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ABSTRACT

The ponderomotive force exerted by a standing electromagnetic wave in an infinite, magnetized plasma containing two ion species is examined. A perturbation analysis is developed to solve the nonlinear wave equation subject to the constraint that the total number of particles of each species is conserved. Detailed calculations are presented for an ion cyclotron wave, for which large modifications of the ion densities result from the influence of the ponderomotive force. To determine the range of validity of the perturbation analysis, the results are compared with those obtained by numerical integration of the nonlinear wave equation.

1. INTRODUCTION

It is well known that gradients of an electromagnetic wave can give rise to a nonlinear, time-independent (ponderomotive) force in a plasma. This force causes modification of both the wave and the plasma properties; for example, particles may be attracted, or repelled, from regions of high electric field strength. Several practical applications of the ponderomotive force have been proposed. In particular, for a multi-ion species plasma, preferential radio-frequency plugging as a means of impurity control (HIDEKUMA et al., 1974; HIROE et al., 1975), and isotope separation (WEIBEL, 1980) have been suggested. These applications utilize the dependence of the ponderomotive force, exerted by a wave with frequency in the vicinity of the ion cyclotron frequency, on the ion charge-to-mass ratio.

It has been previously shown (FESTEAU-BARRIOZ and WEIBEL, 1980) that the ponderomotive force of an ion cyclotron wave in a two ion species plasma can cause substantial spatial modification of the ion densities for an appropriate choice of wave frequency and electric field strength. FESTEAU-BARRIOZ and WEIBEL integrated numerically the nonlinear wave equation to obtain the self-consistent electric field and particle densities for a standing wave (with $\underline{k} = k_{/\!/}\hat{\underline{z}}$) in an infinite, magnetized ($\underline{B} = B_0\hat{\underline{z}}$) plasma. As a boundary condition, a large reservoir of plasma (occupying the half-space, $z < 0$) with each species having its unmodified concentration was assumed. The wave equation was solved in the half-space, $z \geqslant 0$. An injection, or elimination, of particles of each species across the boundary $z = 0$ was required to obtain their steady state solutions. The problem that was solved therefore consists of the convolution of two different particle motions: the movement of particles within the wave between regions of high and low electric field strength, and the particle motion across the boundary z = 0 between the negative half-space (of zero electric field) and the positive half-space (of positive average electric field strength). The total number of particles of each species was allowed to vary, indeed, the negative half-space retained the unmodified concentrations despite a change in the concentrations in the positive half-space.

In the present paper we consider two simplifications to this problem. Firstly, we solve the nonlinear wave equation subject to the physically more reasonable boundary condition that the total number of particles of each species is conserved. This choice ensures that if particles of a particular species accumulate at the maximum (minimum) of the electric field strength, then there is a corresponding depletion of particles from the region of minimum (maximum) electric field strength. Secondly, we consider a perturbation approach which yields analytical solutions of the nonlinear wave equation, and therefore a better physical understanding of the electric field and density modification than can be obtained by numerical integration.

We shall consider in this paper a standing, or cut-off, electromagnetic wave in an infinite, magnetized plasma. The wavefield is assumed to have amplitude gradients only in the direction of the magnetic field. We shall restrict the analysis to a purely electromagnetic wave (no component of oscillating electric field or particle velocity parallel to the steady magnetic field): the electrostatic (acoustic) wave will not be treated. However, both the left and the right circularly polarized waves will be considered, with no restriction on the wave frequency other than wavelength be sufficiently large ($\lambda \gg \lambda_{Debye}$).

2. NONLINEAR EQUATIONS

Consider an oscillatory wavefield which has amplitude gradients along the direction of a constant magnetic field, $\underline{B}_0 = B_0 \,\hat{\underline{z}}$, and an electric field of the form

$$\underline{E}_\omega(z,t) = E(z)\,(\cos\omega t\,,\ \pm\sin\omega t\,,\ 0)\,. \qquad (1)$$

Here, the upper (lower) sign corresponds to a right (left) circularly polarized wave. Using the fluid equation of motion for species σ,

$$n_\sigma m_\sigma\left(\frac{\partial \underline{u}_\sigma}{\partial t} + \underline{u}_\sigma\cdot\nabla\underline{u}_\sigma\right) = n_\sigma q_\sigma\left(\underline{E} + \underline{u}_\sigma\times\underline{B}\right) - \nabla p_\sigma\ , \qquad (2)$$

we obtain, from the perpendicular component, the fluid velocity

$$\underline{u}_\sigma = \frac{q_\sigma E(z)}{m_\sigma(\omega\pm\Omega_\sigma)}\,(\sin\omega t\,,\ \mp\cos\omega t\,,\ 0)\ , \qquad (3)$$

where $\Omega_\sigma = q_\sigma B_0/m_\sigma$ is the cyclotron frequency for species σ. From equation (1), Maxwell's equations yield the magnetic wavefield

$$\underline{B}_\omega = -\frac{1}{\omega}\,\frac{dE(z)}{dz}\,(\pm\cos\omega t\,,\ \sin\omega t\,,\ 0)\,. \qquad (4)$$

