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ABSTRACT

Equilibrium configurations which are stable with respect to ideal magnetohydrodynamic perturbations and the corresponding beta limitations ($\simeq$ 25%) are obtained for reversed field pinches with ideal (delta-function) as well as diffused surface current profiles. Since the surface current can be driven by a radio-frequency wave, such a scheme makes continuous operation of the reversed field pinch feasible.

I. INTRODUCTION

To explain the stable regime of the Zeta operation [1] Taylor [2] introduced a conjecture that the plasma reaches a self-organized state in which the magnetic field energy is minimized with a constraint that the volume integrated magnetic helicity remains constant. This state is found to be a force free equilibrium with the toroidal magnetic field reversed in the plasma. To date, a number of experiments on reversed field pinches (such as ZT-40 at Los Alamos, ETA BETA at Padua, HBTX1 at Culham and TPE1R(M) at Tokyo) have revealed supporting evidence of Taylor's conjecture. [3]

However, the force-free equilibrium obtained by Taylor has a major shortcoming in that the plasma pressure should either be zero or the plasma should be confined by a wall. Attempts have been made to modify the Taylor equilibrium so that a finite pressure gradient is allowed by a careful programming of a perpendicular (with respect to the magnetic field) current $\mathbf{J}_\perp$. [4] However, the realization and maintenance of the desired $J_\perp$ profile in a real experiment is practically impossible unless the profile is of a type which the plasma naturally chooses. One possible remedy for this problem is to drive the current by means of appropriate radio frequency waves. Recent results of successful rf current drive in Tokamaks [5,6] warrant such a possibility. In addition, this scheme is attractive also as a dc operation of the reversed field pinch. If an rf is used, the current can be deposited in an appropriate location in a plasma using the local resonant condition. In particular, if an Alfvén wave is used, both $\mathbf{J}_\perp$ and $\mathbf{J}_\parallel$ are produced at the resonance surface because of the local heating of electrons. [7] If the plasma density has a relatively flat profile, the resonance occurs on the plasma surface. Hence, it is interesting to study the properties of a reversed field pinch confined by a surface current.

In this manuscript, we present equilibrium and stability properties of a reversed field pinch with a surface current and obtain a scheme for achieving a maximum beta for the plasma which is appropriately separated from the wall.

In Sec. II, we present an example of an ideal equilibrium with a delta-function surface current in a cylindrical plasma. Here we first obtain the condition for which the scheme is compatible with the requirement of the minimum energy condition. We then obtain two important necessary conditions for the stability of the pinch. First we show that the polarity of the surface current should be such that it enhances the bulk current (inequality (19) and Fig. 1). Second we show that when a $\mathbf{J}_\perp$ is introduced on the surface $\mathbf{J}_\parallel$ should be introduced simultaneously (inequality (20')). In Sec. III, we present the stability analysis and show the range of stability in the plane of pinch parameter (λa) and b/a using the numerical code THALIA, a one-dimensional magnetohydrodynamic stability program. [8] Here b and a are the wall and the plasma radius, respectively.

The maximum average beta achievable assuming a poloidal surface current is found to be approximately 25% with both ideal and diffused current profile for $b/a \simeq 1.5$. The average beta value is found to increase with the introduction of a toroidal surface current with appropriate polarity and magnitude with respect to the poloidal surface current.

II. EQUILIBRIUM AND NECESSARY CONDITION OF STABILITY

To study the compatibility condition of the presence of a surface current and the condition of minimum energy, let us first derive the Taylor state [2] with particular emphasis on the existence of a plasma boundary.

The variation which minimizes the magnetic field energy under a constraint that the magnetic helicity is kept constant reads,

$$\delta \int B^2 dV - \lambda\delta \int \mathbf{A} \cdot \mathbf{B} dV = 0 \quad , \tag{1}$$

where **B** is the magnetic flux density, **A** is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$, λ is the Lagrange multiplier, and the integration is to be carried out over the entire volume inside a conductive wall.

For convenience to be seen later, we take a variation with respect to the partial time derivative of the vector potential $\delta_t \mathbf{A}$,

$$\int \delta_t \mathbf{A} \cdot (\nabla \times \mathbf{B} - \lambda \mathbf{B}) dV + \oint [\delta_t \mathbf{A} \times (\mathbf{B} - \frac{\lambda}{2}\mathbf{A})] \cdot d\mathbf{S} = 0 \quad . \tag{2}$$

Here $\oint d\mathbf{S}$ is the surface integral at any surface(s) on which the fields become discontinuous and $\int dV$ is the principal value integral, that is, the volume integral which excludes the discontinuous surface(s) within the entire volume.

If the plasma is in contact with a perfectly conducting wall, $\delta \mathbf{A}_t \times d\mathbf{S} = 0$ on the wall. Therefore, Taylor argues, to satisfy Eq. (2) for any $\delta_t \mathbf{A}$,

$$\Delta \times \mathbf{B} - \lambda \mathbf{B} = 0 \tag{3}$$

should be true.

However, two remarks should be made here. First, if the wall conductivity is finite, the tangential component of the electric field, $\delta A_t \times dS$, is not zero, thus Eq. (3) is no longer the solution of the variation (1). Second, even for a case of a perfectly conductive wall, if the plasma conductivity were also infinite, both B^2 and $\mathbf{A}\cdot\mathbf{B}$ are exactly conserved, thus Eq. (1) becomes an identity. Namely, in this case Eq. (1) is satisfied without demanding Eq. (3). Therefore, the Taylor solution is valid only when the wall conductivity is much larger than the plasma conductivity. When the plasma is not in contact with a material wall, the surface integral should be evaluated also on the plasma surface. In this case the plasma surface conductivity becomes an important parameter. To understand the role of the plasma conductivity, let us write δA_t in terms of the current density variation $\delta\mathbf{J}$,

$$\nabla A_t = \mathbf{v}\times\mathbf{B} + \nabla\phi - \eta\cdot\delta\mathbf{J} \quad , \tag{4}$$

where ϕ is a scalar function, $\mathbf{v}$ is the velocity field and η is the (anisotropic) resistivity. As is expected, in the absence of the resistivity, Eq. (2) becomes an identity relation if Eq. (3) is used together with the incompressibility condition, $\nabla\cdot\mathbf{v} = 0$. In the presence of a resistivity, Eq. (2) becomes

$$\int \eta_{\parallel}\delta\mathbf{J}\cdot(\nabla\times\mathbf{B}-\lambda\mathbf{B})dV \tag{5}$$

$$- \oint \eta_{\perp}\delta\mathbf{J}_{\perp}\times\mathbf{B}\cdot d\mathbf{S} = 0 \quad ,$$

where $\eta_{\perp}$ and $\eta_{\parallel}$ are the perpendicular and parallel resistivities and the surface integral is evaluated on the plasma surface. Equation (5) clearly indicates that if $\eta_{\parallel} >> \eta_{\perp}$, the force-free solution, Eq. (3) still gives the minimum energy state.

Experimentally, the force free configuration is established within a certain time period which is much shorter than the classic resistive or diffusive time scale. This is considered to be the consequence of the self-organization of turbulence. [9] Therefore, if the perpendicular resistivity $\eta_\perp$ remains classical, the second term in Eq. (5) is negligible on the time scale of the self-organization. Hence if $\eta_\parallel$ remains much larger than $\eta_\perp$, the presence of a surface current is still compatible to the requirement of the minimum energy. Whether such a condition is satisfied or not depends on experiments. However, if a strong shear is present there seems to exist no micro instability due to the $\mathbf{J}_\perp$ [10] thus $\eta_\perp$ may remain classical and the self-organization with the surface current can be achieved.

