1.04$ ,  $q(a) \rightarrow \infty$ ,  $p(0)=0.3$  ATM, and  $\beta_{\text{TOR}}(0) \approx 1.9$  %.

In addition, for flat current profiles, higher values of  $\beta_{\text{pol}}$  were explored. The general effect is to shift the magnetic axis outwards, shrink the central flux surface, and increase the ratio of  $q_{\text{edge}}/q_{\text{central}}$ . At very high  $\beta_{\text{pol}}$ , reverse currents begin to flow on the inner edge of the plasma.

The important role of the stabilizing field is shown in Fig. 7a, b, c. In Fig. 7(a), an equilibrium is shown which has converged with the help of quadrupole stabilization. In Fig. 7(b), the stabilizing field was fixed at a slightly different value than the equilibrium value. Both the plasma current and elongation decreased strongly. In Fig. 7(c), the stabilizing field was changed slightly in the opposite direction, and the plasma bifurcated into two halves. In these two examples of bifurcation, neither poloidal flux nor plasma current was kept constant.

The equilibria presented here all have a single magnetic axis.

With slightly different shaping, the upper and lower plasma lobes can bulge outwards, and two magnetic axes can form, leading to the doublet-type configurations. These types of configurations are not examined here.

Finally, we have explored a plasma with a somewhat higher aspect ratio to allow more volt-seconds and increased plasma-coil spacing for antennae. The equilibrium is shown in Fig. 8. The parameters are  $R_{\text{coil inner}}=0.50$  m,  $R_{\text{coil outer}}=1.14$  m,  $R_0=0.80$  m,  $a=0.18$  m,  $b/a=4/1$ . The shaping coil currents (1-12) are (-84.7, -83.8, -80.5, -70.5, -75.5, 60.4, 9.9, 236.7; 20.3, -112.9, -281.5, -244.1). The maximum stabilizing current is only 3 kA. The RACETRACK input current has been arbitrarily chosen to be 2500 kA, and the plasma current calculated by the FBT code was 2576 kA. The plasma profile was of the form  $p' \sim \tau\tau' \sim \Phi + L\Phi^2$  (suitable for input to the ERATO stability code) with  $L=-0.5$ . This gives similar profiles to  $p' \sim \tau\tau' \sim \Phi^{0.5}$ .

If the toroidal field is chosen to be 21 kG, with  $I_p = 2.5$  MA, then  $q \approx 2$  at the boundary. The  $q$  profile is shown in Fig. 9. The shape of the  $q$  profile depends on  $B_T$ , due to the  $\tau\tau'$  term. The magnetic axis is at  $R=0.82$  m, the central flux surface has an elongation of 8/1 and is just on the verge of forming two magnetic axes. The outer flux surface is again a nearly perfect racetrack, with only a 1 cm outward bulge on the upper and lower lobes. Other parameters are  $\beta_{\text{pol}}^{\text{input}}=1.0$ ,  $\beta_I=0.44$ ,  $J(0)=8.1 \times 10^6$  A/m<sup>2</sup>,  $P(0)=7$  ATM,  $P=3$  ATM and, at 21 kG,  $\beta_{\text{TOR}}(0)=40$  %,  $\beta_{\text{TOR}}=17$  %, at a plasma aspect ratio of  $A=80/18=4.4/1$ , which is similar to the suggested aspect ratio for a tokamak fusion reactor.

## CONCLUSIONS

In this paper, we have shown that the methods implemented in the RACETRACK code provide a very accurate guess for the shaping coil currents to be input to the free boundary MHD equilibrium code. The MHD code, with quadrupole stabilization, is able to produce 4/1 elongated racetrack equilibria, in which the central flux surface elongation can reach 8/1. The ratio  $I/B_T$  scales approximately as  $(1+K^2)/2$ . The questions of bifurcation, plasma shape control, and stability of actual tokamak plasmas have been commented on and initial calculations presented. A flat current profile is crucial for obtaining these highly elongated equilibria, and quadrupole stabilization is necessary for elongations  $K > 2$ .

