

LRP 225/83

June 1983

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This is a preprint of an invited paper to be presented at the Tenth Annual Conference on Plasma Physics, Bangor, 29 June - 1 July 1983, The Institute of Physics, Plasma Physics Group.

## RUNAWAY AND WAVE-INDUCED CURRENTS

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### ABSTRACT

The present understanding (May 1983) of current drive by lower hybrid waves in Tokamak plasmas is reviewed with particular stress on the theory. First, the "classical" model of Fisch and its variants are discussed in light of recent experimental data. It is argued that these simple models do not account for most of the features of the results obtained. Next, effects of runaways on current generation are considered. It is shown that runaways, resulting from a residual ohmic electric field, may play an important role. Further, the mechanism of runaway-current sustainment, as proposed by Liu et al., is re-examined. It is shown that, for a realistic range of parameters, this mechanism does not allow a significant current to be sustained. Finally, some results of a recently-developed numerical code, which include the evolving electron distribution function, ray tracing and transport, are briefly noted. It is concluded that none of the existing theories can be used to interpret satisfactorily all the experimental observations.

## 1. Introduction

Various methods have been proposed for producing continuous toroidal currents in Tokamak plasmas (Fisch, 1980). A method which has been given extensive consideration, both theoretically and experimentally, is based on the following principle: Any wave with net momentum can generate a current by transferring its momentum, through appropriate damping mechanisms, to the charged particles in the plasma. The idea of a steady-state Tokamak, based on this principle, was first suggested by Wort (1971) who envisioned the use of low-parallel-phase-velocity magnetohydrodynamic waves to achieve current generation. His rather heuristic analysis was subsequently pursued by Klima (1973) and by Klima and Sizonenko (1975) using more general theoretical approach.

An alternative approach of using waves with high-phase velocities to drive currents was suggested by Fisch (1978). In this scheme lower hybrid waves, which have phase velocities parallel to the magnetic field that are several times the electron thermal velocity, dissipate their momentum on the electrons by the Landau damping. The resulting net toroidal momentum causes the electrons to drive toroidally, creating a current. As the electrons move they lose momentum to the ions via the Coulomb collisions, so that a steady-state current is eventually achieved. A simple theoretical analysis of this current drive mechanism was carried out by Fisch (1978) and by Klima and Longinov (1979). These studies have been followed by a number of numerical and analytical works which either improved on the accuracy of the original calculations (Karney and Fisch, 1979; Fisch and Boozer, 1980; Fisch and Karney, 1981; Harvey et al., 1981; Kritz et al., 1981; Belikov et al., 1982a; Cordey et al., 1982; Belikov et al., 1982b; Rolland, 1982) or

are concerned with various novel facets of the problem (Liu et al., 1982a; Liu et al., 1982b; An et al., 1982; Muschietti et al., 1982; Bonoli et al., 1983; Muschietti et al., 1983). Several experiments have also demonstrated the viability of lower-hybrid current drive (Luckhardt et al., 1982; Bernabei et al., 1982; Porkolab et al., 1982; Tonon et al., 1982; Tanaka et al., 1982; Alladio et al., 1982; and the references therein).

It is the scope of this paper to give an overview of the theoretical work related to the problem of current generation by lower hybrid waves and to show its relevance to the experimental observations.

The paper is structured as follows: In Section 2, the "classical" model of Fisch is reviewed and results of the model are compared with recent experimental data. Effects of runaways on current generation are discussed in Section 3. The mechanism of runaway-current sustainment by lower hybrid waves is re-examined in Section 4. Finally, the main conclusions are presented in Section 5.

## 2. "Classical" Theory of Fisch and its Variants

### 2.1 One-dimensional model

A magnetized, homogeneous plasma is considered in the presence of lower hybrid waves traveling parallel to the magnetic field in one direction. The wave spectrum, with parallel phase velocities much higher than the electron thermal velocity and perpendicular wavelengths much larger than the electron Larmor radius, is assumed to be spatially uniform and given. The