Let us consider a cylindrical plasma with radius a surrounded by a coaxial cylindrical conductor with radius b(>a). If the plasma pressure is uniform, ∇p exists only at r = a where **B** has a jump. Such an equilibrium is given by

$$\nabla \times \mathbf{B} - \lambda \mathbf{B} = \mu_0 \mathbf{J}_S \delta(r-a) + f(r)\mathbf{B} \quad , \tag{6}$$

where f(r) may be chosen such that **J** = 0 in the vacuum region, a < r < b, and force-free in the plasma, r < a, i.e.

$$\begin{aligned} f(r) &= 0 \qquad r < a \\ &= -\lambda \quad a < r < b \quad . \end{aligned}$$

The surface current $\mathbf{J}_s$ may be chosen to satisfy the equilibrium condition,

$$\mathbf{J} \times \mathbf{B} = \nabla p \quad , \tag{7}$$

where $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, as

$$\mathbf{J}_s \delta(r-a) \times \mathbf{B} = \nabla p = -\, p_0 \delta(r-a)\hat{r} \quad . \tag{8}$$

An appropriate solution of Eq. (6) is given by, at r < a,

$$B_z(r) = B_0 J_0(\lambda r) \tag{9}$$

$$B_\theta(r) = B_0 J_1(\lambda r) \quad , \tag{10}$$

which is the Taylor solution. Here J_0 and J_1 are Bessel functions. In the vacuum region $a < r < b$, $\nabla \times \mathbf{B} = 0$ and $\nabla \cdot \mathbf{B} = 0$ give

$$B_z = \mu_0 J_{w\theta} \tag{11}$$

$$B_\theta = -\frac{b}{r}\mu_0 J_{wz} \quad , \tag{12}$$

where $\mathbf{J}_w$ is the wall current. Note that $\mathbf{J}_w$ consists of externally applied current, $\mathbf{J}_w^e$, and induced current, $\mathbf{J}_w^i$. Clearly if $\mathbf{J}_w^e = 0$, the jump in $\mathbf{B}$ disappears and p_0 vanishes. At $r = a$, the jump conditions for the magnetic field give

$$-\frac{b}{a} J_{wz} - \frac{B_0}{\mu_0} J_1(\lambda a) = J_{sz} \quad ,$$

and (13)

$$J_{w\theta} - \frac{B_0}{\mu_0} J_0(\lambda a) = - J_{s\theta} \quad .$$

The F-θ relation for the above equilibrium is obtained readily using

$$\bar{B}_z = \frac{1}{\pi b^2}\int_0^b B_z\, 2\pi r\, dr$$

$$= \frac{2}{\lambda b}\left[\frac{a}{b} B_o J_1(\lambda a) + \frac{\lambda b}{2}\left(1-\frac{a^2}{b^2}\right)\mu_o J_{w\theta}\right] ,$$

$$F = \frac{B_{zw}}{\bar{B}_z} = \frac{\mu_o J_{w\theta}}{\bar{B}_z} ,$$

and

$$\theta = \frac{B_{\theta w}}{\bar{B}_z} = \frac{-\mu_o J_{wz}}{\bar{B}_z} . \tag{14}$$

Figure 2 shows the F$-\theta$ diagram with (a/b = 0.7) and without (a/b = 1.0) the surface current for the various values of $J_{s\theta}$ but with $J_{sz} = 0$. It is seen that the field reversal occurs at larger values of θ in the presence of a surface current.

The toroidal flux density at the center, B_0, is related to the plasma pressure p_0 through the equilibrium condition (8) and is given by

$$B_0^2 = \mu_0^2 \frac{\frac{b^2}{a^2} J_{wz}^2 + J_{w\theta}^2 - \frac{2p_0}{\mu_0}}{J_1^2(\lambda a) + J_0^2(\lambda a)} \quad . \tag{15}$$

The plasma beta may be defined in two ways. One is the external beta, β_{ext}, which is

$$\beta_{ext} = \frac{2p_0}{\mu_0 (J_{wz}^2 b^2/a^2 + J_{w\theta}^2)} < 1 \quad , \tag{16}$$

and the other is the internal beta, β_{int},

$$\beta_{int} = \frac{2p_0}{[J_0^2(\lambda a) + J_1^2(\lambda a)] B_0^2/\mu_0} = \frac{\beta_{ext}}{1-\beta_{ext}} \quad . \tag{17}$$

The necessary polarity and the magnitude of the surface current can be decided from necessary conditions for ideal magnetohydrodynamic stability. One is the Newcomb condition [11] which states that q(r) defined as

$$q(r) = \frac{rB_z(r)}{B_\theta(r)} \quad , \tag{18}$$

is a monotonic function. This determines the polarities of $\mathbf{J}_w$ to be negative for both θ and z components as shown in Fig. 1. In other words, the polarity of the surface current should be such that it enhances the bulk current. In this choice of J_w, there exist two different cases. Case one, which we call the forced-reversed field (FRF), corresponds to $\lambda a < 2.4$ (= first root of J_o) where the field reversal occurs on the plasma surface, $r = a$ as shown by solid line in Fig. 1. Case two, which we call the self-reversed field (SRF), corresponds to $\lambda a > 2.4$ in which the field reversal occurs inside the plasma, $r < a$ as shown by dotted line in Fig. 1. We note that the forced reversed case with $\lambda a \simeq 2.4$ corresponds to the spheromac condition. The corresponding q-profile is shown in Fig. 3 where the solid (dotted) line corresponds to FRF (SRF) case. In either case $q(a-0) \geqslant q(a+0)$ is required for stability, i.e.,

$$\frac{J_{w\theta}}{J_{wz}} \geqslant - \frac{bJ_0(\lambda a)}{aJ_1(\lambda a)} \quad . \tag{19}$$

The other necessary condition for MHD stability is the Suydam condition [12] which states that

$$\left(\frac{q'}{q}\right)^2 > \frac{-8\mu_0 p'}{rB_z^2} \quad , \tag{20}$$

where the primes denote derivatives with respect to r. Using Eqs. (7) and (18), we can express q'/q as well as p' in terms of **J** and **B**. Then the Suydam condition can be written

$$\left(\frac{2}{r} - \mu_0 \frac{\mathbf{J}\cdot\mathbf{B}}{B_z B_\theta}\right)^2 > - \frac{8\mu_0}{rB_z^2} (\mathbf{J}\times\mathbf{B})_r \quad . \tag{20'}$$

Referring Fig. 2 and Eq. (19), if the inequality of (19) is used, q' becomes a delta-function, and the Suydam condition is always satisfied (since $\delta^2(x) \geqslant \delta(x)$). It is important to note that inequality (20') shows that the presence of *a parallel* component of the surface current is essential to satisfy the Suydam condition.

