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K. Appert, S. Succi, J. Vaclavik and

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K. Appert, S. Succi, J. Vaclavik, and L. Villard
Centre de Recherches en Physique des Plasmas
Association Euratom - Confédération Suisse
Ecole Polytechnique Fédérale de Lausanne
CH-1007 Lausanne, Switzerland

ABSTRACT

An introduction to the basic physics of Alfvén Wave Heating is presented. First, the resonant absorption of global modes of the fast magnetosonic (compressional Alfvén) wave is compared to the resonant absorption of laser light. Second, the spectral approach, known from ideal MHD stability studies, is described. Third, a brief discussion of the global modes and their potential for heating is presented on the basis of a cylindrical model. The resonantly-absorbed magnetosonic surface mode and the Landau-damped global eigenmodes of the (shear) Alfvén wave are discussed. Finally, non-cylindrical plasmas are considered and possible effects due to the loss of symmetry are shortly described. References to recent papers are given.

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1. INTRODUCTION

By the term Alfvén Wave Heating (AWH) one designates RF-heating of a plasma in the frequency range of the Alfvén wave. The method[1),2)] originally proposed, relies on the resonant absorption of a so-called surface eigenmode[3)] which is, in fact, an eigenmode of the fast magnetosonic wave. In this method, an antenna-excited fast magnetosonic wave is linearly mode-converted into a kinetic Alfvén wave[4),5)] which is subsequently Landau-damped by the electrons. In recent experiments [TCA] it has been shown that the Alfvén wave possesses global eigenmodes[6),7)] which can easily be excited directly by an external antenna. Landau damping leads to electron heating in this case too[8)].

In a plasma containing one ion species, a clear definition of Alfvén Wave Heating is "heating at frequencies below the ion-cyclotron frequency". In a plasma containing heavy, partially-ionized impurities, however, Alfvén Wave Heating may have many features of ICRF heating. A certain amount of ion heating has been indeed observed in an experiment[9)] and can possibly be explained by ion-cyclotron heating of impurities.

2. RESONANT ABSORPTION

Resonant absorption can most easily be understood in the case of a laser beam impinging obliquely on a plasma layer with a gradually-increasing density[10),11)]. The beam of frequency ω will be reflected before it reaches the point x_r in the plasma, defined by $\omega = \omega_{pe}(x_r)$, where ω_{pe} is the local plasma frequency. Some amount of the field, however, tunnels through the evanescent region to the resonance point x_r and excites resonantly a local plasma oscillation (in the cold plasma picture) or an outwards-traveling Langmuir wave (if temperature effects are considered). The essential point is that the amount of energy converted from the electromagnetic wave into the electrostatic field does not depend on the model used to describe this field (wave or oscillation) or on the type of damping from which this field suffers. This last statement is correct as long as the physical damping mechanism is strong enough to prevent the converted wave from reaching the plasma boundary.

Theoretically, resonant absorption of a laser beam can be adequately treated by the WKB-method since the laser wavelength λ is usually much smaller than the density scale length L of the plasma on which it is impinging. In this picture, energy is transported into the plasma by an electromagnetic wave which is partly reflected and partly absorbed (or converted) at the spatial Langmuir resonance.

The resonant absorption of a fast magnetosonic wave at the spatial Alfvén resonance has many common features with the laser light absorption. In the cold plasma picture, the shear Alfvén oscillations which, at the resonance point, are predominantly electrostatic satisfy an equation which is identical in form to the one describing Langmuir oscillation in an inhomogenous plasma[12]. In the warm plasma picture, the converted energy appears once more in the form of a propagating wave, the kinetic Alfvén wave, which is partly electrostatic[4].

(The main difference between AWH and laser light absorption lies in the relevant scale lengths of the two problems. Whereas in the laser problem $\lambda \ll L$, in AWH usually $\lambda \sim L$, making a WKB-approach impracticable. It is, therefore, not possible to separate the wave motion into in-, outgoing and converted waves. The wave function has to be calculated globally. A conceptually simple picture can, however, be obtained by the so-called spectral method, i.e. looking for eigen-oscillations of the bounded plasma in question.

Before discussing the spectral method in more detail, let us make one remark concerning ICRF heating. In a two-ion-component plasma, a fast magnetosonic wave may be resonantly absorbed at the Alfvén or at the ion-ion-hybrid resonance. The basic mechanism in both cases is once again linear mode conversion. The characteristic lengths λ and L in this case are such that WKB is at the limit of its applicability. Some parameter combinations might indeed require a spectral treatment as in the AWH case.