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FIGURE CAPTIONS

- Fig. 1. Poloidal Field System for Elongated Equilibria.
- Fig. 2. Coil Currents versus Plasma Elongation and Current. This graph illustrates one possible method of establishing the full plasma current.
- Fig. 3. Free Boundary MHD Equilibrium of 4/1 Racetrack with a relatively flat current profile. Input values : Limiters  $0.68 \pm 0.18$  m,  $p' \sim r r' \sim \Phi^{0.5}$ ,  $\beta_p = 1.0$ . Output values :  $I_p = 1000$  kA,  $p(0) = 1.1$  ATM. For  $B_T = 0.8$  T :  $q(nc) = 2.0$ ,  $q(a) = 2.0$ ,  $\beta_T(0) = 43$  % . The tenth flux contour is the limiter surface.
- Fig. 4. Free Boundary MHD Equilibrium of nearly Circular Plasma with a relatively flat current profile. Input values : Limiters  $0.68 \pm 0.18$  m,  $p' \sim r r' \sim \Phi^{0.5}$ ,  $\beta_p = 0.3$ . Output values :  $I_p = 330$  kA,  $p(0) = 0.4$  ATM. For  $B_T = 2.2$  T :  $q(nc) = 1.17$ ,  $q(a) = 2.0$ ,  $\beta_T(0) = 2.1$  % .
- Fig. 5. Free Boundary MHD Equilibrium of 4/1 Racetrack with a moderately peaked current profile. Input values : Limiters  $0.68 \pm 0.18$  m,  $p' \sim r r' \sim \Phi^{1.0}$ ,  $\beta_p = 1.0$ . Output values :  $I_p = 993$  kA,  $p(0) = 2.1$  ATM. For  $B_T = 1.2$  T :  $q(nc) = 1.0$ ,  $q(a) = 5.0$ ,  $\beta_T(0) = 36$  % .
- Fig. 6. Free Boundary MHD Equilibrium with a separatrix near to the limiter surface and with a very peaked current profile. Input values : Limiters  $0.68 \pm 0.18$  m,  $p' \sim r r' \sim \Phi^{1.5}$ ,  $\beta_p = 0.3$ . Output values :  $I_p = 286$  kA,  $p[0] = 0.3$  ATM. For  $B_T = 2.0$  T :  $q(nc) = 1.0$ ,  $q(a) \rightarrow \infty$ ,  $\beta_T(0) = 1.9$  % .
- Fig. 7. a) Converged MHD Equilibrium, using quadrupole stabilization.  
b) Collapsing MHD Equilibrium, using fixed quadrupole fields perturbed from the optimum.  
c) Bifurcation of MHD Equilibrium into two halves, using fixed quadrupole fields perturbed in opposite direction.



Fig. 8. Free Boundary MHD Equilibrium of 4/1 Racetrack at higher Aspect ratio with a relatively flat current profile. Input values : Limiters  $0.80 \pm 0.18$  m,  $p' \sim \pi r' \sim \Phi - 0.5 \Phi^2$ ,  $\beta_p = 1.0$ . Output values :  $I_p = 2576$  kA,  $p(0) = 7$  ATM,  $p = 3$  ATM. For  $B_T = 2.1$  T,  $q(nc) = 1.8$ ,  $q(a) = 1.9$ ,  $\beta_T(0) = 40$  %,  $\beta_T = 17$  %.

Fig. 9. Safety factor  $q_\psi$  profile versus major radius along the mid-plane for  $B_T = 2.1$  T and  $B_T = 1.0$  T, for MHD equilibrium shown in Fig. 8, with  $I_p = 2576$  kA,  $K = 4/1$ ,  $R(\text{mag. axis}) = 0.82$ .

