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AERH - A NEW RF HEATING SCHEME ?

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ABSTRACT

Recent experimental and theoretical studies have revealed the existence, in a current-carrying plasma, of hitherto unknown global eigenmodes of the Alfvén wave (discrete Alfvén spectrum). The potential for a new RF heating scheme using the resonant excitation of such modes in Tokamaks, the "Alfvén Eigenmode Resonance Heating" (AERH) is explored. It is concluded that such a heating scheme is most efficient for axisymmetric excitation (toroidal mode number $n = 0$). In this case the antenna load is one to two orders larger than that of the resonant absorption scheme of Alfvén Wave Heating.

1. INTRODUCTION

Recent experimental [1] and numerical [2] results concerning the antenna loading for Alfvén wave heating of Tokamak plasmas show evidence of resonance peaks at frequencies just below the lower edge of the Alfvén continuum. We have provided a simple interpretation [3] of this phenomenon by means of the ideal MHD theory. In particular, we have shown that the resonance peaks can be related to the excitation of a new class of eigenmodes of the Alfvén wave which we call "global eigenmodes of the Alfvén wave" (GEAW) to distinguish them from the well-known singular eigenmodes.

The objective of this paper is to explore the potential for a new heating scheme using the resonant excitation of GEAW.

2. GLOBAL EIGENMODES OF THE ALFVEN WAVE

We consider low-frequency ($\omega \ll \omega_{ci}$) small-amplitude perturbations in a cold current-carrying plasma. The plasma motion can then be described by the linearized ideal MHD equations

$$\rho c \frac{\partial \underline{v}}{\partial t} = \underline{j} \times \underline{B}_o + \underline{j}_o \times \underline{B}, \tag{1}$$

$$c\underline{E} + \underline{v} \times \underline{B}_o = 0, \tag{2}$$

where $\underline{v}$ is the plasma velocity, $\underline{j}$ and $\underline{j}_o$ are the perturbation and equilibrium current densities, $\underline{B}$ and $\underline{B}_o$ are the perturbation and equilibrium magnetic fields, $\underline{E}$ is the electric field and ρ is the equilibrium mass density. We adopt a cylindrical geometry and assume that the equilibrium quantities are functions of radius r only. We may then take the time and space dependence of the perturbation quantities as $\exp[i(kz+m\Theta-\omega t)]$. Moreover, we introduce a local coordinate system with $\hat{r}$, $\hat{e}_\perp = \hat{e}_\parallel \times \hat{r}$, $\hat{e}_\parallel = \underline{B}_o/B_o$ and assume $|B_{o\Theta}/B_{oz}| \ll 1$. On combining Eqs. (1) and (2) with the Faraday law we can determine a relation between $\underline{j}$ and $\underline{E}$, and hence obtain the expression for the plasma dielectric tensor $\underline{\underline{\varepsilon}}$. Up to the first order in $B_{o\Theta}/B_{oz}$ and for $c/c_A \gg 1$ we find

$$\varepsilon_{rr} = \varepsilon_{\perp\perp} = \left(\frac{c}{c_A}\right)^2, \tag{3}$$

$$\varepsilon_{\perp r} = - \varepsilon_{r\perp} = i\left(\frac{c}{\omega}\right)^2 k_\parallel \frac{1}{r} \frac{d}{dr} \left(r \frac{B_{o\Theta}}{B_{oz}}\right), \tag{4}$$

where c_A is the Alfvén speed and $k_\parallel B_o = kB_{oz} + (m/r)B_{o\Theta}$.