The parallel component of equation (2) describes the force balance along the steady magnetic field:

$$n_\sigma q_\sigma\left(E_\parallel + \underline{u}_\sigma\times\underline{B}_\omega\right) - \frac{\partial p_\sigma}{\partial z} = 0\ . \qquad (5)$$

Since, for a circularly polarized wave, the fluid velocity and magnetic wavefield are always mutually perpendicular, the vector product $q_\sigma(\underline{u}_\sigma \times \underline{B}_\omega)$ is independent of time. This constant, nonlinear (ponderomotive) force may be written, using equations (3) and (4), as

$$q_\sigma \left(\underline{u}_\sigma \times \underline{B}_\omega \right) = - \frac{\partial \Phi_\sigma}{\partial z} \hat{\underline{z}} , \tag{6}$$

where the ponderomotive potential Φ_σ is given by

$$\Phi_\sigma = \frac{q_\sigma^2 E^2(z)}{2 m_\sigma \omega (\omega \pm \Omega_\sigma)} . \tag{7}$$

In steady state, the ponderomotive force exerted on a particular species is balanced by axial pressure gradients and the time-independent parallel electrostatic field, $E_{\parallel}$, which arises from the spatial separation of the different species. Writing

$$E_{\parallel} = - \frac{\partial U}{\partial z} \qquad \text{and} \qquad p_\sigma = n_\sigma T_\sigma ,$$

we obtain from equation (5), assuming constant T_σ,

$$n_\sigma \frac{\partial}{\partial z} \left(q_\sigma U + \Phi_\sigma \right) + T_\sigma \frac{\partial n_\sigma}{\partial z} = 0 . \tag{8}$$

We therefore obtain for each species a Boltzmann distribution of density,

$$n_\sigma(z) = n_{\sigma o} \exp \left[- \frac{q_\sigma U + \Phi_\sigma}{T_\sigma} \right] . \tag{9}$$

The particle densities and the electric field strength of the wave are related via the wave equation. Using equation (3) to obtain the current densities for each species, Maxwell's equations combine to yield the following wave equation:

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2}\left[1 - \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega(\omega \pm \Omega_\sigma)}\right]E = 0 , \qquad (10)$$

where $\omega_{p\sigma} = (n_\sigma q_\sigma{}^2/m_\sigma \varepsilon_o)^{1/2}$ is the plasma frequency for species σ.

The electrostatic potential U must satisfy Poisson's equation. However, as noted by FESTEAU-BARRIOZ and WEIBEL (1980), if the wavelength is much larger than the Debye length, the plasma may be assumed to be charge neutral,

$$\sum_\sigma n_\sigma q_\sigma = 0 . \qquad (11)$$

If we neglect very high frequencies ($\omega \gg |\Omega_e|$), the wavelength of an electromagnetic wave propagating parallel to the steady magnetic field is small only in the vicinity of the cyclotron frequencies ($\omega \simeq |\Omega_\sigma|$). However, it is in these regions where kinetic effects, for example, cyclotron damping (STIX, 1962) and transit time effects (CONSOLI and HALL, 1963; DIMONTE et al. 1983) which are not treated by our fluid approach, play a dominant role. Hence, we may assume the validity of charge neutrality for the cases of interest in the present study. Equation (11) may thus be used to eliminate the electrostatic potential from equation (9).

We shall assume that the total number of particles of each species is conserved, that is, the average density $\overline{n_\sigma}$ is constant. This condition allows the determination of the constants $n_{\sigma 0}$ in equation (9).

In this paper, we shall consider a plasma consisting of two ion species (σ = 1,2) having the same temperature ($T_1 = T_2 = T_i$). The electrons (σ = e) have a possibly different temperature, T_e.

3. PERTURBATION ANALYSIS

Equations (7) and (9) when substituted into (10) yield a non-linear differential equation for the electric field strength, E(z). We shall solve this equation using a perturbation approach, assuming that

$$\delta \equiv \max \left| \frac{\Phi_1}{T_i} , \frac{\Phi_2}{T_i} , \frac{\Phi_e}{T_e + T_i} \right|$$

is small.