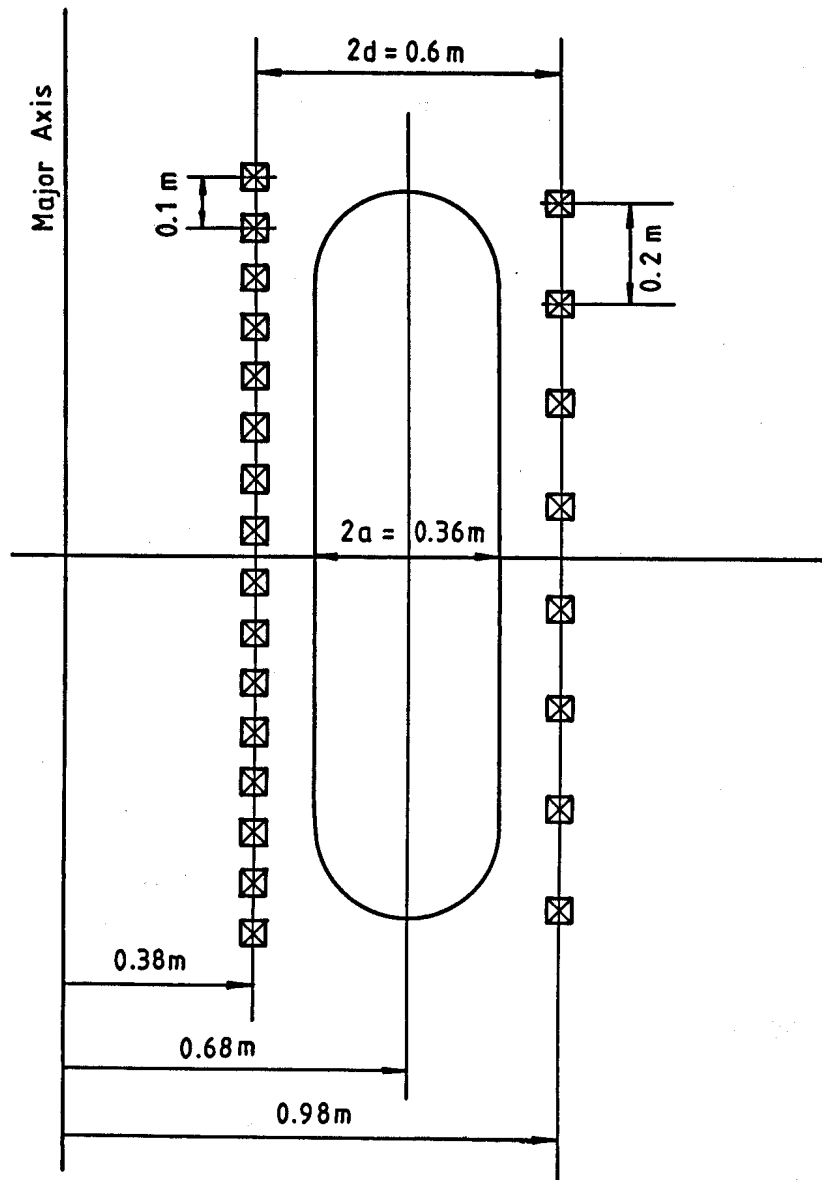


Fig. 1

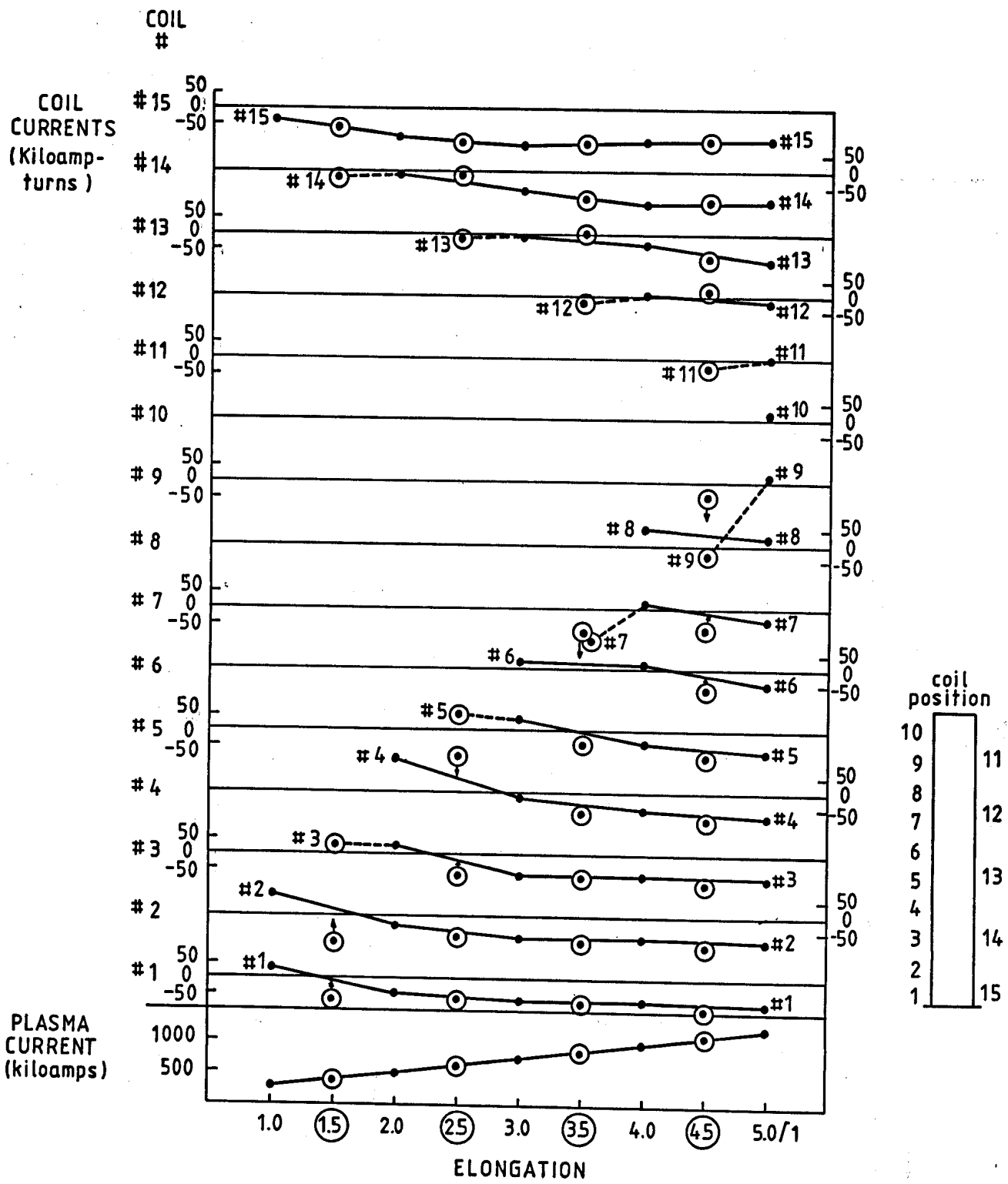