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Cerenkov interaction between the plasma electrons and the waves results in parallel velocity diffusion which competes with the collisional relaxation. Thus, the evolution of the electron distribution function is governed by

$$\frac{\partial f}{\partial t} = \left( \frac{\partial f}{\partial t} \right)_{LH} + \left( \frac{\partial f}{\partial t} \right)_c, \quad (1)$$

where

$$\left( \frac{\partial f}{\partial t} \right)_{LH} = \frac{\partial}{\partial v_{\parallel}} D_{LH} \frac{\partial f}{\partial v_{\parallel}}, \quad (2)$$

$$D_{LH}(v_{\parallel}) = 2\tilde{\omega} \int_{k_{\parallel} > 0} \frac{d^3 k}{(2\tilde{\omega})^3} W_{LH}(\vec{k}) \delta(\omega_k - k_{\parallel} v_{\parallel}) \quad (3)$$

is the quasi-linear diffusion term,  $W_{LH}(\vec{k})$  being the wave spectrum, and where

$$\left( \frac{\partial f}{\partial t} \right)_c = (2+Z) \nu_0 \frac{\partial}{\partial v_{\parallel}} \frac{1}{v_{\parallel}^3} \left( v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}} \right), \quad (4)$$

$$\nu_0 = \frac{\log \Lambda \omega_{pe}^3}{4\tilde{\omega} m v_{te}^3} \quad (5)$$

is the one-dimensional Fokker-Planck term representing the electron-electron and electron-ion collisional effects.  $Z$  denotes the ion charge state. Since the collision term is linear, it simulates situations in which dissipated energy is removed into a thermal reservoir. All the quantities in

equations (1) - (5) and throughout the paper are normalized according to  $k \rightarrow k\lambda_D$ ,  $v \rightarrow vv_{te}$ ,  $t \rightarrow t/\omega_{pe}$ ,  $f \rightarrow fn/v_{te}^3$  and  $W \rightarrow W4\pi nT_e\lambda_D^3$ .

The situation of interest for current generation occurs when  $D_{LH}$  is very large in a finite velocity interval,  $v_1 < v_{\parallel} < v_2$ , and vanishes elsewhere. The steady-state solution of equation (1) is then Maxwellian outside this interval and relatively flat in the resonant region. For  $D_{LH}^2 v_1^2 / v_0 \gg \Delta > 1/v_1$ , where  $\Delta = v_2 - v_1$ , the plateau in the resonant region acts as a fast beam with current density

$$j = \frac{\exp(-v_1^2/2)}{(2\pi)^{1/2}} \cdot \frac{v_2^2 - v_1^2}{2} \quad (6)$$

In the steady state, the wave power, dissipated by the resonant electrons, balances the power that these electrons dissipate in the bulk particles through collisional slowing down. The power density dissipated is thus given by

$$P_d = (2+z)v_0 \frac{\exp(-v_1^2/2)}{(2\pi)^{1/2}} \log\left(\frac{v_2}{v_1}\right) \quad (7)$$

In order to appreciate the power cost for generating the current a figure of merit is defined as the ratio of the current density to the power density. From equations (6) and (7) one finds

$$\eta_{LD} \equiv \frac{j}{P_d} = \frac{1}{(2+z)v_0} \langle v^2 \rangle, \quad (8)$$

where

$$\langle v^2 \rangle \equiv \frac{v_2^2 - v_1^2}{2 \log(v_2/v_1)} . \quad (9)$$

For a narrow spectrum, equation (9) reduces to

$$\langle v^2 \rangle \approx v_1^2 . \quad (10)$$

Equations (6) and (8) are the main results of this model.

## 2.2 Three-dimensional model

Axial symmetry about the magnetic field allows a reduction in the complexity of the problem from three dimensions to two dimensions in velocity space. The reduction from two to one velocity dimension in the Fokker-Planck term (used in the previous section) is achieved under the following assumption: The dependence of the electron distribution function on the perpendicular velocity is assumed to be a Maxwellian with the bulk electron temperature. In order to assess the validity of this assumption, Karney and Fisch (1979) carried out numerical studies of equation (1) with the two-dimensional Fokker-Planck term, i.e.,  $(\partial f/\partial t)_c$  given by Kulsrud et al. (1973). Also this term is linear since the collision integrals are evaluated for Maxwellian electron and ion distributions. It includes, however, the pitch-angle scattering of test electrons by ions and electrons, a fea-

ture which is missing in the one-dimensional model. The results of these studies may be summarized as follows: The numerically-obtained values of  $j$  are in excellent agreement with those given by equation (6); whereas, the figure of merit is found to be 2.5 larger than that given by equation (8) for  $Z = 1$ . This enhancement of the efficiency is a result of the perpendicular flattening of the distribution function in the resonant region. The electrons in this perpendicular region, which are the ones that lose power to the bulk distribution through collisions, have higher absolute velocities than in the one-dimensional model where a perpendicular Maxwellian is assumed. Thus, these electrons collide less frequently, dissipating less power, though carrying the same amount of current. Moreover, the numerical results reveal that the scaling of the figure of merit with  $Z$  is close to  $(5 + Z)^{-1}$  rather than  $(2 + Z)^{-1}$ . The same scaling was deduced analytically from simple physical arguments by Fisch and Boozer (1980) who have shown that

$$\eta_{2D} = 4 \frac{2+Z}{5+Z} \eta_{1D} . \quad (11)$$

However, this equation yields a value which is 1.3 times smaller than the numerical one given above. The point is that equation (11) is expected to be accurate only for values of  $D_{LH}$  which are not too large. This was not the case in the numerical studies. Indeed, subsequent numerical work by Fisch and Karney (1981) confirmed the validity of equation (11) only for smaller values of  $D_{LH}$ .