III. STABILITY ANALYSES

Here we discuss the stability problem for the equilibrium configuration with the surface current considered in the previous section. To find the stable beta limit we first obtain the qualitative dependence of β on $J_{w\theta}/J_{wz}$, λa and b/a for an ideal surface current using the energy integral, [13]

$$W(\xi) = \frac{\pi}{2} \int_0^b dr \left[f(r) \left(\frac{d\xi}{dr} \right)^2 + g(r)\xi^2 \right] \quad , \tag{21}$$

where

$$f(r) \equiv r \, \frac{(krB_z+mB_\theta)^2}{m^2+k^2r^2} \geqslant 0 \quad , \tag{22}$$

$$g(r) \equiv \frac{2k^2r^2}{m^2+k^2r^2} \frac{dp}{dr} + \frac{1}{r} (krB_z+mB_\theta)^2 \, \frac{2k^2r^2+m^2-1}{m^2+k^2r^2} \quad ,$$

$$+ \frac{2k^2r}{(m^2+k^2r^2)^2} (k^2r^2B_z^2-m^2B_\theta^2) \quad . \tag{23}$$

Here m and k are the azimuthal and axial mode numbers. In evaluating the integral (21), we follow Robinson's argument that the stability criteria can be found fairly accurately by choosing a trial function [4] $\xi(r)(=\xi_r) = \xi_0$ (const.) in $0 \leqslant r < b$ and $\xi(r) = 0$ at $r = b$, since such a function eliminates the contribution of the first term in Eq. (21) (which is positive otherwise). To further eliminate the contribution of the first term at $r = b$, we choose the wavenumber k such that f(b) vanishes i.e., $kq(b) = -m$ (hereafter we discuss the case m=1 which is known to be most unstable[8]). Then the stability condition (Eq. (21) >0) for the equilibrium configuration with Eqs. (9), (10), (11) and (12) becomes

$$\left[\int_0^1 \phi(x;\alpha,\lambda a,b/a)\,dx\right.$$

$$\left.+\frac{\alpha^2}{\alpha^2+(b/a)^2}[J_0^2(\lambda a)+J_1^2(\lambda a)]\Theta(\alpha,b/a)\right]/\Delta(\alpha,b/a) \tag{24}$$

$$>\frac{2\mu_0 p_0}{B_0^2} \geqslant 0 \quad ,$$

where $\alpha = J_{w\theta}/J_{wz}(>0)$ and the functions $\phi(x;\alpha,\lambda a,b/a)$, $\Theta(\alpha,b/a)$ and $\Delta(\alpha,b/a)$ are given by

$$\phi(x;\alpha,\lambda a,b/a) = \frac{x}{x^2+\alpha^2(b/a)^2}\left\{(\frac{a}{b})xJ_0(\lambda ax)+\alpha J_1(\lambda ax)\right\}^2$$

$$+\frac{2\alpha^2 x}{\{x^2+\alpha^2(b/a)^2\}^2}\left\{x^2J_0^2(\lambda ax)-(\frac{b}{a})^2\alpha^2J_1^2(\lambda ax)\right\} \quad , \tag{25}$$

$$\Theta(\alpha,b/a) = \frac{(1-\alpha^2)^2}{2\alpha^2}\log\frac{1+\alpha^2}{\alpha^2+(b/a)^2}-\frac{1}{\alpha^2}\log\frac{b}{a}$$

$$+\frac{1-(a/b)^4}{\alpha^2+(a/b)^2}-\frac{1}{2}\left\{1-(\frac{a}{b})^2\right\} \quad , \tag{26}$$

$$\Delta(\alpha,b/a) = \frac{\alpha^2}{1+\alpha^2(b/a)^2}-\frac{\alpha^2}{\alpha^2+(b/a)^2}\Theta(\alpha,b/a) \quad . \tag{27}$$

Using the relation between β_{ext} in (16) and $2\mu_0 p_0/B_0^2$ derived from Eq. (16), i.e., $\beta_{ext} = (2\mu_0 p_0/B_0^2)/\ \{J_0^2(\lambda a)+J_1^2(\lambda a)+(2\mu_0 p_0/B_0^2)\}$, the stability condition (24) can be written in terms of β_{ext},

$$\Phi(\alpha,\lambda a,b/a) > \beta_{ext} \geqslant 0 \quad , \tag{24'}$$

where $\Phi(\alpha,\lambda a,b/a)$ is given by the following form with the function of the left-hand side in (24), say $F(\alpha,\lambda a,b/a)$,

$$\Phi(\alpha,\lambda a,b/a) = \frac{F(\alpha,\lambda a,b/a)}{J_0^2(\lambda a)+J_1^2(\lambda a)+F(\alpha,\lambda a,b/a)} \leqslant 1 \quad . \tag{28}$$

From the numerical integration of the function of the left-hand side in (24), $F(\alpha,\lambda a,b/a)$, one can see that

A_1) $\Phi(\alpha,\lambda a,b/a) \rightarrow 1$, as $\alpha \rightarrow 0$,

A_2) there exists $\alpha_{max} > 0$ (which is a function of λa and b/a) such that

$$\Phi(\alpha_{max},\lambda a,b/a) = 0 \quad ,$$

A_3) $\frac{\partial}{\partial\alpha}\Phi(\alpha,\lambda a,b/a) < 0$, for $0 < \alpha < \infty$,

A_4) $\frac{\partial}{\partial(b/a)}\Phi(\alpha,\lambda a,b/a) < 0$.

The beta value β_{ext} in (24) corresponding to the ideal equilibrium configuration (i.e., the fields given by (9) to (12)) can be given explicitly by

$$\beta_{ext} = \frac{1}{\gamma^2}\left(\gamma^2-1+\frac{\alpha^2-\alpha_0^2}{\alpha^2+(b/a)^2}\right) \quad , \tag{29}$$

where $\alpha_0 = \alpha(J_{s\theta}=J_{sz}=0) = -(b/a)J_0(\lambda a)/J_1(\lambda a)$ and $\gamma = B_\theta(r=a+0)/B_\theta(r=a-0) = -(b/a)\mu_0 J_{wz}/(B_0 J_1(\lambda a))$ (note $\gamma=1$ implies no surface current in z-component). It should be noted that a higher beta value can be obtained by increasing J_{wz} (i.e., increasing γ). One can easily see from (29) that

B_1) $\beta_{ext} \rightarrow 1$ as $\alpha \rightarrow \infty$,

B_2) $\beta_{ext} = 0$ for some $\alpha_{min} > 0$ (e.g., for $\gamma=1$, $\alpha_{min}=|\alpha_0|$),

B_3) $\frac{\partial}{\partial\alpha}\beta_{ext} > 0$, for $0 < \alpha < \infty$.

B_4) $\dfrac{\partial}{\partial(b/a)}\beta_{ext} < 0$.

From those properties of $\Phi(\alpha,\lambda a,b/a)$ and $\beta_{ext}(\alpha,\lambda a,b/a;\gamma)$, there is a finite region (i.e., $\alpha_{min}<\alpha<\alpha_{max}$) in which the plasma configuration is stable and has a finite beta value as shown in Fig. 4 for the case $\gamma = 1$. The boundary of this stable region in the plane of λa and b/a is given by $\alpha_{min} = \alpha_{max}$. In Fig. 5 the stable region and the beta value corresponding to a point in the plane $\lambda a - b/a$ for the case $\gamma = 1$ ($J_{sz}=0$) is shown. It should be noted that the stable region extends below the $\lambda a = 2.4$ line, while in Taylor's configuration $\lambda a = 2.4$ is the stability boundary. The region below $\lambda a = 2.4$ corresponds to the forced reversed case in which the z component of the magnetic field reverses on the plasma surface due to the θ component of the surface current (solid lines in Figs. 1 and 3).

From Fig. 5 (also A_4 and B_4), we see that the maximum β_{ext} is a decreasing function of b/a. For an example of $b/a = (0.7)^{-1} = 1.43$, the maximum β_{ext} produced only by the poloidal surface current becomes 0.25, which corresponds to $\beta_{int} = 0.33$.