Fig. 2

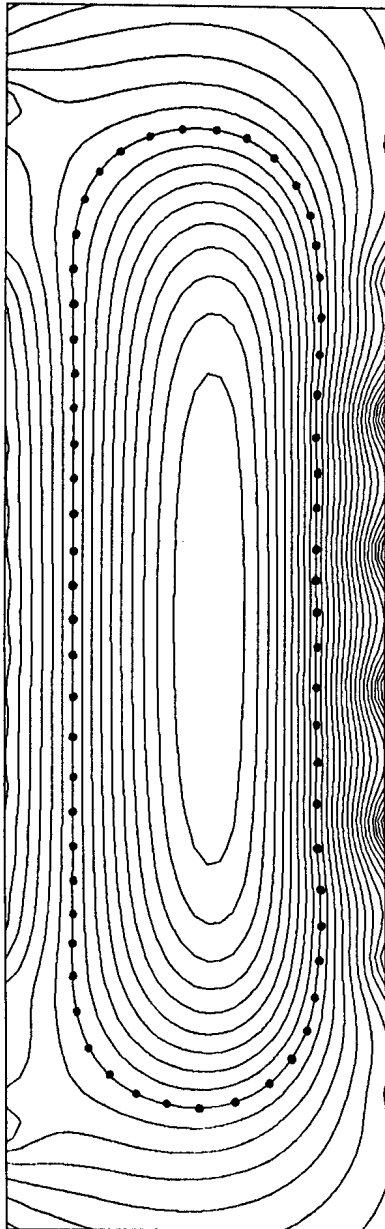
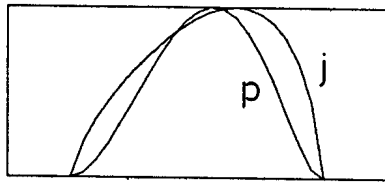