### 2.3 Further extensions of Fisch's theory

In this section we briefly discuss some additional studies. These studies include some new physical aspects of the problem but do not result in any major modifications of equation (11).

By using the full non-linear Fokker-Planck term to treat the electron-electron collisions, the problem of current generation was re-examined numerically by Harvey et al. (1981) and analytically by Cordey et al. (1982), Rolland (1982) and Belikov et al. (1982a). The conclusion, which can be drawn from these studies, is that equation (11) underestimates the current by about 15 - 20% for  $v_1 \gtrsim 3$ .

The effect of trapped electrons on current generation in Tokamak plasmas in the banana regime was investigated by Cordey et al. (1982) and Belikov et al. (1982b). It was concluded that for high-phase-velocity waves the toroidal effects are insignificant. The reason is that at high phase velocity the waves interact mainly with passing particles.

Nonlinear resonance broadening effects on the wave spectrum have been considered by Kritz et al. (1981). It was shown that when resonance broadening is treated consistently, the narrowing of the spectrum is counteracted by the resonance broadening effects on the particles. As a consequence, the wave spectrum does not narrow significantly, and the uniform spatial deposition of wave power is maintained.

## 2.4 Practical formulae and comparison with experiments

In order to compare the theoretical predictions with the experimental observations it is useful to define the following figure of merit in convenient units

$$\eta \equiv \frac{I[kA] n [10^{13} \text{ cm}^{-3}] R[\text{cm}]}{P[kW]} , \quad (12)$$

where R is the plasma major radius. On combining equations (8), (11) and (12), one finds

$$\eta_F = \frac{8.3 \times 10^3}{5+Z} \left\langle \frac{1}{N_{||}^2} \right\rangle , \quad (13)$$

where

$$\left\langle \frac{1}{N_{||}^2} \right\rangle = \left( \frac{1}{N_1^2} - \frac{1}{N_2^2} \right) \frac{1}{2 \log(N_2/N_1)} . \quad (14)$$

$N_1$  and  $N_2$  denote the lower and upper edge of the wave spectrum in terms of the parallel index of refraction.

For the total current, equation (6) yields

$$I_F [kA] = 6.8 \times 10^2 \frac{a^2 [cm] n [10^{13} cm^{-3}]}{T_e^{1/2} [keV]} \times \left( \frac{1}{N_1^2} - \frac{1}{N_2^2} \right) \exp \left( - \frac{256}{T_e [keV] N_2^2} \right), \quad (15)$$

where  $a$  is the plasma minor radius.

We have evaluated the theoretical figure of merit,  $\eta_F$  in eq. (13), using the data from experiments on several Tokamaks: Versator II (Luckhardt et al., 1982), PLT (Bernabei et al. 1982), Alcator C (Porkolab et al., 1982), Wega (Tonon et al., 1982), WT-2 (Tanaka et al., 1982) and FT (Alladio et al. 1982). We have found that the values of  $\eta_F$  are about 2 - 10 times larger than the corresponding experimental values of  $\eta$  obtained using equation (12). This can be regarded as rough agreement between theory and experiment if one takes into account the uncertainties in determining the wave spectrum and the value of  $Z$ .

We also calculated the total current, eq. (15), for the same experiments. In this case, however, it turned out that no reasonable comparison is possible. Equation (15) yields values that are negligibly small compared to the values of the total current observed in the experiments. The reason is that the experimental value of the quantity  $T_e [keV] N_2^2$ , appearing in the

argument of the exponential in equation (15), is in all the cases too small. Thus, this theory does not explain the mechanism by which the super-thermal electrons are generated by high phase-velocity waves in a relatively cold plasma. Some other features, observed in the experiments, which cannot be accounted for by this theory will be discussed later.