The total magnetic field energy E corresponding to the configuration given by Eqs. (9) to (12) can be shown to assume a minimum value for the plasma radius a in $0 < a < b$ with proper choice of other parameters. For example, if b, λa, B_0 and J_w are fixed, E becomes minimum when

$$\frac{a}{b} = \left(\frac{C_3}{C_1-C_2}\right)^{1/2} , \tag{30}$$

where C_1, C_2 and C_3 are the positive quantities given by

$$C_1 = \frac{b^2}{a^2}\int_0^a B^2 dV = 2\pi L_z b^2 B_0^2 (J_0^2 + J_1^2 - \frac{J_0 J_1}{\lambda a}) \quad ,$$

where the arguments of J_0 and J_1 are λa and L_z is the length in z direction.

$$C_2 = \pi L_z B^2 \mu_0^2 J_{w\theta}^2 \quad ,$$

and

$$C_3 = \pi L_z b^2 \mu_0^2 J_{wz}^2 \quad .$$

Clearly from Eq. (30),

$$C_1 \geqslant C_2 + C_3 \quad , \tag{31}$$

is required to have a between zero and b.

Having obtained the qualitative parameter dependencies of β, let us now evaluate the effect of diffused surface current using the numerical code THALIA developed by Appert *et al.* [8] which enables us to evaluate the energy integral given by Bernstein *et al.* [13] using an exact eigenfunction for the appropriate magnetic field configuration.

For the diffused current profile, we use

$$\mu_0 J_\theta$$

$$= B_0 \lambda J_1(\lambda r) + \frac{1}{2}\mu J_{0\theta} \mathrm{sech}^2 \left| \frac{r-a+\ell\delta}{\delta} \right| , \quad \text{at } r \leqslant a \ ,$$

$$= 0 \quad , \quad \text{at } a < r < b \ , \tag{32}$$

and

$$
\begin{aligned}
&\mu_0 J_z \\
&= B_0 \lambda J_0(\lambda r) + \frac{1}{2}\mu_0 J_{0z} \operatorname{sech}^2\left(\frac{r-a+\ell\delta}{\delta}\right), \quad \text{at } r \leqslant a \ , \\
&= 0\,, \quad \text{at } a < r < b \ . \qquad (33)
\end{aligned}
$$

The corresponding magnetic field is

$$
\begin{aligned}
&B_z \\
&= B_0 J_0(\lambda r) - \frac{1}{2}\mu_0 J_{s\theta}\left[\tanh\left(\frac{r-a+\ell\delta}{\delta}\right) + \tanh\left(\frac{a-\ell\delta}{\delta}\right)\right], \quad \text{at } r \leqslant a \ , \\
&= B_0 J_0(\lambda a) - \frac{1}{2}\mu_0 J_{s\theta}\left[\tanh\left(\frac{a-\ell\delta}{\delta}\right) + \tanh\ell\right] \ (= \text{const.}), \quad \text{at } a < r < b \ (34)
\end{aligned}
$$

$$B_\theta$$

$$= B_0 J_1(\lambda r) + \frac{1}{2}\mu_0 J_{sz} \left[\tanh(\frac{r-a+\ell\delta}{\delta}) \right.$$

$$\left. - \frac{\delta}{r} \ln \left\{ \frac{\cosh(\frac{r-a+\ell\delta}{\delta})}{\cosh(\frac{-a+\ell\delta}{\delta})} \right\} \right] \text{, at } r \leqslant a$$

$$= \frac{a}{r} B_0 J_1(\lambda a) + \frac{1}{2}\mu_0 \frac{aJ_{sz}}{r} \left[\tanh\ell - \frac{\delta}{a} \ln \left\{ \frac{\cosh\ell}{\cosh(\frac{\ell\delta - a}{\delta})} \right\} \right] , \quad (35)$$

at $a < r < b$.

Here, $J_s = \delta J_0$, δ is the width of the surface current, and ℓ denotes the location of the surface current i.e., if $\ell = 0$, the surface current peaks at $r = a$ while if $\ell > 0$ the peak exists inside the plasma, at $r = a - \ell\delta$. The pressure and the magnetic field profiles for an example of $J_{sz} = 0$, $\ell = 1$, $\delta = 0.1$ and $a/b = 0.7$ are shown in Fig. 6.

The numerical code is designed such that for a fixed value of J_s and λa, the value of b/a is searched such that the growth rate goes to zero for m = 1 mode. The corresponding value of average $\beta(=\beta_{ave})$ as well as that of k is printed out. Simultaneously, the Suydam criterion for stability, Eq. (20), is checked at each point in the radius. β_{ave} is calculated by averaging β inside the plasma. Hence it corresponds to β_{int} in Eq. (17) for the ideal case.

Figure 7 shows the results of the numerical evaluation for a case with $J_{sz} = 0$, $\ell = 1$ and $\delta = 0.1$, that is, the width of the surface current is 10% of the minor radius with the current peak located at $r = 0.9a$. It is interesting to note that the stability range as well as the corresponding values of β is almost the same as the case of the delta-function surface current obtained using a simple trial function.

We varied parameters ℓ and J_{sz} to see their effects. When ℓ is increased, an unstable region appears in $\lambda a < 2.4$ due to the violation of the Suydam criterion. When ℓ is reduced to zero, the negative pressure region was expanded at $\lambda a < 2.4$. When the toroidal surface current J_{sz} is introduced $J_{s\theta}$ should be increased simultaneously to satisfy the Suydam criterion. Then the β increases accordingly. However, the stable region is found to shrink.

In the case of a diffuse surface current, the Suydam criterion expressed in Eq. (20') should particularly be recognized such that a sufficiently large parallel current exists at the region of field reversal, $B_z = 0$. Other than this, the overall qualitative parameter dependencies of β on J_s are the same as in the case of the ideal surface current.

IV. CONCLUSION

A finite beta reversed field pinch configuration using the surface current is presented. The configuration is found to be stable within the framework of the ideal magnetohydrodynamic perturbation for an average beta of twenty to thirty percent. Since the surface current can easily be induced by a radio frequency wave, the scheme is an interesting candidate not only for a finite beta reversed field pinch but also for its continuous operation.

Nonideal magnetohydrodynamic instabilities such as the resistive instability may contribute to diffuse the surface current at an anomalous rate. However, it may relatively be easy to inject the surface current at a rate faster than the diffusion rate if an appropriate radio-frequency wave is used. Although the resistive mode is expected not to be crucial for the confinement (since it may be localized on the surface) it remains an important future investigation in this project.

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FIGURE CAPTIONS

Fig. 1 Magnetic field (**B**) and current density (**J**) profile of the equilibrium obtained in the text for FRF (solid) and SRF (dotted) cases.

Fig. 2 F$-\theta$ diagram with (a/b = 0.7) and without (a/b = 1.0) the surface current $\bar{J}_s(=\mu_o J_{S\theta}/B_o)$, $J_{sz} = 0$.