3. THE SPECTRAL METHOD

The essence of this method can most easily be understood in the framework of ideal MHD. The generalisation to cold plasma theory is not difficult[13].

Let us start with the equation of motion of small amplitude oscillations in a low-β plasma[14]

$$\rho_o \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}(\vec{\xi}), \tag{1}$$

where
$$\vec{F}(\xi) = \frac{1}{\mu_o} \text{ rot } \vec{B}_o \times \vec{B} + \frac{1}{\mu_o} \text{ rot } \vec{B} \times \vec{B}_o, \tag{2}$$

and
$$\vec{B} = \text{rot } \vec{\xi} \times \vec{B}_o \tag{3}$$

Here $\rho_o(\vec{r})$ and $\vec{B}_o(\vec{r})$ are the equilibrium mass density and magnetic field respectively. The wave quantities are denoted by ξ and B, the displacement and the magnetic field respectively.

Equation (1) is the usual starting point for ideal-MHD stability studies. Once a geometry is specified eq. (1) is solved, subject to appropriate boundary conditions, in the form of an eigenvalue problem:

$$-\omega^2 \rho_o \vec{\xi} = \vec{F}(\vec{\xi}). \tag{4}$$

Here the time dependence is assumed to be $\exp(-i\omega t)$. Whereas in stability studies only negative eigenvalues for ω^2 are of interest, in the heating context we consider positive values of ω^2.

In general, it is impossible to solve eq. (4) analytically. In the last 10 years, powerful computer codes have been developed to solve eq. (4) in cylindrical[15] and toroidal geometries[16,17]. These codes have been used for extensive AWH studies[18,19]. For details we refer to these papers. In the context of the present introductory paper, we have more benefit from searching an analytic solution in a simple limiting case.

To this end, let us discuss a currentless plasma cylinder with radius a and $\vec{B}_o = B_o\vec{e}_z$, B_o = const. The axial and azimuthal dependence of the wave quantities can be assumed to be $\exp(ikz+im\theta)$. Further simplifying assumptions are $ak \ll 1$ and $\omega \ll C_A/a$, where $C_A^2 = B_o^2/(\mu_o\rho_o)$. Equation (4) may then be written in the following simple form[20)]

$$\frac{1}{r}\frac{d}{dr} r\,\xi_r = \frac{m^2}{r^2}\left(\frac{\omega^2}{C_A^2} - k^2\right)^{-1}\frac{B_z}{B_o}\,, \tag{5}$$

$$\frac{d}{dr}\frac{B_z}{B_o} = \left(\frac{\omega^2}{C_A^2} - k^2\right)\xi_r\,. \tag{6}$$

If we assume now that $m = 1$ and ρ_o = const, it is straightforward to obtain the lowest radial eigenmode of the fast magnetosonic wave, the so-called surface eigenmode[21)]. First we remark that $\xi_r = C_1$ and $B_z/B_o = ((\omega^2/C_A^2) - k^2)C_1 r$ is a solution of eqs. (5) and (6). If the plasma is surrounded by a vacuum extending to infinity, we also know the vacuum field $B_z = C_2K(k_r)$, where K denotes the modified Bessel function and C_1 and C_2 are constants. Matching the plasma and the vacuum magnetic fields B_z and B_r($B_r = ikB_o\xi_r$ in the plasma, and $B_r = 1/ik\ dB_z/dr$ in the vacuum) at the plasma-vacuum interface, r = a, yields the eigenfrequency of the mode

$$\omega = \sqrt{2}\,C_A k. \tag{7}$$

Equations (5) and (6) have another, quite peculiar solution: For $\omega^2 = k^2C_A^2$ and $B_z = 0$ an arbitrary $\xi_r(r)$ is a solution. This is the shear Alfvén wave with its infinitely degenerated eigenvalue $\omega^2 = k^2C_A^2$.

The mode with the eigenfrequency, eq. (7), is the one which may be absorbed resonantly in the Alfvén wave heating scheme. Resonant absorption in the present spectral approach can also be discussed with the help of eqs. (5) and (6). Assume a diffuse density profile, say $\rho_o(r) = \rho_{oo}(1-r^2/a^2)$. The Alfvén velocity is then a monotonic function of radius: $C_A^2(r) = B_o^2/\mu_o\rho_{oo}\ (1-r^2/a^2)^{-1}$. Equation (5) contains now a singularity at r_s, where $\omega^2 = C_A^2(r_s)k^2$. As a consequence one finds now, for any given $\omega > C_A(o)k$, a singular eigenfunction, $\xi_r \sim \ln|r-r_s|$, of the shear Alfvén wave. The shear Alfvén wave has a continuous spectrum. The global mode associated with eq. (7), on the other hand, has apparently disappeared.