The other components of $\underline{\underline{\varepsilon}}$ are not needed since $E_\parallel = 0$ in the MHD model. On substituting the expressions (3) and (4) into the Maxwell equations expanded up to the first order in $B_{o\Theta}/B_{oz}$ we obtain two coupled first-order equations for the quantities E and $B_\parallel$

$$A \frac{1}{r}\frac{d}{dr}(rE_\perp) = -\frac{2B_{o\Theta}B_{oz}}{4\pi r\rho} k_\parallel k_\perp E_\perp + \frac{i\omega}{c}(A-k_\perp^2 c_A^2)B_\parallel, \qquad (5)$$

$$A\frac{dB_\parallel}{dr} = \frac{c}{i\omega}\left[\left(\frac{2B_{o\Theta}}{r}\right)^2 \frac{k_\parallel^2 B_{oz}^2}{4\pi\rho} - 4\pi\rho A^2\right] E_\perp + \frac{2B_{o\Theta}B_{oz}}{4\pi r\rho} k_\parallel k_\perp B_\parallel, \qquad (6)$$

where $A = \omega^2 - \omega_A{}^2$, $\omega_A{}^2 = k_\parallel{}^2 c_A{}^2$ and $k_\perp B_o = (m/r)B_{oz} - kB_{o\Theta}$.

These are the basic equations of our model.

We shall now seek a solution to Eqs. (5) and (6) in the WKB approximation. Simple algebra yields the following dispersion relation:

$$(\omega^2-\omega_A^2)\left[\omega^2-c_A^2(k_r^2+k_\perp^2+k_\parallel^2)\right]\left[\omega^2-\omega_A^2+\frac{(2B_{o\Theta}/r)^2 k_\parallel^2 B_{oz}^2}{(\kappa_r^2+k_\perp^2)\ (4\pi\rho c_A)^2}\right] = 0, \qquad (7)$$

where k_r is the radial wavenumber. This dispersion relation has obviously three branches. The first and second ones correspond to the usual Alfvén wave (AW) and the fast magnetoacoustic wave, respectively. The third branch, hitherto unknown, corresponds to the modes which we call "global eigenmodes of the Alfvén wave" (GEAW).

Figure 1 shows schematically the dispersion relation (7) as a function of ρ. For parameters typical of present day Tokamaks the fast magnetoacoustic wave is cutoff, and sometimes called the surface wave (SW). It is worth mentioning that the new modes cannot be found in a slab geometry. Their existence is intrinsically connected with the curvature of magnetic field lines [4].

The global eigenmodes can be found numerically by means of the one-dimensional spectral code THALIA [5]. For this purpose we choose the following profiles of the equilibrium current and density

$$j_{o\Theta} = 0, \; j_{oz} = j(o) \left[1-\left(\frac{r}{a}\right)^2\right]^{\kappa}, \quad \rho = \rho(o) \left[1 - 0.95 \left(\frac{r}{a}\right)^2\right], \tag{8}$$

where a is the plasma radius. The quantities j(o) and κ are free parameters while $\rho(o)$ appears only in the normalizing Alfvén frequency

$$\omega_N = \frac{B_{oz}}{a[4\pi\rho(o)]^{1/2}}. \tag{9}$$

The plasma is surrounded by a vacuum region which is limited by a concentric wall of radius 1.6a.

In Fig. 2 is shown the spectrum associated with the global eigenmodes in the stable region. The distance $\Delta\omega^2/\omega_N^2$ of the eigenfrequencies from the lower edge of the Alfvén continuum is plotted as a function of the toroidal mode number n = kR, where R is the major radius. The parameters used for this figure were $\kappa = 4$, m = 1, $B_{oz} = 1$ and j(o) was determined from the condition that the safety factor at the axis $q_o = 1$. Only the eigenfrequencies with $\Delta\omega^2/\omega_N^2 > 10^{-5}$ are given. At $n = n_c$ (marked by II) we find 13 eigenfrequencies. The number of eigenfrequencies decreases for $n < n_c$ (region I) and for $n > n_c$ (region III). For high (n > 7) all the eigenfrequencies seem to disappear. For negative n numbers, the mode corresponding to the uppermost eigenfrequency becomes unstable and the other global eigenmodes disappear.

3. "ALFVEN EIGENMODE RESONANCE HEATING" SCHEME

The existence of global eigenmodes associated with frequencies below the Alfvén continuum suggests most naturally that, for heating purposes, these modes could be excited resonantly [1,2,3]. A priori there are as many modes for candidates as there are combinations of radial (ℓ), poloidal (m) and toroidal (n) mode numbers. Noting that the excitation of modes with

$m > 1$ usually resembles even quantitatively the excitation of modes having $m = 1$, we may restrict our interest to $m = 1$. The remaining problem is then to find the best combination of ℓ and n. By the "best" (ℓ,n) we mean the eigenmode characterized by (ℓ,n) whose excitation leads to the largest antenna loading impedance, Z.