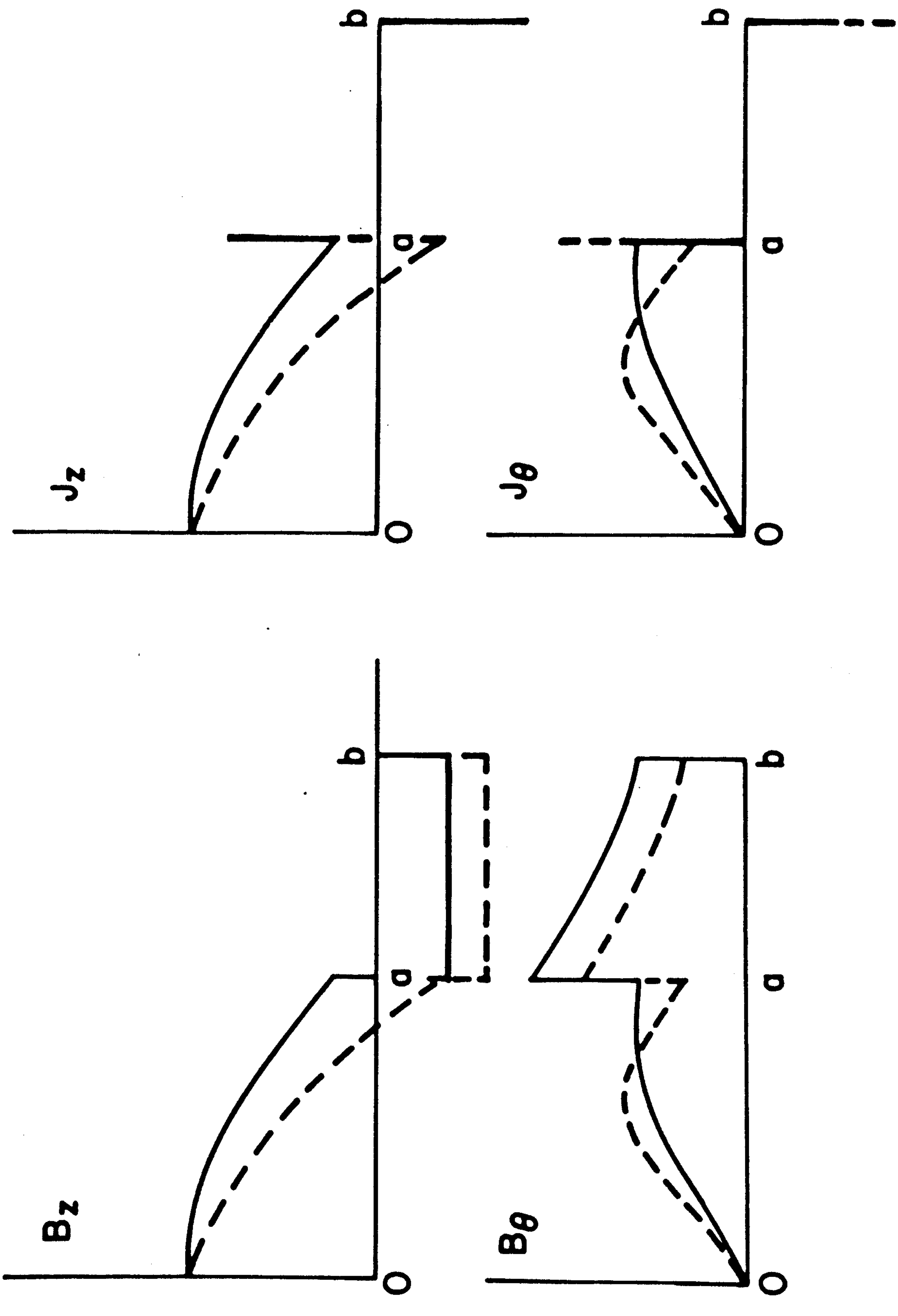
Fig. 3 q profile of the equilibrium for the FRF (solid) and SRF (dotted) cases.

Fig. 4 The functions Φ (Eq. (28)) and β_{ext} (Eq. (29)) are plotted for $\alpha(=J_{w\theta}/J_{wz})$ for the case of a/b $\simeq$ 0.7 and $\gamma = 1$ ($J_{sz} = 0$).

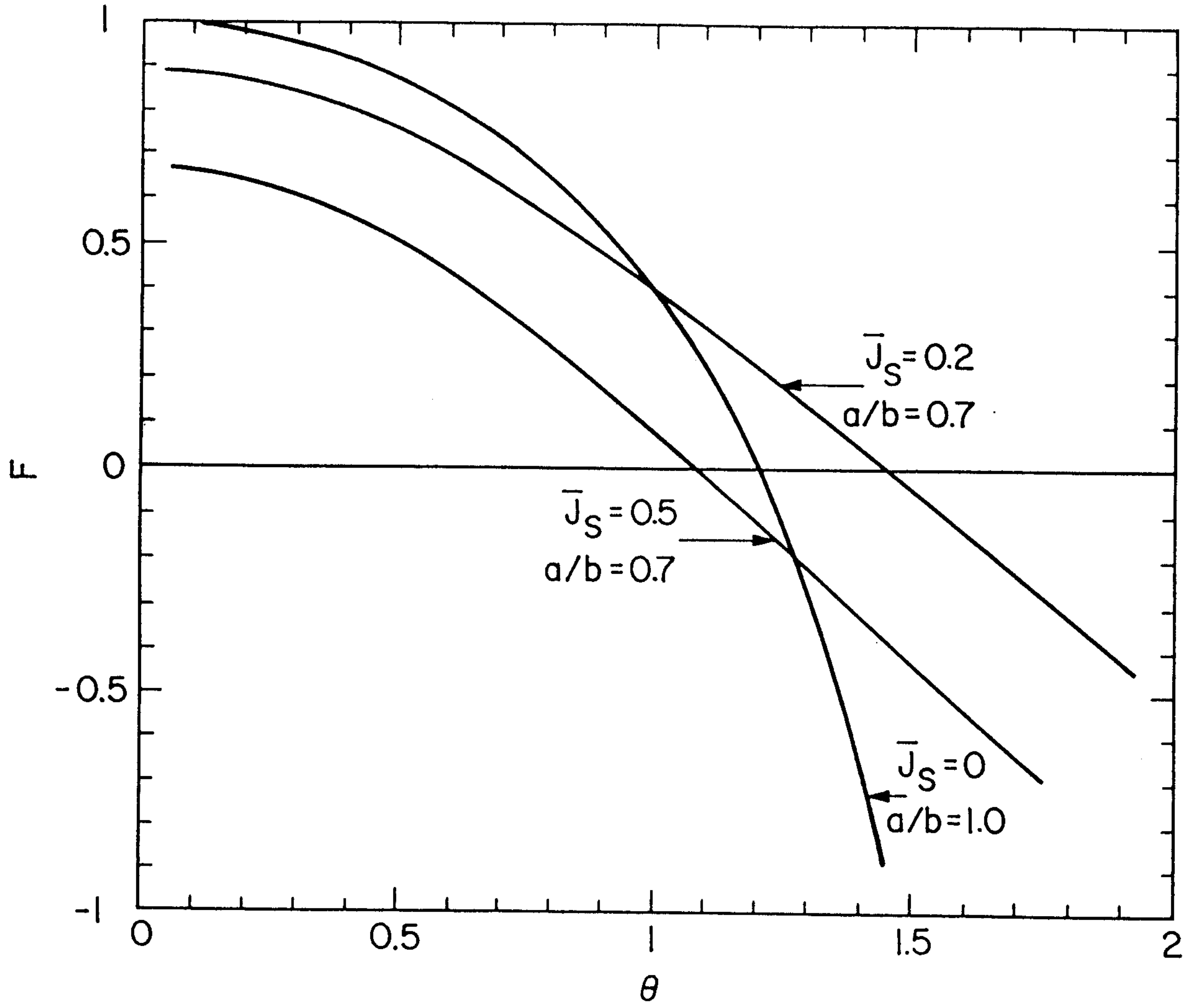
Fig. 5 Stability boundary in the parameter of λa and b/a for an ideal current case. The beta value within the stable region can be changed by changing the $J_{wz}/J_{w\theta}$ ratio. The dot-dash line indicates the boundary at which P_0 becomes negative for $J^e_{wz} = 0$. If $J^e_{wz} < 0$ introduced, this boundary can be expanded to lower value of λa.