Equation (9) may be expanded to give the particle densities correct to $0(\delta^2)$:

$$n_\sigma(z) = \bar{n}_\sigma \Big[1 + A_\sigma (E^2 - \overline{E^2}) - (A_\sigma^2 + B_\sigma^2)\overline{E^2}(E^2 - \overline{E^2}) + \frac{1}{2}(A_\sigma^2 + B_\sigma^2)(E^4 - \overline{E^4}) \Big] , \qquad (12)$$

where the bar denotes the spatial average. The coefficients A_σ and B_σ are defined, for the electron fluid, by

$$A_e = -\frac{1}{T_e + T_i} \left(\frac{\bar{n}_1 \varepsilon_1 + \bar{n}_2 \varepsilon_2 + \bar{n}_e \varepsilon_e}{\bar{n}_e} \right)$$

$$B_e^2 = \frac{T_i}{T_e + T_i} \frac{\bar{n}_1}{\bar{n}_e} \frac{\bar{n}_2}{\bar{n}_e} \frac{(\varepsilon_2 - \varepsilon_1)^2}{T_i^2} , \qquad (13)$$

where

$$\varepsilon_\sigma = \frac{q_\sigma^2}{2 m_\sigma \omega (\omega \pm \Omega_\sigma)} .$$

For the ion species 1,

$$A_1 = A_e + \frac{\bar{n}_2}{\bar{n}_e} \frac{(\varepsilon_2 - \varepsilon_1)}{T_i} \quad ; \qquad B_1^2 = - \frac{T_e}{T_i} B_e^2 ,$$

and similarly for ion species 2.

To calculate n_σ to $0(\delta^2)$, we require E to $0(\delta)$. Taking the first two terms of equation (12), we obtain an expansion of the wave equation (10) to $0(\delta)$:

$$\frac{d^2E}{dz^2} + k_o^2 \left[1 + \alpha \left(E^2 - \overline{E^2} \right) \right] E = 0 , \qquad (14)$$

where

$$k_o^2 = \frac{\omega^2}{c^2} \left[1 + \frac{2}{\varepsilon_o} \bar{n}_e (T_e + T_i) A_e \right]$$

$$\alpha = \frac{2 \mu_o \omega^2}{k_o^2} \bar{n}_e (T_e + T_i)(A_e^2 + B_e^2) . \qquad (15)$$

Note that since terms of $0(\delta^2)$ have been neglected in equation (14), we have assumed

$$| A_\sigma^2 + B_\sigma^2 | \, \overline{E^2} \ll | A_\sigma | .$$

Using equations (15) it may be shown that for the electron fluid, this condition is equivalent to

$$| \alpha \overline{E^2} | \ll 1 . \qquad (16)$$

If in equations (12) and (14), terms proportional to $\overline{E^2}$ and $\overline{E^4}$ are neglected and $\overline{n_\sigma}$ is replaced by $n_\sigma(z=0)$, we regain the equations appropriate for the boundary condition used by FESTEAU-BARRIOZ and WEIBEL (1980). The constraint that the total number of particles of each species is conserved can be shown to have only a slight effect on the electric field profile, but may influence significantly the modification of the particle densities.

To solve equation (14) we must distinguish between two cases: a) $k_0{}^2 > 0$ and b) $k_0{}^2 < 0$.

Case a) Standing wave, $k_0{}^2 > 0$

We shall consider the boundary conditions

$$E(z=0) = E_0$$

$$\frac{dE}{dz}(z=0) = 0 \quad .$$

Here

$$\overline{E^2} = \frac{k}{2\pi}\int_0^{\frac{2\pi}{k}} E^2\, dz \quad ,$$

where k is the parallel wavenumber.

For this case, equation (14) is similar in form to that which governs the motion of an anharmonic oscillator. It may be solved for E(z) by successive approximations, as described by LANDAU and LIFSHITZ (1960), correcting the value of k at each approximation.

To 0(1):

$$E(z) = E_0 \cos k_0 z \quad .$$

To $0(\delta)$:

$$E(z) = E_o\left(1 - \frac{\alpha E_o^2}{32}\right)\cos kz + \frac{\alpha E_o^3}{32}\cos 3kz\ , \qquad (17)$$

where

$$k = k_o\left(1 + \frac{\alpha E_o^2}{8}\right) .$$

From equation (12) we obtain, to $0(\delta^2)$:

$$n_\sigma(z) = \bar{n}_\sigma\left\{1 + \frac{1}{2}A_\sigma E_o^2\cos 2kz + \left[\frac{1}{16}\left(A_\sigma^2 + B_\sigma^2\right) + \frac{\alpha}{32}A_\sigma\right]E_o^4\cos 4kz\right\}. \qquad (18)$$

For a standing wave ($k_0{}^2 > 0$) we note:

i) Since $(A_e{}^2 + B_e{}^2) > 0$, the parameter α, which governs the nonlinearity of E(z), is positive. Hence the finite amplitude of the wavefield causes a decrease in the wavelength.