Concerning the radial mode number ℓ one presumes that the lowest mode number, $\ell = 1$, must be the best. It has indeed been shown [3] that the radial displacement $\xi_r(a)$ at the plasma-vacuum interface, attains a maximum value for $\ell = 1$. The basic or "ground" mode ($\ell = 1$) is therefore the most easily accessible mode and its excitation should lead to the largest antenna loading impedance. That this is the case has been demonstrated by the Texas group in a kinetic calculation concerning PLT [2].

In order to optimize the toroidal mode number n a bit more is needed than just the solutions of the eigenvalue problem in cylindrical geometry. In what follows use is made of the cylindrical [6] and the toroidal [7,8] Alfvén Wave Heating codes which treat the wave excitation problem in the framework of ideal MHD,

$$-(\omega+i\nu)^2\rho\underline{\xi} = \underline{F}(\underline{\xi}). \tag{10}$$

In the resonant absorption studies the artificial damping coefficient ν simply ensured causality. In the present investigation we use ν to simulate electron Landau damping and obtain in this way reasonable estimates for the antenna loading impedance.

Since the GEAW have much of the physical properties of the usual Alfvén waves it is reasonable to assume that their Landau damping is the same. In the limit $\omega \ll \omega_{ci}$ and $k_\parallel < (k_\perp^2 + k_r^2)^{1/2}$ one obtains [9]

$$\frac{\nu}{\omega} \approx \frac{\pi}{8}\frac{m_e}{m_i}\frac{v_{the}}{c_A}\frac{\omega^2}{\omega_{ci}}\frac{k_\perp^2+k_r^2}{k_\parallel^2}\exp -\frac{1}{2}\left(\frac{\omega}{k_\parallel v_{the}}\right)^2 \tag{11}$$

The radial structure of modes with $n > 0$ may roughly be approximated by $k_r \approx 2\pi\ell/a \gg k_\perp$. With $k_\parallel \approx n/R$ we then find $(k_\perp^2+k_r^2)^{1/2}/k_\parallel \approx 2\pi(R/a)(\ell/n)$
The frequencies of GEAW are of the order of $k_\parallel c_A$.

Using these estimates and assuming $\beta m_i/m_e = 1$ or $v_{the}^2/c_A^2=0.5$, we have calculated ν/ω for the modes n=2, ℓ=1,2 in PLT. The parameters were taken from Ref. 2. We find the values $4\cdot10^{-5}$ and $1.6\cdot10^{-4}$. With these values of ν/ω the cylindrical MHD model yields impedances within a factor of 2 of those obtained with the kinetic model [2] assuming $\omega/\omega_{ci} \ll 1$.

Restricting our investigation to $\ell = 1$, we now show that the mode n=0 leads to the largest impedance. The estimate $(k_\perp^2 + k_r^2)^{1/2}/k_\parallel \approx (R/a)q_s$ for the case n=0 is, with an assumed safety factor at the plasma surface $q_s \approx 3$, of the same order of magnitude as for n=2. Applying the calculation to a specific TCA equilibrium (B_{oz} = 12 kG, q_o = 1, $n_o = 3\cdot10^{13}$ cm^{-3} Deuterium, T_{eo} = 800 eV, R/a = 3.6, κ = 4) we obtain as a rough estimate $\nu/\omega \approx 0.01$ irrespective of n. In Figure 3 we show the exciting frequency ω and the antenna loading impedance Z as a function of n for a helical antenna of radius 1.2a. The impedance clearly reaches a maximum for n=0. In reality this maximum would even be more pronounced because, in a better estimate, ν/ω would tend to smaller values with decreasing n instead of remaining constant. The mode n=0 is therefore the best!