### 3. Effects of runaways

While it is true that in a steady-state Tokamak of the future, the inductive electric field may no longer play a role, in experiments that are underway to examine theoretical predictions for current drive, it is generally not correct to ignore the inductive field and the runaway electrons produced by this field. Before discussing the effects of runaways on current generation, however, it may be useful to recall some aspects of the "classical" runaway phenomenon.

It is well known that when a uniform electric field  $E$  is applied to a uniform plasma, a certain fraction of the electrons will run away; that is, they will gain an energy such that the electric force on them exceeds the collisional drag force. Consequently, they will accelerate indefinitely. The critical velocity for this to happen is

$$v_c = \left( \frac{E_c}{E} \right)^{1/2}, \quad (16)$$

where  $E_c$  is the so-called critical electric field which is defined as that field for which the electrons having thermal velocity become runaways.

Notice that, in the units used,

$$E_c = v_0, \quad (17)$$

where  $v_0$  is given by equation (5). In convenient units, the critical field can be evaluated using the following formula:

$$E_c [V/cm] = 4 \times 10^{-2} \frac{n [10^{13} \text{ cm}^{-3}]}{T_e [\text{keV}]} \frac{\log \Lambda}{15}. \quad (18)$$

If  $E \ll E_c$ , the situation typical for the experiments, then  $v_c \gg 1$ , and only an exponentially small fraction of electrons run away. The number of such runaways produced per unit collision time, the so-called runaway production rate, has been calculated by a number of authors (Kulsrud et al., 1973; and the references therein). The generally accepted expression for the runaway production rate is

$$A_K = 0.35 \left( \frac{E_c}{E} \right)^{3/8} \exp \left[ - \frac{E_c}{4E} - \left( \frac{2E_c}{E} \right)^{1/2} \right], \quad (19)$$

where  $Z = 1$  is assumed. We note, though, that for the purpose of the present discussion, the principal dependence of the runaway production rate on  $E$  can be obtained from a simple one-dimensional model (Parail and Pogutse, 1978). Indeed, consider the following equation for the electron distribu-

tion function

$$\frac{\partial f}{\partial t} = -E \frac{\partial f}{\partial v_{\parallel}} + \nu_0 \frac{\partial}{\partial v_{\parallel}} \frac{1}{v_{\parallel}^3} \left( v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}} \right). \quad (20)$$

A steady-state solution of this equation satisfies

$$E f = \frac{\nu_0}{v_{\parallel}^3} \left( v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}} \right) + A, \quad (21)$$

where the constant of integration A is identified as the flux of runaway electrons in the limit  $v_{\parallel} \rightarrow \infty$ , i.e.,  $A = E f(v_{\parallel} \rightarrow \infty)$ . Thus, the runaway distribution function is not normalizable. Of course, a steady-state distribution function involving a loss of particles at  $\infty$  implies an equivalent source at  $v = 0$ . A simple way to cope with this problem is to ensure a constant number of particles by keeping the Maxwellian value of f at  $v = 0$ . This assumption provides the boundary condition which is necessary for integrating equation (21). After some manipulations, one obtains the following expression for the runaway production rate:

$$A_{1D} \equiv \frac{A}{\nu_0} = \frac{1}{\sqrt{2} \mathcal{I}} \left( \frac{E}{E_c} \right)^{3/2} \exp \left( -\frac{E_c}{4E} \right). \quad (22)$$

Although the expression in equations (19) and (22) are formally different, they yield approximately the same value for the production rate (Muschietti, 1982).

Consider now a situation where the current generation, by means of lower hybrid waves, occurs in the presence of a weak d.c. electric field,  $E \ll E_C$ , which is parallel to the magnetic field. If the phase velocities of the waves are in the vicinity of the critical velocity,  $v_C$ , the accompanying quasilinear diffusion enhances the flow of electrons across  $v_C$ . As a result, an increased runaway production rate is expected. In order to quantify this assertion in a simple way, the one-dimensional model described above may be used. Equation (20) is supplemented by the quasilinear diffusion term  $(\partial f / \partial t)_{LH}$ , eq. (2), with the following simplifying assumption:

$$D_{LH}(v_{||}) = \begin{cases} \frac{D}{v_{||}^3} & , \quad v_1 \leq v_{||} \leq v_2, \\ 0 & , \quad \text{elsewhere} \end{cases} \quad (23)$$

where  $D$  is a constant, and where  $v_1$  and  $v_2$  are such that  $v_1 < v_C < v_2$ .