Fig. 6 The magnetic field profile (top) and the corresponding pressure profile (bottom) are shown as a function of radius for a case of diffused current profile with $\delta = 0.1$, $\ell = 1$ and a/b = 0.7.

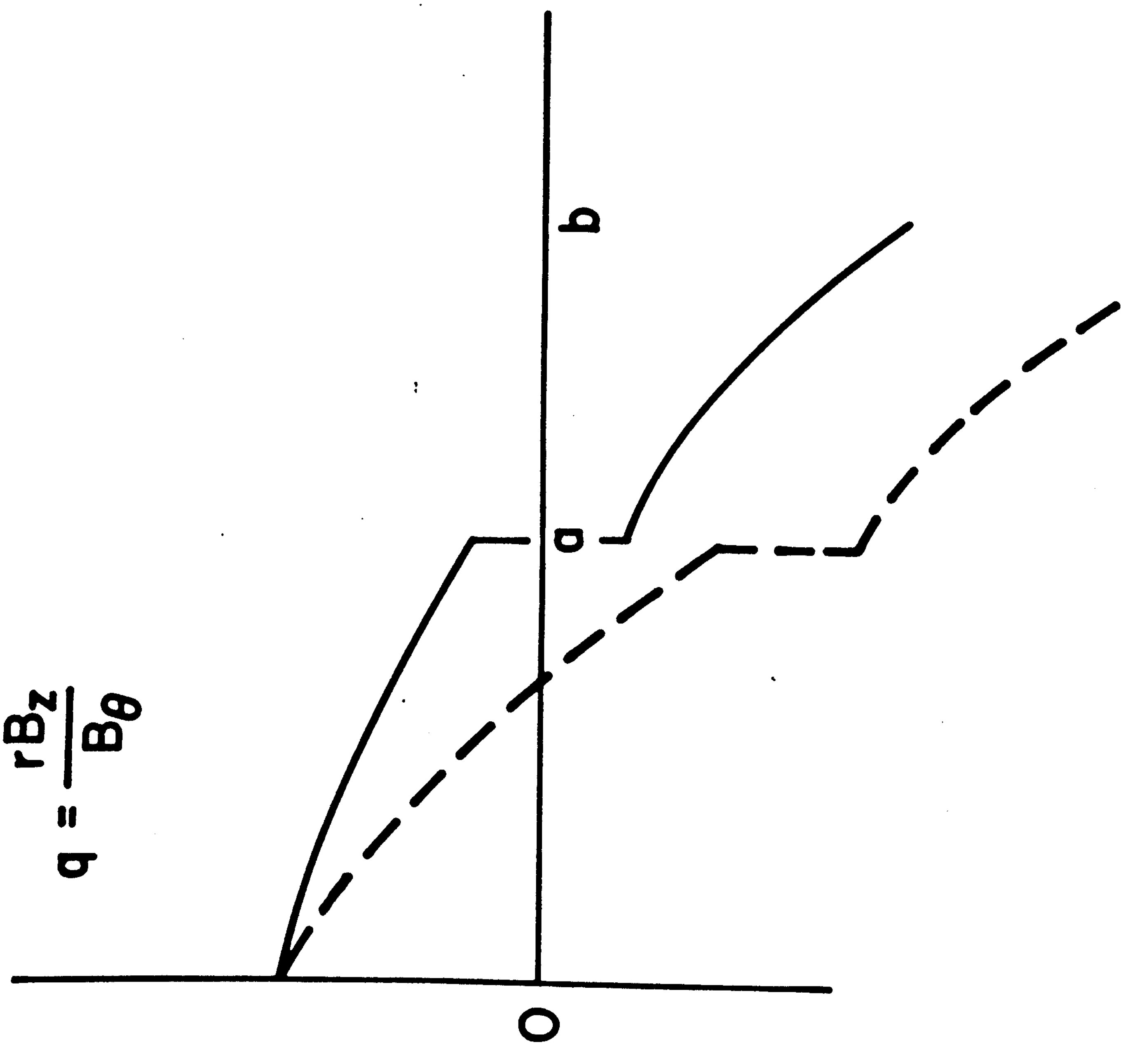
Fig. 7 The stability boundary in the parameter space of λa and b/a for a diffusive current profile is shown with contours of different $J_{s\theta}$ and resultant β_{ave}. $J_{sz} = 0$, $\delta = 0.1$ and $\ell = 1$ are used here. The wavy line is the boundary due to the violation of the Suydan condition.