ii) Since the displacement current is negligible for cases of interest (i.e., wavelength much greater than the vacuum wavelength), from equation (15), $A_e > 0$, and therefore the coefficients of the expansion (18) for $n_e(z)$ are positive. Hence the electrons move from regions of low electric field strength to regions of high electric field strength under the influence of the ponderomotive force. This behaviour is independent of the wave frequency and polarization: for example, electrons in the standing field of a left circularly polarized, ion cyclotron wave move in the same direction as in a right circularly polarized, electron cyclotron wave. This effect is a result of the different nature of the ponderomotive effect on electrons for different frequency regimes. For high frequencies (Ω_1, $\Omega_2 \ll \omega < |\Omega_e|$), the electrons move as a result of the ponderomotive force exerted upon them, while the ions do not respond to the oscillating fields but only to the zero-frequency electrostatic field which results from charge separation. At low frequencies ($\omega \lesssim \Omega_1, \Omega_2$), the ponderomotive force on the ions is dominant, and the electrons move to maintain charge neutrality.

iii) For very low frequencies ($\omega \ll |\Omega_\sigma|$), the single particle ponderomotive potential given by equation (7) becomes large. However, charge neutrality ensures that $A_e \simeq 0$ and for a singly ionized plasma $\varepsilon_1 \simeq \varepsilon_2$. Therefore no modification of the electric field profile or the particle densities results from the finite amplitude of the wave.

Case b) Cut-off wave, $k_0^2 < 0$

For this case, we shall consider only the positive z half-space, with the boundary conditions

$$E(z = 0) = E_0$$

$$E(z \to \infty) = 0 .$$

Here,

$$\overline{E^2} = \lim_{L \to \infty} \frac{1}{L} \int_0^L E^2 \, dz .$$

Writing $\gamma_0^2 = -k_0^2$, we obtain from equation (14)

To O(1):

$$E(z) = E_0 \, e^{-\gamma_0 z}$$

To O(δ):

$$E(z) = E_0 \left(1 - \frac{\alpha E_0^2}{8}\right) e^{-\gamma_0 z} + \frac{\alpha E_0^3}{8} e^{-3\gamma_0 z} . \qquad (19)$$

From equation (12) we obtain to $0(\delta^2)$:

$$n_\sigma(z) = \bar{n}_\sigma\left\{1 + \left(1 - \frac{\alpha E_0^2}{4}\right)A_\sigma E_0^2 e^{-2\gamma_0 z} + \left[\frac{1}{2}\left(A_\sigma^2 + B_\sigma^2\right) + \frac{\alpha}{4}A_\sigma\right]E_0^4 e^{-4\gamma_0 z}\right\}. \tag{20}$$

For a cut-off wave ($k_0{}^2 < 0$) we note:

i) Equation (19) yields a decay length $\simeq (1 - \alpha E_0{}^2/8)/\gamma_0$. Therefore, since $\alpha < 0$, the finite amplitude of the wave causes an increase in the decay length.

ii) $A_e < 0$, and therefore the coefficient of the expansion (20) to $0(\delta)$ is negative. Hence the electrons move from regions of high electric field strength (small values of z) to regions of low electric field strength. This is the opposite behaviour to that obtained for a standing wave. The difference results from the fact that at high frequencies (Ω_1, $\Omega_2 \ll \omega < |\Omega_e|$) for which the electrons respond directly to the ponderomotive force, the cut-off wave is left circularly polarized. The ponderomotive force is therefore opposite in direction to that of the right circularly polarized, standing wave.

iii) As for a standing wave, at very low frequencies ($\omega \ll |\Omega_\sigma|$), there is no modification of the electric field profile or particle densities in a singly ionized plasma.

4. SPECIFIC EXAMPLES

From the analysis presented in section 3, it may be seen that the effect of the ponderomotive force is greatest if the wave frequency is in the vicinity of the cyclotron frequencies (however, not so close that the kinetic effects mentioned in section 2 become important). We shall consider specific examples of a left circularly polarized wave with frequency in the vicinity of the ion cyclotron frequency (i.e., an ion cyclotron wave). For this choice the ponderomotive effect on the ions is large, and different for the two different species. The wave nonlinearity therefore causes spatial separation of the two ion species (WEIBEL, 1980).

4.1 Comparison of perturbation and complete nonlinear analyses

To determine the range of validity of the perturbation solutions, the results calculated using the analysis presented in section 3 have been compared with those computed by numerical integration of the nonlinear wave equation (10). We show here the results obtained for a neon plasma (composed of 90% Ne^{20} and 10% Ne^{22}) with $B_0 = 0.3$ T, $\bar{n}_e = 10^{16} m^{-3}$, $T_e = 5$ eV and $T_i = 0.1$ eV. The wave frequency was chosen to lie midway between the two ion cyclotron frequencies, $\omega/2\pi = 218$ kHz. For this set of parameters, $k_0{}^2 > 0$, and we therefore consider a standing ion cyclotron wave. Associating subscript 1 with Ne^{22} and subscript 2 with Ne^{20}, we find

$$\frac{\Phi_e}{T_e + T_i} \ll \frac{\Phi_1}{T_i} \lesssim -\frac{\Phi_2}{T_i} ,$$

and therefore $\delta = |\Phi_2|/T_i$.