The resulting equation is then analysed in the same way as in the case without the quasilinear term, and the runaway production rate in the steady state can be obtained (Liu et al., 1982a). The general expression, which is rather cumbersome, was evaluated numerically. It was shown that even for moderate values of  $D/v_0$ , the runaway production rate is enhanced many orders of magnitude over that without the waves ( $D = 0$ ). With  $D \neq 0$ , even a very small electric field  $E/E_C = 10^{-3}$  can result in a significant rate of runaway production. For various limiting cases, analytical formulae were also derived. As an example, if  $v_1 \ll v_C \lesssim v_2$ , the expression for the production rate becomes

$$A_W = \frac{1}{\sqrt{2} \tilde{\kappa}} \left( \frac{E}{E_c} \right)^{3/2} \exp \left\{ - \frac{1 + 2 (N_1/N_c)^2 D/v_0}{4 (E/E_c) (1 + D/v_0)} \right\}. \quad (24)$$

It is just the factor  $(1 + D/v_0)^{-1}$  in the argument of the exponential which makes  $A_W$  significantly enhanced over  $A_{1D}$ , eq. (22), when  $D/v_0 > 1$ .

The model described above is limited by an assumption about the wave spectrum, given in eq. (23). This wave spectrum is not self-consistent with the distribution function but is given a priori. Therefore, a direct estimation of the strength of the r.f. source involved is impossible. A more comprehensive study, which relaxes this limitation, was provided by Muschietti et al. (1982a). In that work the following set of equations is considered

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_{\parallel}} \left[ D_W \frac{\partial f}{\partial v_{\parallel}} - E f + \frac{\gamma_0}{v_{\parallel}^3} \left( v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}} \right) \right], \quad (25)$$

with

$$D_W = \frac{1}{2\tilde{\kappa} v_{\parallel}^3} \int_0^{\tilde{\kappa}/2} d(\cos \theta) \cos \theta W(k = v_{\parallel}^{-1} \theta), \quad (26)$$

and

$$\frac{\partial W}{\partial t} = 2 \left( \gamma - \frac{\gamma_0}{2} \right) W + S(k, \theta), \quad (27)$$



with

$$\gamma = \frac{\pi}{2} \frac{\cos \theta}{k^2} \left. \frac{\partial f}{\partial v_{||}} \right|_{v_{||} = k^{-1}}, \quad (28)$$

where the dispersion relation of the waves is assumed to be  $\omega = \cos \theta$ ,  $\theta$  being the angle between the magnetic field and the wave vector  $\vec{k}$ . The term  $S(k, \theta)$ , in equation (27), represents an external source that drives the waves. For simplicity, its form is assumed to be

$$S(k, \theta) = \begin{cases} S \delta(\cos \theta - \cos \theta_0), & k_2 < k < k_1, \\ 0 & \text{elsewhere,} \end{cases} \quad (29)$$

where  $S$  is a constant.

A search for a steady-state solution of equations (25) and (27) requires that two limiting cases be distinguished:  $|\gamma| \gg v_0$  (weak source) and  $|\gamma| \ll v_0$  (strong source). In a weak source limit one finds a simple expression for the runaway production rate in terms of the source strength:

$$A_w = \frac{S}{2\pi^2 v_0} \left( \frac{E}{E_c} \right)^{5/2}. \quad (30)$$

This formula holds if

$$S \ll S_c \equiv 2\tilde{\mu}^2 \nu_0 \nu_1^3 f(\nu_1), \quad (31)$$

where

$$f(\nu) = \frac{1}{\sqrt{2\tilde{\mu}}} \exp \left[ \frac{\nu^2}{2} \left( \frac{\nu^2}{2\nu_c^2} - 1 \right) \right] \quad (32)$$

and  $\nu_1 = k_1^{-1} \ll \nu_c$ .

In the case of a strong source,  $S \gg S_c$ , one obtains the results previously found with the inconsistent treatment of the wave spectrum (Liu et al., 1982a) described above. In fact, the runaway production rate saturates at

$$A_w = \frac{1}{\sqrt{2\tilde{\mu}}} \frac{\left[ (\nu_2/\nu_c)^2 - 1 \right] \left[ 1 - (\nu_1/\nu_c)^2 \right]}{\nu_2^2 - \nu_1^2} \exp \left[ -\frac{\nu_1^2}{2} \left( 1 - \frac{\nu_1^2}{2\nu_c^2} \right) \right] \quad (33)$$

for

$$S > S_s \equiv \frac{\tilde{\mu} \nu_0^2}{\cos \theta_0} \left( \frac{\nu_2^2}{\nu_c^2} - 1 \right) \left( 1 - \frac{\nu_1^2}{\nu_c^2} \right) \nu_c^2 \quad (34)$$

which is typically much smaller than  $S_c$ .