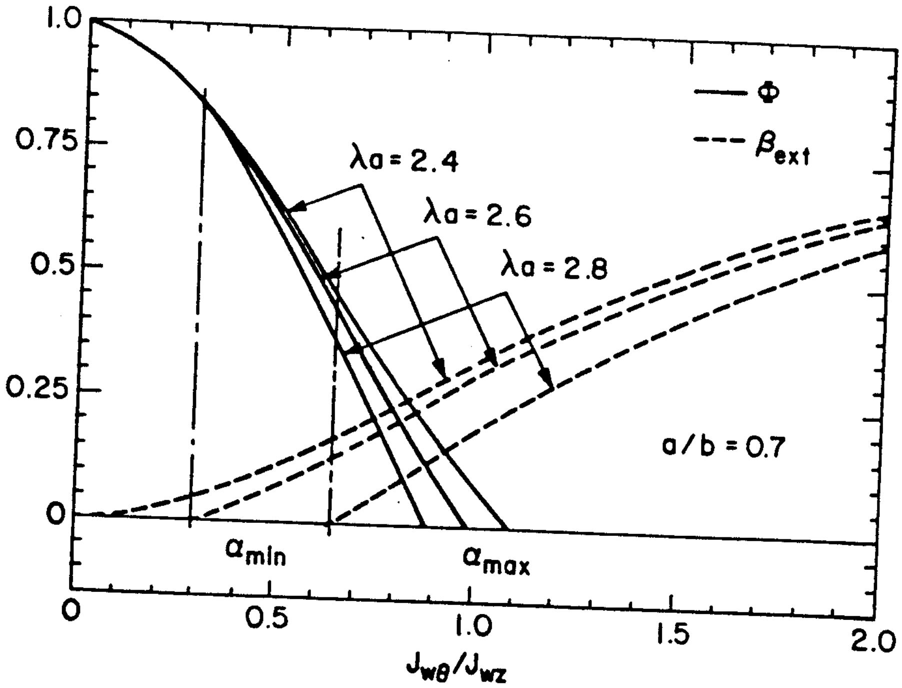


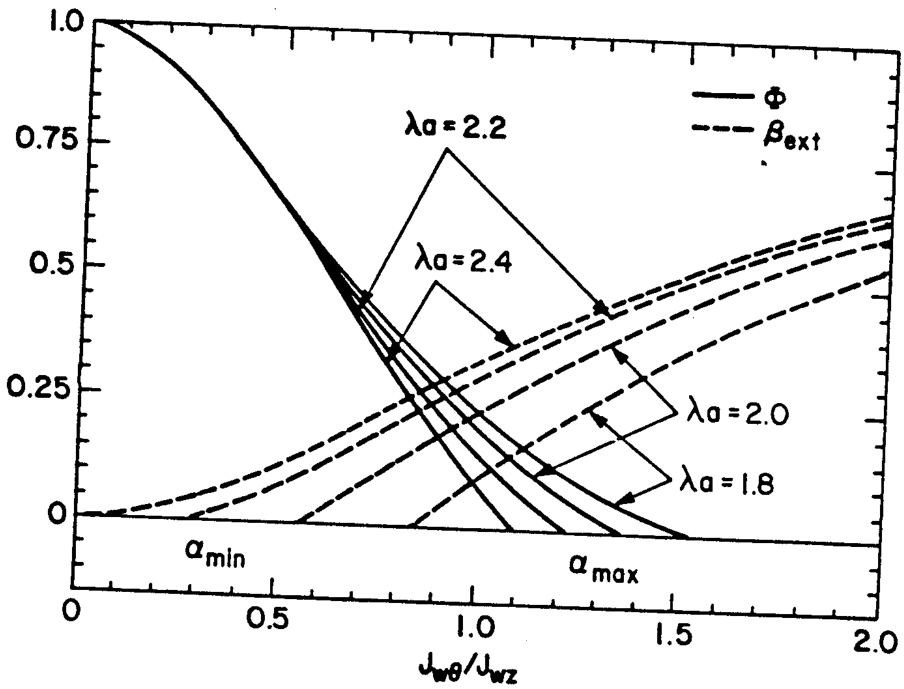
Jz
Jθ
Bz
Bθ
0
a
b

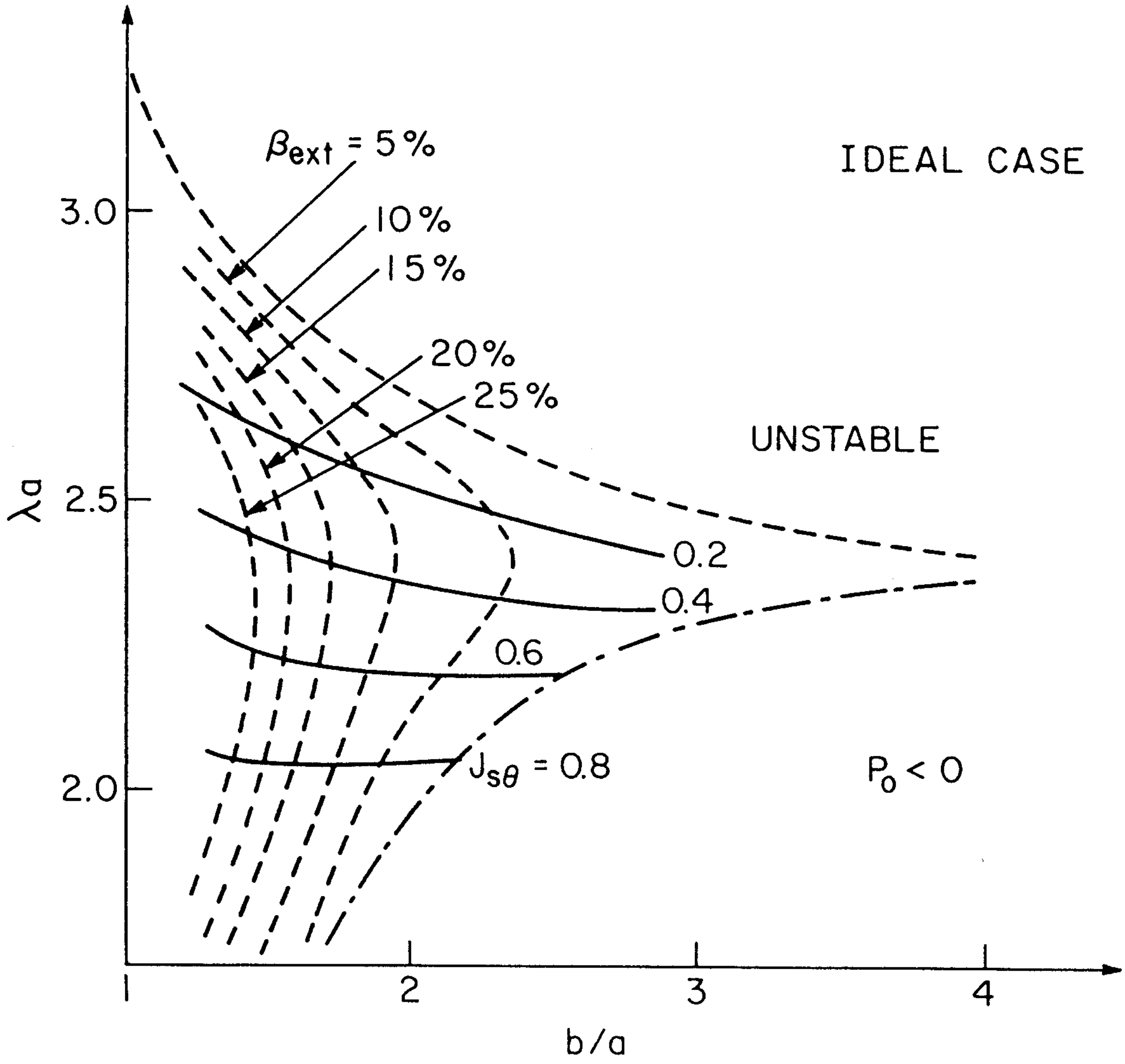


1
0.5
0
-0.5
-1
F
0
0.5
1
1.5
2
θ
J̄S = 0.2
a/b = 0.7
J̄S = 0.5
a/b = 0.7
J̄S = 0
a/b = 1.0

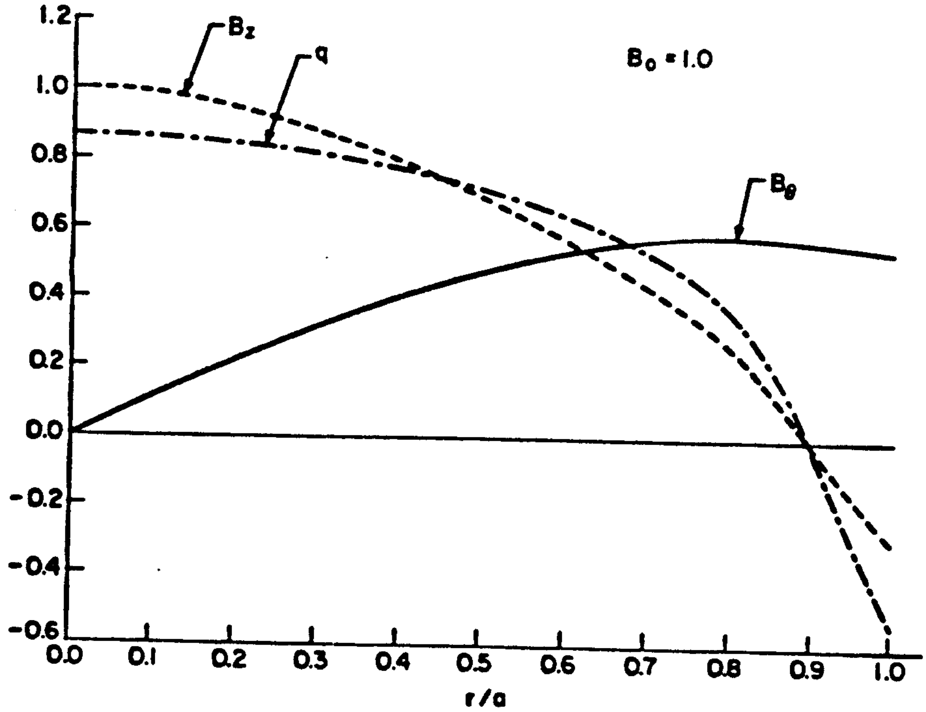

$q = \frac{rB_z}{B_\theta}$
b
a
0


1.0
0.75
0.5
0.25
0
Φ
β_{ext}
λa = 2.4
λa = 2.6
λa = 2.8
a/b = 0.7
α_{min}
α_{max}
0
0.5
1.0
1.5
2.0
$J_{w\theta}/J_{wz}$