Figure 1 a shows plots of the normalized increase in the wave-number, $(k-k_0)/k_0$, as a function of $|\Phi_2|/T_i$, calculated using the perturbation analysis and also by numerical integration of the nonlinear wave equation (10). In Fig. 1 b, similar curves for the normalized amplitude of the third harmonic of the electric field, $E^{(3)}/E_0$, are presented. The corresponding modification of the particle densities are shown in Figs. 2 a-c. Here is plotted the normalized change in density, $[n_\sigma(z)-\bar{n}_\sigma] / \bar{n}_\sigma$, for two of the values of z where $n_\sigma(z)$ is maximum and minimum.

Figures 1 and 2 show that within the range of validity of the perturbation analysis, $\delta = |\Phi_2|/T_i \ll 1$, there is agreement between the values calculated using the perturbation analysis and those obtained from numerical integration of equation (10). Indeed, the two analyses yield similar values for the changes in wavenumber and particle densities at higher values of electric field strength, $|\Phi_2|/T_i \lesssim 1$, for which it is to be expected that the perturbation approach is no longer valid.

From Figs. 1a and 1b it can be seen that both the change in wavenumber and the third harmonic amplitude are very small even at large values of $|\Phi_2|/T_i$. Fig 2c shows that there is also little modification of the electron density. For the example considered, with $T_e \gg T_i$, the strong electron pressure resists any spatial modification of the electron density (as seen from equation (8)). However, Figs. 2a and 2b show that there is a substantial modification of the ion densities. In particular, there is a large percentage change in the minority (Ne^{22}) species. This can be readily seen from the perturbation analysis, since for $A_e \ll |A_1|$, A_2 we obtain

$$\left[\frac{n_1 - \bar{n}_1}{\bar{n}_1}\right]_{max} \simeq \frac{1}{2}|A_1| E_0^2 \simeq \frac{1}{2}\frac{\bar{n}_2}{\bar{n}_e}\frac{(\Phi_1 - \Phi_2)}{T_i},$$

$$\left[\frac{n_2 - \bar{n}_2}{\bar{n}_2}\right]_{max} \simeq \frac{1}{2} A_2 E_0^2 \simeq \frac{1}{2}\frac{\bar{n}_1}{\bar{n}_e}\frac{(\Phi_1 - \Phi_2)}{T_i};$$

that is, the density change for one ion species is proportional to the density of the other ion species. Physically, for the example considered, the electron density remains approximately uniform and the two ion species move in opposite directions under the influence of the ponderomotive force, while still maintaining plasma charge neutrality.

In Figs. 3 a-c is shown for three values of $|\Phi_2|/T_i$ (i.e., 0.5, 1.0 and 1.5 corresponding to E_0 = 43.5, 61.5 and 75.3 Vm^{-1}, respectively) the spatial profiles of the normalized electric field and particle densities, calculated using the perturbation analysis and also by numerical integration. These curves are shown for a complete wavelength of the electric field.

It can be seen from Fig. 3 that, for the set of wave and plasma parameters considered, the perturbation approach yields values for the electric field and electron density that are very close to those calculated using the full nonlinear analysis, even for the highest value of electric field strength. The ion densities are also well represented by the perturbation approach for $|\Phi_2|/T_i \leq 1.0$. However, for $|\Phi_2|/T_i$ = 1.5 (well out of the range of validity of the perturbation analysis), significant differences between the values calculated by the two methods are evident.

4.2 Different plasma mixtures

Calculations have been made, using the perturbation analysis described in section 3, for various plasma mixtures. In Table 1 we present the results obtained for four different mixtures. The first is the Ne^{22}/Ne^{20} plasma considered in section 4.1. The second is a mixture of the same ion species but with a majority of Ne^{22} ions. The other two examples consider argon/neon and helium/neon mixtures with Ne^{20} as the majority ion species. For all four cases the calculations were made assuming B_0 = 0.3 T, $\overline{n_e}$ = $10^{16}m^{-3}$, T_e = 5 eV and T_i = 0.1 eV. The wave frequency was chosen to be the same for all cases, $\omega/2\pi$ = 218 kHz, as was the electric field strength, E_0 = 43.5 Vm^{-1} (corresponding to $|\Phi_2|/T_i$ = 0.5, where subscript 2 denotes the Ne^{20} species).

We first note that the second mixture (neon mixture with Ne^{22} majority) yields $k_0{}^2 < 0$ for the parameters considered: we therefore examine for this case a cut-off wave. For the other three cases, as $k_0{}^2 > 0$, a standing wave is considered.

As a quantitative measure of the effect of the ponderomotive force on each particle species, we have defined the parameter $\Delta n_\sigma = n_\sigma^{max} - n_\sigma^{min}$. In Table 1 we present the normalized values, $\Delta n_\sigma / \overline{n}_\sigma$, calculated for the electron and two ion species of each plasma mixture.