The two-dimensional effects of collisional pitch-angle scattering are neglected in the work described up to this point. Some consideration has been given to these effects for the case of an infinite range spectrum (An et al., 1982). The case of a finite range spectrum is presently being studied by the authors. Some preliminary results were reported recently (Kritz et al., 1983).

A comparison between the theoretical results, reviewed in this Section, and the experimental results is not straightforward. For a brief qualitative comparison of theory and experiment the reader is referred to a discussion by Muschietti et al. (1982a). A quantitative comparison remains to be carried out. To this end, however, the theoretical model must be improved. The physical description should be sufficiently complete so that the resulting distribution function can be normalized and the corresponding current, calculated. It should be noted that present current-drive experiments always involve at least a small residual electric field which, together with the applied r.f. power, may be sufficient to generate a significant number of (runaway, slide-away) superthermal electrons.

#### 4. Runaway-current sustainment by lower-hybrid waves

One of the principal problems in lower-hybrid current-drive experiments is the explanation of the mechanism by which the superthermal electrons are generated by high phase-velocity waves in relatively cold target plasmas. Recently, Liu et al. (1982b) proposed that the combination of a small population of runaway electrons, together with nonlinearly excited plasma

waves, can provide the mechanism that bridges the velocity gap between the high phase-velocity lower-hybrid waves and the low parallel-velocity electrons. Before discussing this mechanism in more detail, however, it is necessary to recall some aspects of the nonlinear theory of a runaway distribution.

The "classical" runaway distribution is a very flat function of  $v_{\parallel}$  for  $v_{\parallel} > v_c$  with electrons that have more parallel energy than perpendicular. It has been shown by Kadomtsev and Pogutse (1968) that this anisotropy can destabilize magnetized plasma waves that have phase velocity  $\omega_k/k_{\parallel} = 1/k > v_c$  through the anomalous Doppler resonance  $\omega_{ce} = k_{\parallel}v_{\parallel}$ , where  $\omega_{ce}$  is the electron cyclotron frequency. The nonlinear stage of this instability has been considered by <sup>a</sup>number of authors (Muschiatti, 1982; and the references therein). For the purpose of the present discussion the relevant theory is that developed by Muschiatti et al. (1982b). In that work the evolution of the runaway kinetic instability is described by the following set of equations

$$\begin{aligned} \frac{\partial f}{\partial t} = v_0 \frac{\partial}{\partial v_{\parallel}} \frac{1}{v_{\parallel}^3} \left( v_{\parallel} f + \frac{\partial f}{\partial v_{\parallel}} \right) - E \frac{\partial f}{\partial v_{\parallel}} \\ + \frac{\partial}{\partial v_{\parallel}} D_0 \frac{\partial f}{\partial v_{\parallel}} + \left( \frac{\partial}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \right) D_1 \left( \frac{\partial f}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}} \frac{\partial f}{\partial v_{\perp}} \right), \end{aligned} \quad (35)$$

where

$$D_0 = 2\tilde{\pi} \int_{k_{\parallel} > 0} \frac{d^3 k}{(2\tilde{\pi})^3} \left( \frac{k_{\parallel}}{k} \right)^2 W_k \delta \left( \frac{k_{\parallel}}{k} - k_{\parallel} v_{\parallel} \right) \quad (36)$$

and

$$D_1 = \frac{\tilde{\nu}}{2} \int_{k_{\parallel} > 0} \frac{d^3 k}{(2\tilde{\nu})^3} \left(\frac{k_{\parallel}}{k}\right)^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}}\right)^2 W_k \delta(\omega_{ce} - k_{\parallel} v_{\parallel}) \quad (37)$$

are the quasilinear diffusion coefficients due to the Čerenkov and anomalous Doppler interactions, respectively. The spectrum of the magnetized plasma waves,  $W_k$ , is governed by the equation

$$\frac{\partial W_k}{\partial t} = 2 \left( \gamma_0 + \gamma_1 - \frac{\nu_0}{2} \right) W_k, \quad (38)$$

where

$$\gamma_0 = \frac{\tilde{\nu}}{2} \frac{k_{\parallel}^2}{k^3} \int d^3 v \frac{\partial f}{\partial v_{\parallel}} \delta\left(\frac{k_{\parallel}}{k} - k_{\parallel} v_{\parallel}\right), \quad (39)$$

and

$$\gamma_1 = \frac{\tilde{\nu}}{8} \frac{k_{\parallel}^2}{k^3} \int d^3 v \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}}\right)^2 \left( \frac{\partial f}{\partial v_{\parallel}} - \frac{v_{\parallel}}{v_{\perp}} \frac{\partial f}{\partial v_{\perp}} \right) \delta(\omega_{ce} - k_{\parallel} v_{\parallel}). \quad (40)$$