This is, however, not the whole truth. A global oscillation of the plasma column may be excited by an antenna situated in the vacuum region most easily when the dispersion relation for the surface modes is satisfied. Equation (7) is somewhat modified by the density profile.

Now the emerging picture of AWH is the following: The antenna couples well to the surface mode which is a global oscillation of the plasma column and which transports energy into the plasma interior. Just as in the laser case, the energy is dumped into local oscillations, represented here by the singular eigenoscillations of the Alfvén wave.

In general, a global mode whose frequency lies within the Alfvén continuum $[\omega > C_A(o)\ k]$ suffers from resonant absorption.

4. GLOBAL MODES

Any global mode of a plasma column which may easily be excited by an external antenna is a candidate to be used for heating. Once the energy is coupled to the plasma, it may be absorbed either resonantly or directly by Landau damping. For an optimal heating method, we would, however, require that the absorption takes place in the plasma centre. If the absorption relies on Landau damping we require that the wave function of the global mode peaks in the centre. In the resonant absorption case we try to place the resonant surface as near to the centre as possible. This last criterion might be somewhat too restrictive, since the converted kinetic wave might transport the energy a substantial distance from the resonant layer towards the centre.

We have recently investigated the question of finding the optimal modes in the framework of cold plasma theory, i.e. ideal MHD including the Hall-term[13] which accounts for finite ω/ω_{ci} effects. We have found that the best candidate for the resonant-absorption scheme is the previously-discussed surface mode (for $|m| = 1$) which is in fact the lowest radial eigenmode of the fast magnetosonic wave. We have shown that the higher radial eigenmodes of this wave have their resonant layers always near to the plasma boundary and are discarded for this reason. In a current-carrying plasma and for values of ω/ω_{ci} not too small a further choice has to be made. There are, in fact, four different surface modes depending on the signs of m and k (++, +−, −+, −−). For details we refer to Ref. 13.

In a current-carrying plasma and/or at frequencies which are a substantial fraction of ω_{ci} (say $\omega \geq 0.1\ \omega_{ci}$), the Alfvén wave has global eigenmodes[7),13)] in addition to the local (singular) eigenmodes associated with the continuum and discussed in the previous section. These eigenmodes are identical with the ion-cyclotron modes known for frequencies $\omega < \omega_{ci}$. The eigenfrequencies of these modes lie densely packed just below the Alfvén continuum. For increasing radial mode numbers, the frequencies accummulate at the lower boundary of the Alfvén continuum. Experimentally, it has been found to be possible to excite the two lowest radial eigenmodes[6)] by the external antenna. For a well-chosen parameter set the lowest radial eigenmode may peak in the centre of the plasma and is, therefore, a good candidate for the Landau damping scheme of AWH[8)]. High antenna loads have been found experimentally. This scheme suffers, however, from a serious difficulty. The frequency response to this mode is extremely narrow and its frequency evolves with the plasma parameters. It seems, therefore, that a system of frequency tracking would be necessary which represents a non-trivial experimental problem.

5. NON-CYLINDRICAL PLASMAS

Up to this point the selection of suited global modes relied on results obtained for cylindrical symmetry. The main problem arising from the deviation from cyclindrical symmetry is that m or even k cease to be good quantum numbers. As an example, a global mode with an m = 1 azimuthal dependence in an ayisymmetric torus may loose some of its energy via resonant absorption at m = 0, -1, -2, etc. surfaces which are situated near the plasma boundary.

We have found[19)] that no serious effect of this kind is to be expected in an axisymmetric torus with a circular or a mildly elliptical cross-section. Elongations of the order of 1.5 and higher, however, pose a problem. More than 25% of the energy is deposited near the plasma boundary. Triangular cross-sections and stellarator-like equilibria which might arise in a Tokamak quite naturally due to island formation, have not yet been investigated. Our feeling, however, is that a substantial amount of energy could be absorbed near the boundary. In order to minimize this kind of coupling one should excite low-frequency global modes only[19)]. Good candidates are the surface mode with toroidal wavenumber n = -1 and the global Alfvén eigenmodes with n = 0 and n = -1.

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