For each of the mixtures considered, Table 1 shows that there is a substantial modification of the minority ion density. The modification of the majority ion density is more modest, as was discussed in section 4.1. For the mixtures of argon/neon and helium/neon the density modification is approximately half of that calulated for the Ne^{22}/Ne^{20} mixture. This is a result of the increase in $(\omega-\Omega_1)$: the wave frequency is sufficiently distant from the minority ion species for the ponderomotive potential of this species to be small. The ponderomotive force then acts directly on the majority Ne^{20} species only, with the modification of the minority species being a result of the plasma charge neutrality.

5. CONCLUSIONS

A perturbation analysis has been developed to solve the nonlinear wave equation in a multi-species plasma subject to the constraint that the total number of particles of each species is conserved. It has been shown that this analysis yields good approximations for the electric field profile and particle density modification, even for substantial values of electric field strength. Using the perturbation approach, a deeper physical understanding of the effect of the ponderomotive force of an electromagnetic wave on a multi-species plasma may be obtained. Taking the ion cyclotron wave as an example, large modifications of the ion densities result from the ponderomotive effect in a two ion species plasma, even for cases for which there is a significant difference of ion mass for the two species.

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FIGURE CAPTIONS

Fig. 1 The normalized increase in (a) wavenumber and (b) amplitude of the third harmonic of the electric field as a function of electric field strength. The solid lines are calculated from the perturbation analysis and the dotted curves from numerical integration.

Fig. 2 The modification of the particle densities as a function of electric field strength at the maximum and minimum of the electric field calculated from the perturbation analysis (solid curves) and numerical integration (dotted curves).

Fig. 3 Profiles of the normalized electric field and particle densities for three values of electric field strength corresponding to values of $|\Phi_2|/T_i$: (a) 0.5, (b) 1.0, and (c) 1.5.

ion species	Ne^{22}/Ne^{20}	Ne^{22}/Ne^{20}	Ar^{40}/Ne^{20}	He^{4}/Ne^{20}
% $conc^{n}$	10:90	90:10	10:90	10:90
k	0.390	i0.395	0.411	0.414
$\Delta n_1 / \bar{n}_1$	0.86	0.14	0.46	0.42
$\Delta n_2 / \bar{n}_2$	0.10	1.19	0.06	0.06
$\Delta n_e / \bar{n}_e$	7.5×10^{-3}	6.8×10^{-3}	8.3×10^{-3}	8.4×10^{-3}

Table 1 : Comparison of wavenumber and density modification for four different plasma mixtures