Equations (35) - (40) were solved numerically using the procedure described by Muschietti et al. (1981). It was found that the system reaches a quasi-steady turbulent state, accessible from different initial conditions. Moreover, it was demonstrated that this state can be described by a simple analytical model. In this model, the distribution function is assumed to be of the form

$$f = \frac{F(v_{\parallel})}{2\pi T_{\perp}} \exp\left(-\frac{v_{\perp}^2}{2T_{\perp}}\right), \quad (41)$$

where the quantity  $T_{\perp}$  is a slowly increasing function of time and its value is determined by the initial conditions. The parallel velocity space is divided into three regions that correspond to three different mechanisms which may balance the acceleration of the electrons caused by the electric field. In the region  $v_{\parallel} < v_c$ , the Coulomb collisions balance the electric field so that the reduced parallel distribution  $F$  remains quasi-Maxwellian. In the region  $v_c < v_{\parallel} < v_d \equiv v_c \omega_{ce}$ , the flux due to the electric field is balanced by the backward flux caused by the Čerenkov interaction so that

$$EF = D_0 \frac{\partial F}{\partial v_{\parallel}}, \quad v_c < v_{\parallel} < v_d. \quad (42)$$

Since  $D_0$  is much larger than  $E$ , the solution for  $F$  is approximately

$$F \approx F(v_c), \quad v_c < v_{\parallel} < v_d. \quad (43)$$

Although  $F$  is nearly constant, it should be noted that equation (42) implies  $\partial F / \partial v_{\parallel} > 0$ . Finally, in the region  $v_{\parallel} > v_d$ , the flux due to the electric field is balanced by the backward flux due to the anomalous Doppler interaction. Therefore,

$$EF = \bar{D}_1 \left( T_{\perp} \frac{\partial F}{\partial v_{\parallel}} + v_{\parallel} F \right), \quad v_{\parallel} > v_d, \quad (44)$$

where the bar denotes the average over the perpendicular velocities (using the Maxwellian in equation (41)). Since  $D_{\perp}$  is much larger than  $E$ , the approximate solution for  $F$  is

$$F = F(v_c) \exp\left(-\frac{\bar{v}_{\parallel}^2 - \bar{v}_d^2}{2T_{\perp}}\right), \quad \bar{v}_{\parallel} > \bar{v}_d. \quad (45)$$

Thus, the quasi-steady-state distribution is characterized by the three parameters:  $v_c$ ,  $v_d$  and  $T_{\perp}$ .

Following Liu et al. (1982b), let us assume that the electric field is switched off, but that before the distribution function has time to relax significantly, lower hybrid waves of very high parallel phase velocity,  $v_1 < v_{\parallel} < v_2$ , are launched. Optimal runaway-current sustainment should occur when  $v_1 = v_d$  and  $v_2 = \omega_{ce}v_d$ . Under these conditions, the lower hybrid waves can quasilinearly diffuse the energetic electrons in  $v_{\parallel}$ -space through the Čerenkov resonance and, thereby, effectively increase the electron parallel energy. With this increment in parallel energy, the originally isotropic distribution for  $v_{\parallel} > v_d$  becomes anisotropic, and the magnetized plasma waves can again be destabilized. Liu et al. assume, however, that a new quasi-steady state will be achieved. Below we show how they utilize the model of Muschietti et al., outlined above, to describe the new quasi-steady state.

The term  $EF$  in equation (44) is replaced by  $D_{LH}(\partial F/\partial v_{\parallel})$  with  $D_{LH}$  given by equation (23). They assume that the same term in equation (42) can

be replaced by the collision term  $-(v_{\perp}/v_{\parallel}^3)(v_{\parallel} F + (\partial F/\partial v_{\parallel}))$ . The resulting equation implies, however,  $\partial F/\partial v_{\parallel} < 0$  which, as noted above, is contrary to the situation when the electric field is present. In order to be able to solve for the distribution function, a spectrum for the waves is required. They assume that the spectrum is  $W_K \sim \delta(\theta - \theta_0)$ . The modified equations (42) and (44), combined with the marginal stability condition,  $\gamma_1 = \nu_0/2 - \gamma_0$ , yield the distribution function corresponding to the new quasi-steady state. This distribution function results in the following current and power densities:

$$j = F(\nu_c) \frac{\nu_z^2}{2} \left( 1 - \frac{\nu_0 \omega_{ce} \nu_z^2}{4 \cos \theta_0 D} \right), \quad (46)$$

$$P_d = F(\nu_c) \frac{\nu_0 \omega_{ce}}{\cos \theta_0} \log \left( \frac{\nu_z}{\nu_d} \right). \quad (47)$$

It turns out, however, that the new steady state, associated with the proposed mechanism for current sustainment, leads to an inconsistency. This is indicated below.