From preliminary toroidal calculations using n $\geqslant$ 1 we arrive at the same conclusion. These calculations show that a large fraction of the power is deposited near the plasma surface. The situation appears to be similar to that encountered with the usual Alfvén wave heating scheme for which toroidal coupling leads to surface heating [8]. It therefore appears that AERH using an n $\geqslant$ 1 antenna is not advantageous compared to resonant absorption. On the other hand, for the case n=0, AERH has attractive features. The low frequency involved does not appear in any Alfvén continuum. Therefore, there is no possibility of coupling to the plasma surface. Since only the cases n $\geqslant$ 1 can be treated with the present version of our toroidal heating code, we cannot unfortunately corroborate this assertion by a numerical calculation.

Another advantageous feature of AERH is the simplicity of an n=0 antenna structure. In the resonant absorption scheme the n=0 antenna is not well coupled to the plasma since the global eigenmode is far from the continuum.

The most attractive point of the AERH scheme, however, is the fact that the antenna loads turn out to be 10-100 times larger than those in the usual Alfvén Wave Heating scheme.

The AERH scheme might encounter certain difficulties due to the sharpness of the resonances ($\gamma/\omega \sim 10^{-4} \div 10^{-2}$). It is known that the frequency of the n=0 modes are affected by the equilibrium profiles, by the ratio of the plasma radius to that of the conducting wall, or by the elongation of the plasma cross-section. If, during the heating pulse, the profiles evolve either by themselves or under the influence of the heating, frequency tracking might be necessary.

In order to illustrate this problem, we calculate the frequency and the loading impedance for the mode (n=0, ℓ=1) in two simple families of equilibria. We vary j(o) and κ, Eq. (8), in such a manner that in the 1st family of equilibria $q_s = 5$ and $q_o = 2.5 \div 0.5$ (Fig. 4) and in the second family $q_o = 1$ and $q_s = 2 \div 9$ (Fig. 5). TCA parameters lead to the estimation $\gamma/\omega \approx 4.10^{-4}\ q_s^2$.

The equilibria investigated in Fig. 4 have all the same total current. We remark that in this family the sensitivity of ω and Z to profile changes is weak. In contrast, we find a strong dependence of ω and Z in the case where the change in profiles results in a change of the total current (Fig. 5). We find roughly $\omega \sim 1/q_s$ and $Z \sim 1/q_s^3$.

In conclusion it may be noted that although definite assurance cannot be given that the AERH scheme with axisymmetric excitation (n=0) is likely to work, there is a certain confort to be taken from the fact that resonance peaks have been observed experimentally [1] and that they have been identified positively [3] as GEAW as described by simple MHD theory.

Note added after the Symposium

At the Symposium A.W. Kolfschoten and T. Hellsten pointed out to us that J.P. Goedbloed was aware of the complexity of the Alfvén spectrum as early as in 1975; see Phys. Fluids **18** (1975) 1258.

References

[1] A. de Chambrier et al., Lausanne Report LRP 195/81, accepted for publication in Plasma Physics.

[2] D.W. Ross, G.L. Chen and S.M. Mahajan, Austin Report FRCR # 227, June 1981, to be published in Phys. Fluids 25. No.4 (1982).

[3] K. Appert, R. Gruber, F. Troyon and J. Vaclavik, Lausanne Report LRP 200/81, December 1981, to be published in Plasma Physics.

[4] K. Appert, B. Balet and J. Vaclavik, Phys. Letters 87A (1982), 233.

[5] K. Appert et al., Comp. Phys. Commun. 10 (1975), 11.

[6] B. Balet, K. Appert and J. Vaclavik, Lausanne Report LRP 188/81, accepted for publication in Plasma Phys.

[7] K. Appert et al., Proc. 2nd Joint Grenoble-Varenna Int. Symp. on Heating in Toroidal Plasmas, Como 1980, Vol. II, p. 643.

[8] K. Appert et al., Lausanne Report LRP 187/81, submitted to Nucl. Fusion.

[9] A.I. Akhiezer et al., "Plasma Electrodynamics", Pergamon, Oxford, 1975, Vol. I, p. 246.