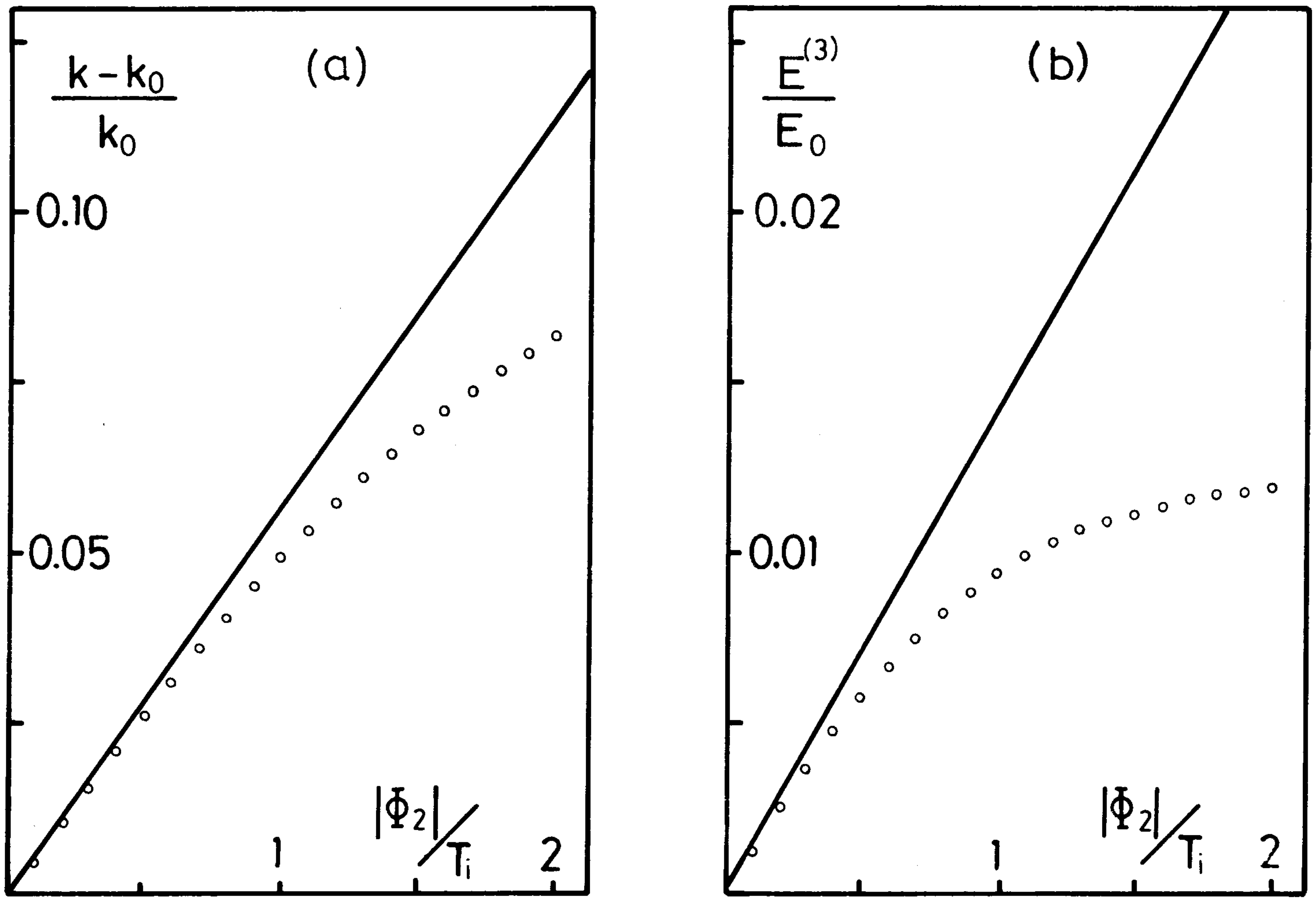
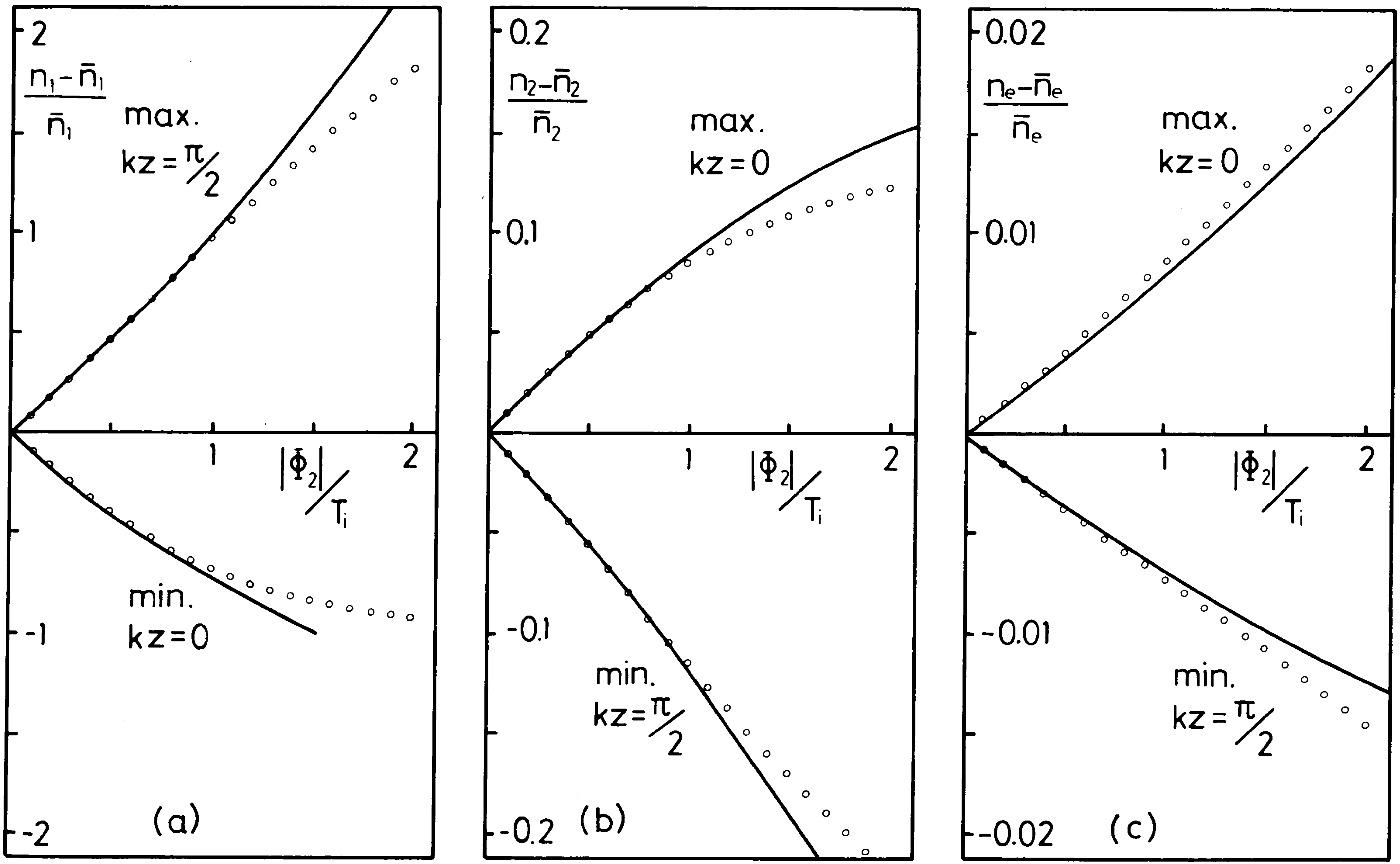