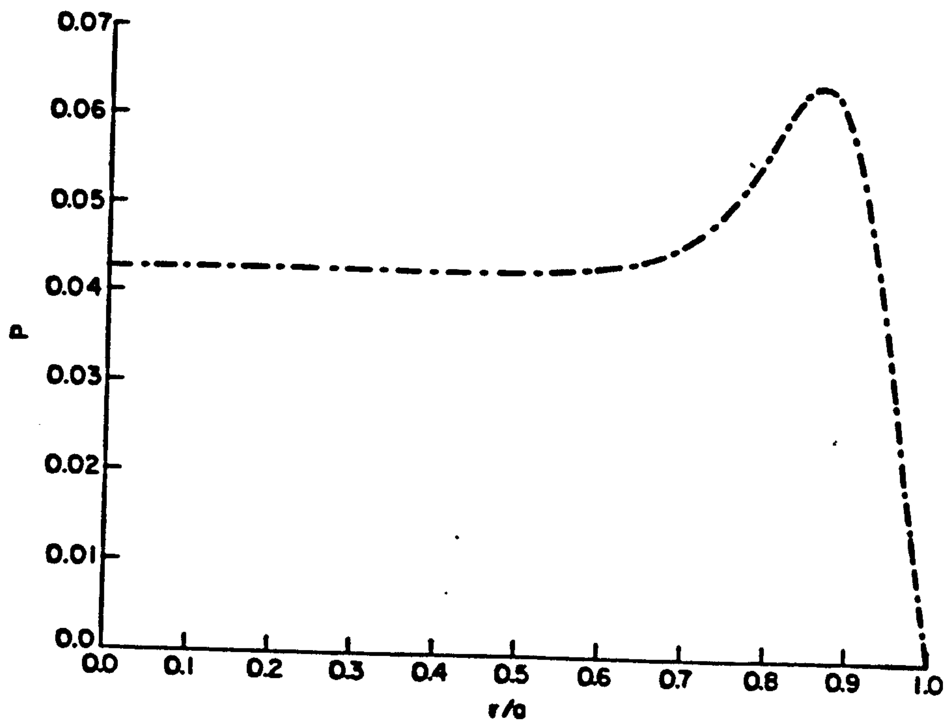

1.0
0.75
0.5
0.25
0
Φ
β_{ext}
λa = 2.2
λa = 2.4
λa = 2.0
λa = 1.8
α_{min}
α_{max}
0
0.5
1.0
1.5
2.0
$J_{w\theta}/J_{wz}$



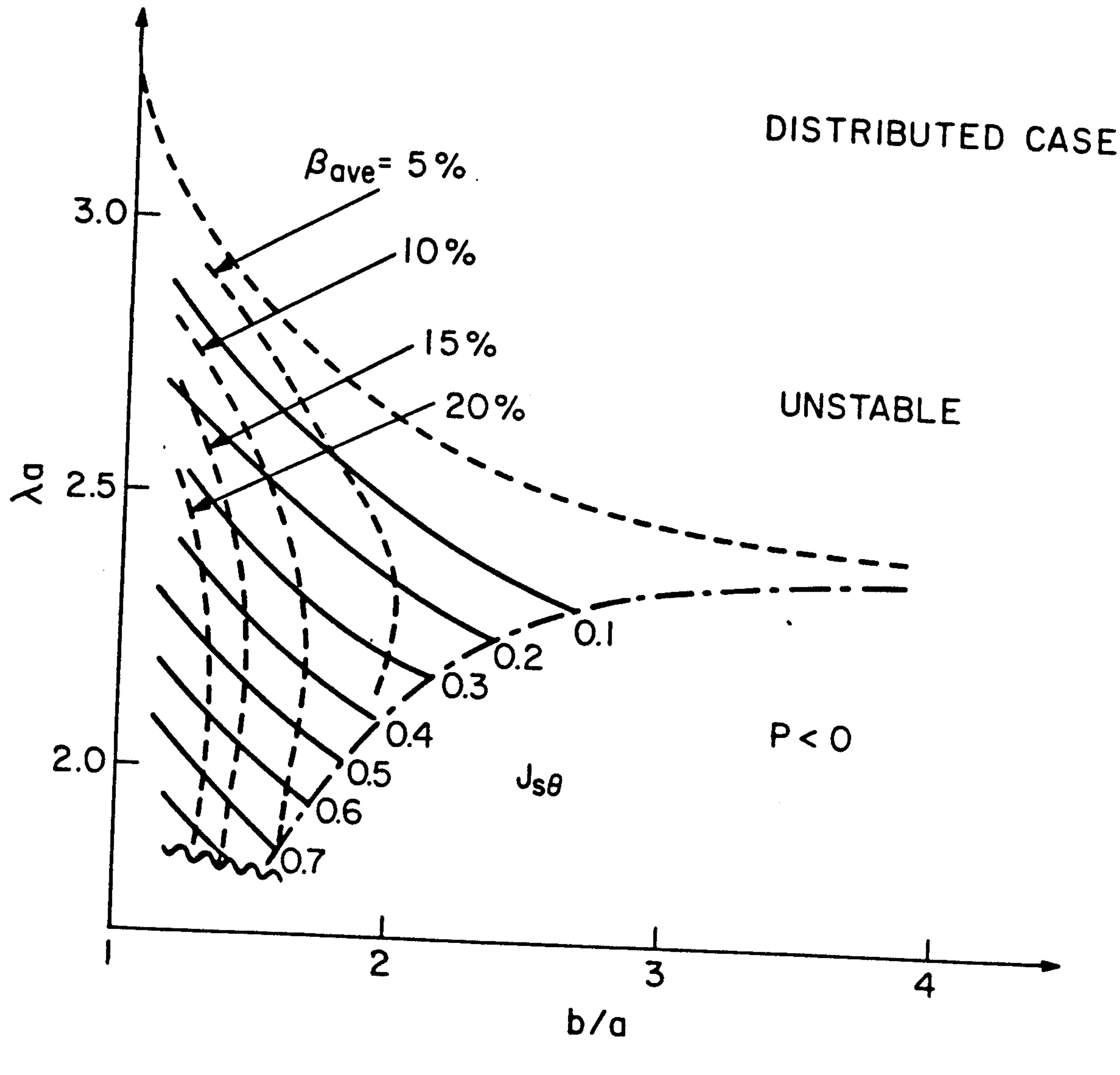
β_{ext} = 5%
10%
15%
20%
25%
IDEAL CASE
UNSTABLE
0.2
0.4
0.6
$J_{s\theta}$ = 0.8
$P_0 < 0$
3.0
2.5
2.0
λa
1
2
3
4
b/a



B_z
q
B_0 = 1.0
B_θ
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
r/a



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.0
p
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
r/a


DISTRIBUTED CASE
β_{ave}= 5%
10%
15%
20%
UNSTABLE
3.0
2.5
2.0
λa
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$J_{s\theta}$
P < 0
1
2
3
4
b/a