If there is no electric field to accelerate the electrons in the region  $\nu_c < v_{\parallel} < \nu_d$ , the slope of the distribution function in this region is negative. However, this result is inconsistent with pitch-angle diffusion due to the anomalous Doppler effect which occurs in the adjacent region  $v_{\parallel} > \nu_d$ . The pitch-angle diffusion must manifest itself in the region  $\nu_c < v_{\parallel} < \nu_d$  resulting in a local positive slope for the distribution function. Consequently, a quasi-steady state cannot be achieved.



In order to verify this assertion, Muschietti et al. (1983) have tested the model numerically.

Equations (35) - (40), in which the electric field was omitted, were solved for a typical set of parameters, suggested by Liu et al. The initial distribution was given by equations (43) and (45). It was found that the pre-formed runaway tail cannot be maintained after E is switched off and that the electrons carrying the current relax towards the bulk of the distribution. The characteristic time of the relaxation is given by the slowest time among all the different processes involved, namely, the collision time. Therefore, the fraction of electrons in the tail evolves according to

$$\frac{\Delta n}{\Delta n_0} = \exp \left[ - \frac{\nu_0 t}{\nu_c^2 (\nu_d - \nu_c + T_{\perp} / \nu_d)} \right]. \quad (48)$$

This formula agrees reasonably well with our computation experiments. With typical parameters  $\nu_c = 3$ ,  $\nu_d = 9$ ,  $T_{\perp} = 50$  it yields an e-folding time of the order 100 collision times. Thus, it is concluded that the mechanism proposed by Liu et al. does not allow a significant steady-state current to be sustained by only the launched lower hybrid waves.

## 5. Conclusions

Before comparing theory and experiment, we first summarize some typical features of the data obtained from experiments on lower-hybrid current drive in Tokamak plasmas:

- 1) An effective generation of a steady-state current, about 1A of current per 1W of incident r.f. power, has been achieved.
- 2) The launched lower hybrid waves have too high phase velocity, as compared to the electron thermal velocity of the target plasmas, so that the direct Čerenkov interaction cannot be operative. Yet there is a large population of superthermal electrons as evidenced by the enhanced nonthermal soft-X-ray and synchrotron radiation.
- 3) For a given frequency, there exists a sharp density threshold above which there is no current drive. This density threshold appears to satisfy an empirical scaling  $\omega/\omega_{LH} \approx 2$ .
- 4) In some cases, the synchrotron radiation, loop voltage, and soft and hard X-rays exhibit relaxation oscillations. These are similar to those observed in very low density Tokamak discharges (Brossier, 1978).

As already mentioned, the Fisch theory (Section 2) can satisfactorily account only for item 1. One may speculate that an improved version of the theory that includes the residual electric field (Section 3) could explain

items 1 and 2. The same is true for the theory of Liu et al. (Section 4) if it is applied to a short-time-scale experiment.

Recently, Bonoli et al. (1983) have developed a computational model that includes a radial transport code in conjunction with a one-dimensional Fokker-Planck calculation and a toroidal ray tracing code. Current drive simulations of PLT and Alcator C have been carried out. The results obtained appear to be in reasonable qualitative agreement with the experimental observations as far as items 1 and 2 are concerned. The physical mechanism in the model which allows the high phase-velocity waves to interact eventually with the lower velocity electrons is the upshift in the parallel wavenumber prior to damping that results from toroidal effects. Typically, the  $k_{\parallel}$  of a ray that is launched with low  $k_{\parallel}$  undergoes moderate periodic variations over the course of several edge reflections until a large upshift occurs and the ray damps.

To the best of our knowledge there are no theoretical models that would reasonably explain items 3 and 4. Thus, one may conclude that none of the existing theories, reviewed in this paper, can be used to interpret satisfactorily all of the experimental observations.

#### Acknowledgements

The author wish to acknowledge the useful discussions with Dr. E. Canobbio and Dr. E. Valeo. This work has been supported by the Ecole Polytechnique Fédérale de Lausanne, by the Swiss National Science Foundation and by Euratom.

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