FIG. 1



2
n1-n̄1 / n̄1
max.
kz=π/2
1
-1
-2
min.
kz=0
|Φ2|/Ti
(a)
0.2
n2-n̄2 / n̄2
max.
kz=0
0.1
-0.1
-0.2
min.
kz=π/2
(b)
0.02
ne-n̄e / n̄e
0.01
-0.01
-0.02
(c)

FIG. 2

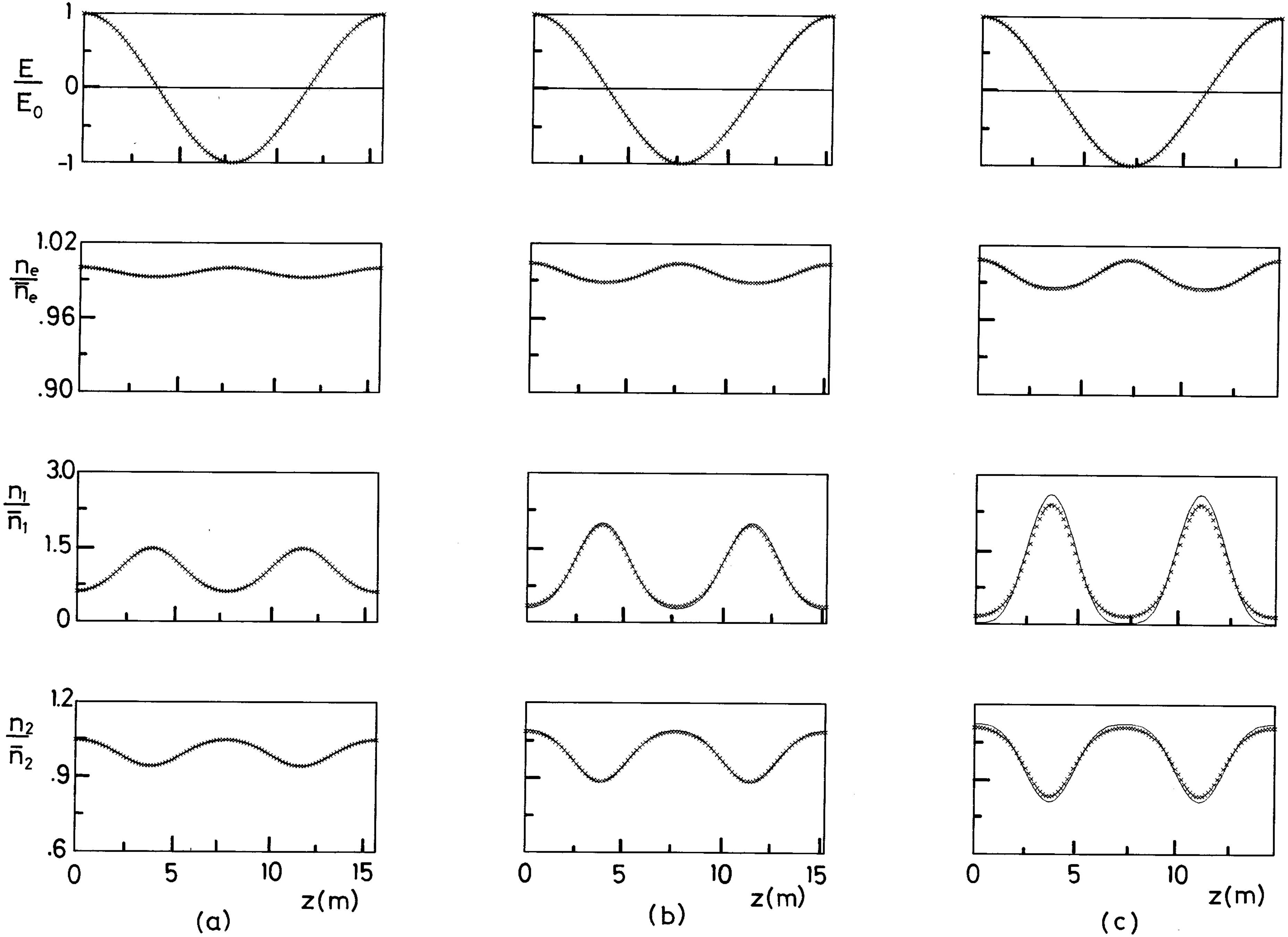


FIG. 3