[10] A. de Chambrier et al., Proc. 3rd Joint Varenna-Grenoble Int. Symposium on Heating in Toroidal Plasmas, Grenoble, March 22 - 26, 1982.

Figures Captions

Figure 1: Schematic diagram of the dispersion relation (7): radial wave number vs. density for a typical Tokamak plasma.

Figure 2: Spectrum associated with the global eigenmodes. The distance $\Delta\omega^2/\omega^2$ of the eigenfrequencies from the lower edge of the Alfvén continuum is plotted versus the toroidal mode number n for different radial mode numbers ℓ. For negative values of n, the mode corresponding to the uppermost eigenfrequency is unstable.

Figure 3: Antenna loading impedance Z/Z_N and frequency ω/ω_N versus the toroidal mode number n for the mode $\ell = 1$. The normalizing impedance $Z_N = (4\pi)^2 \cdot 10^{-9}(R/a)c_A[\text{Ohm}]$.

Figure 4: Antenna loading impedance Z/Z_N and frequency ω/ω_N versus the safety factor at the axis q_o. The safety factor at the plasma surface $q_s = 5$.

Figure 5: Antenna loading impedance Z/Z_N and frequency ω/ω_N versus the safety factor at the plasma surface q_s. The safety factor at the axis $q_o = 1$.