Fig. 3

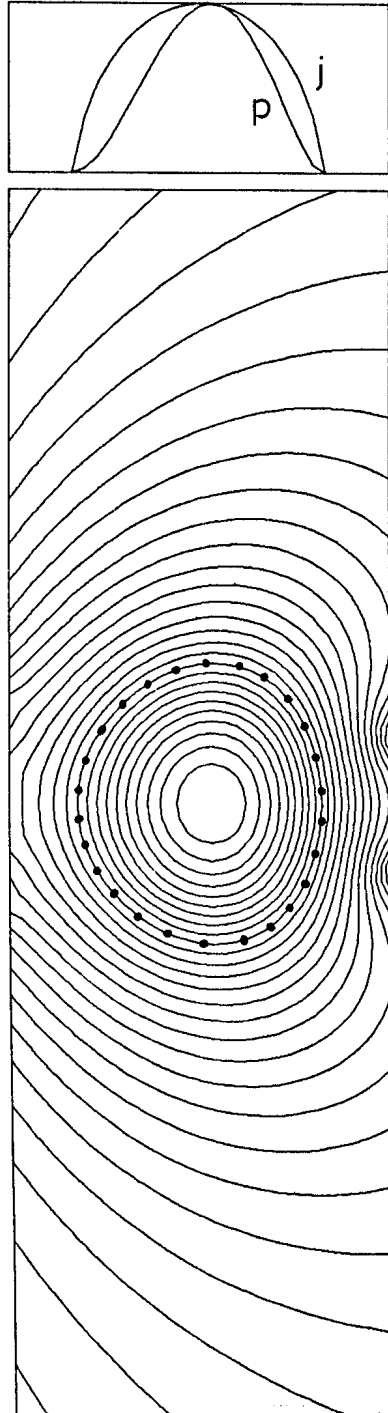


Fig. 4

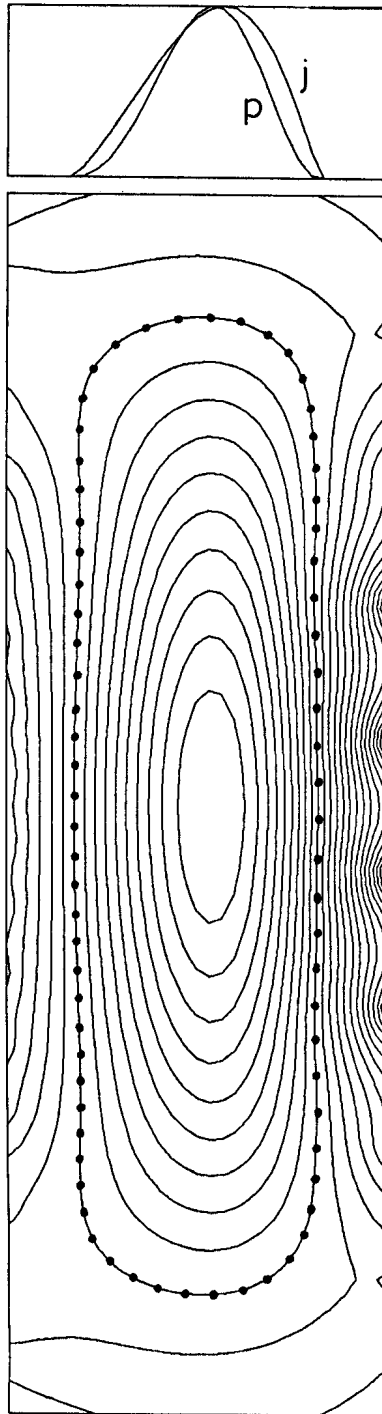


Fig. 5

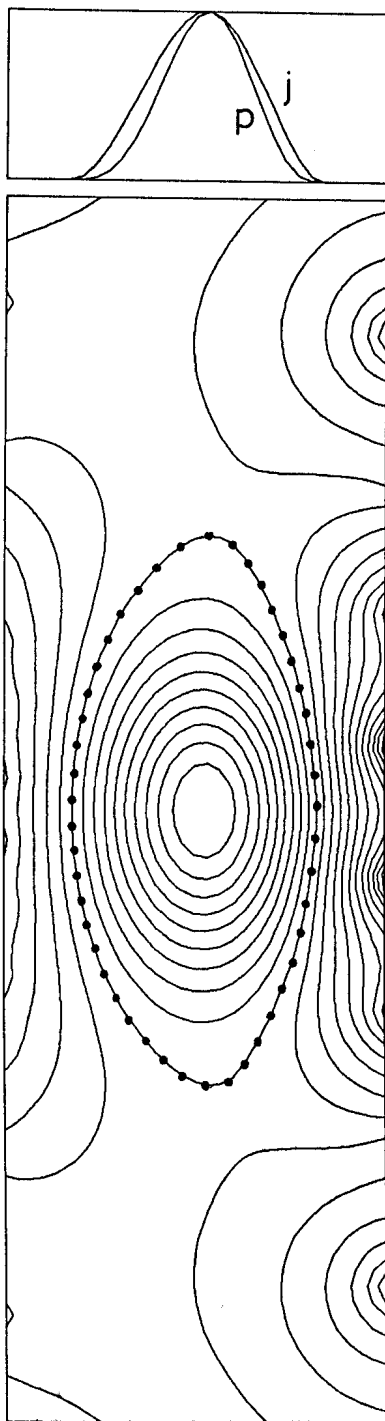


Fig. 6

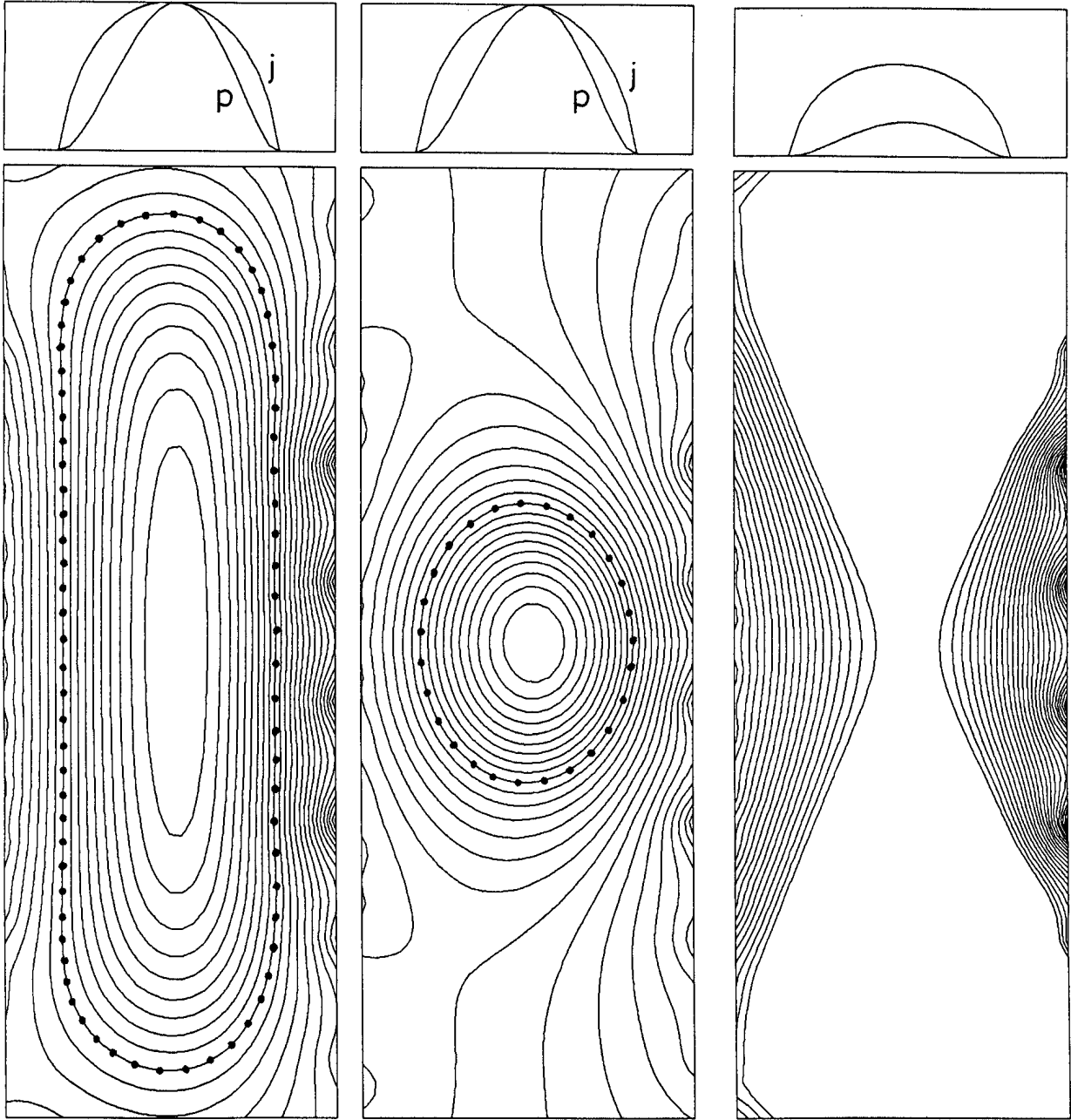


Fig. 7

(a)

(b)

(c)



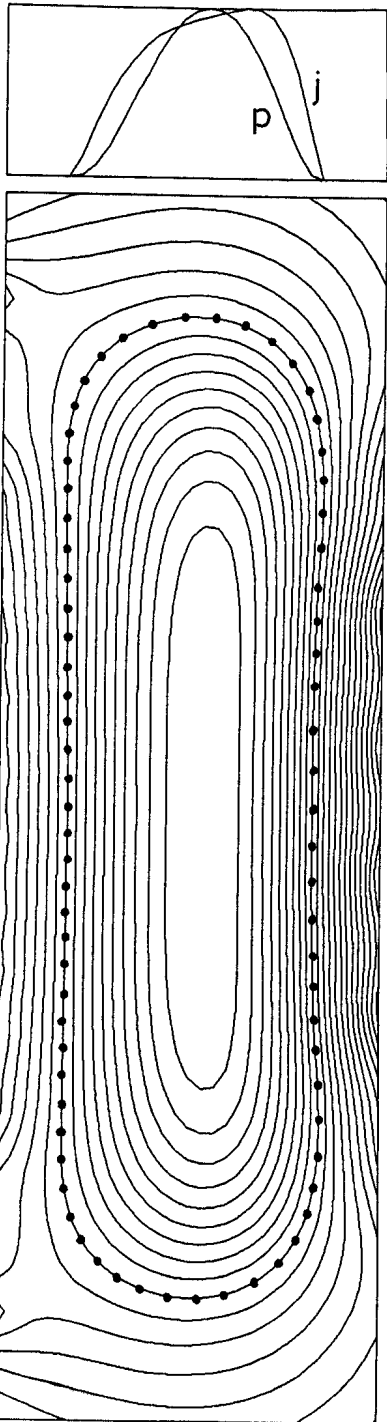


Fig. 8

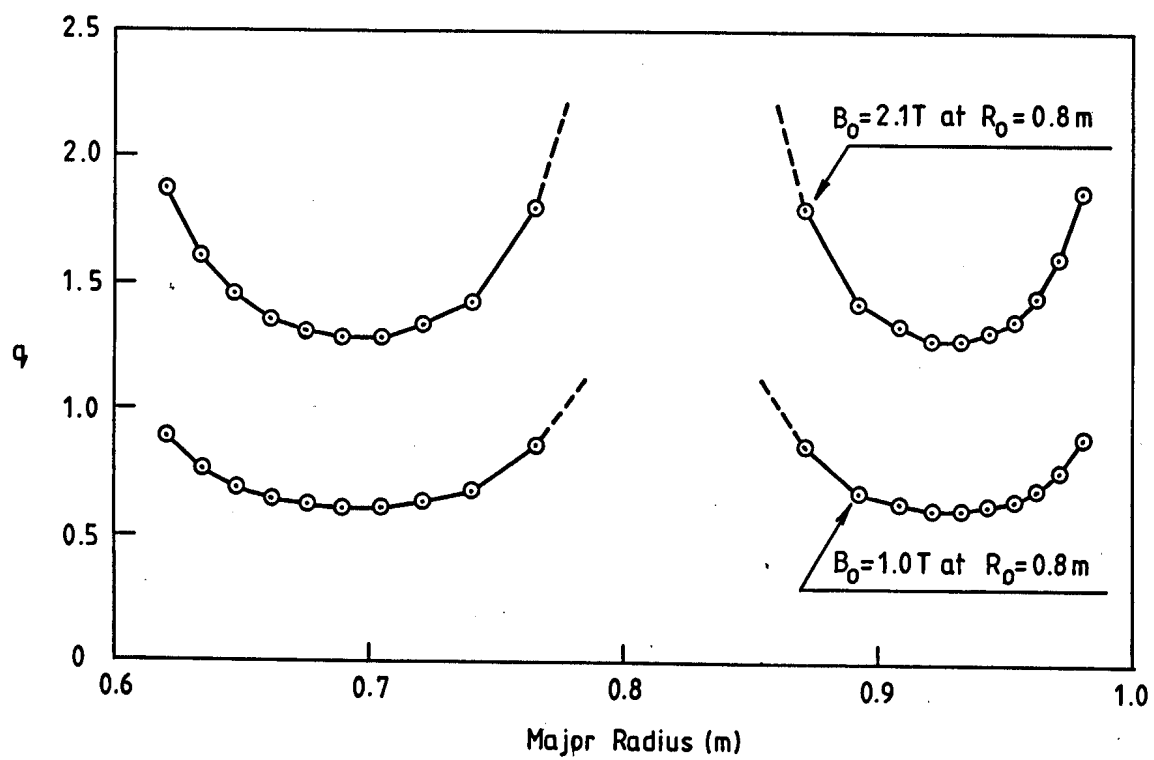


Fig. 9