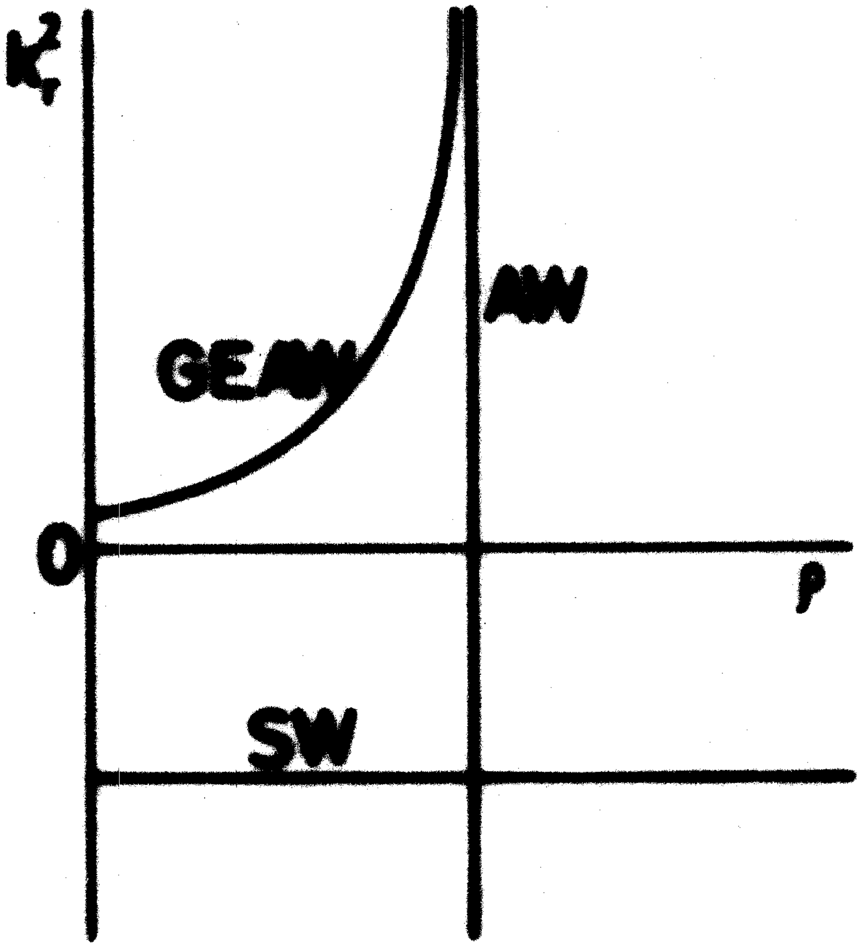
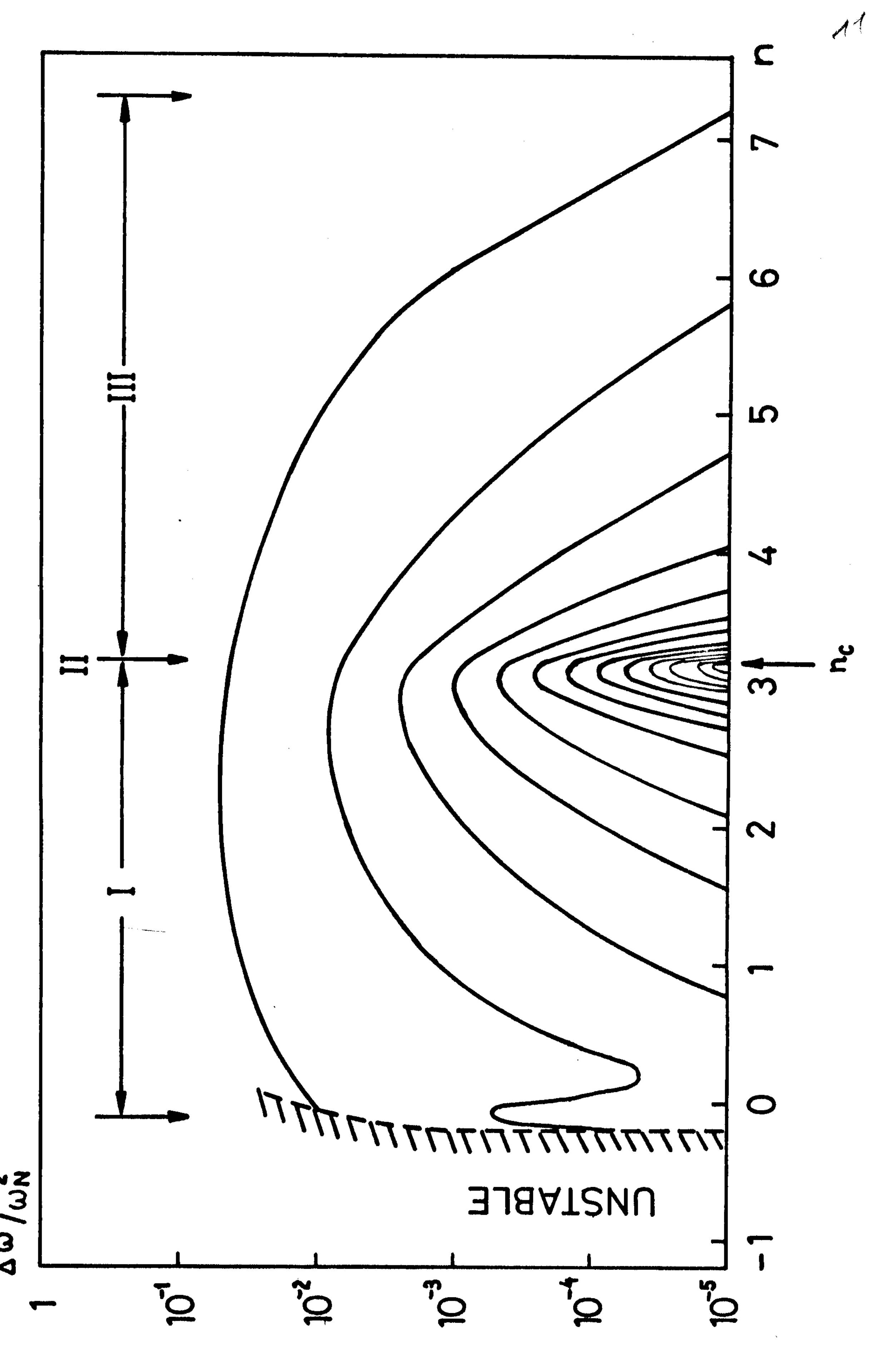


FIG. 1



$\Delta\omega^2/\omega_N^2$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
−1
0
1
2
3
4
5
6
7
n
n_c
I
II
III
UNSTABLE

FIG. 2

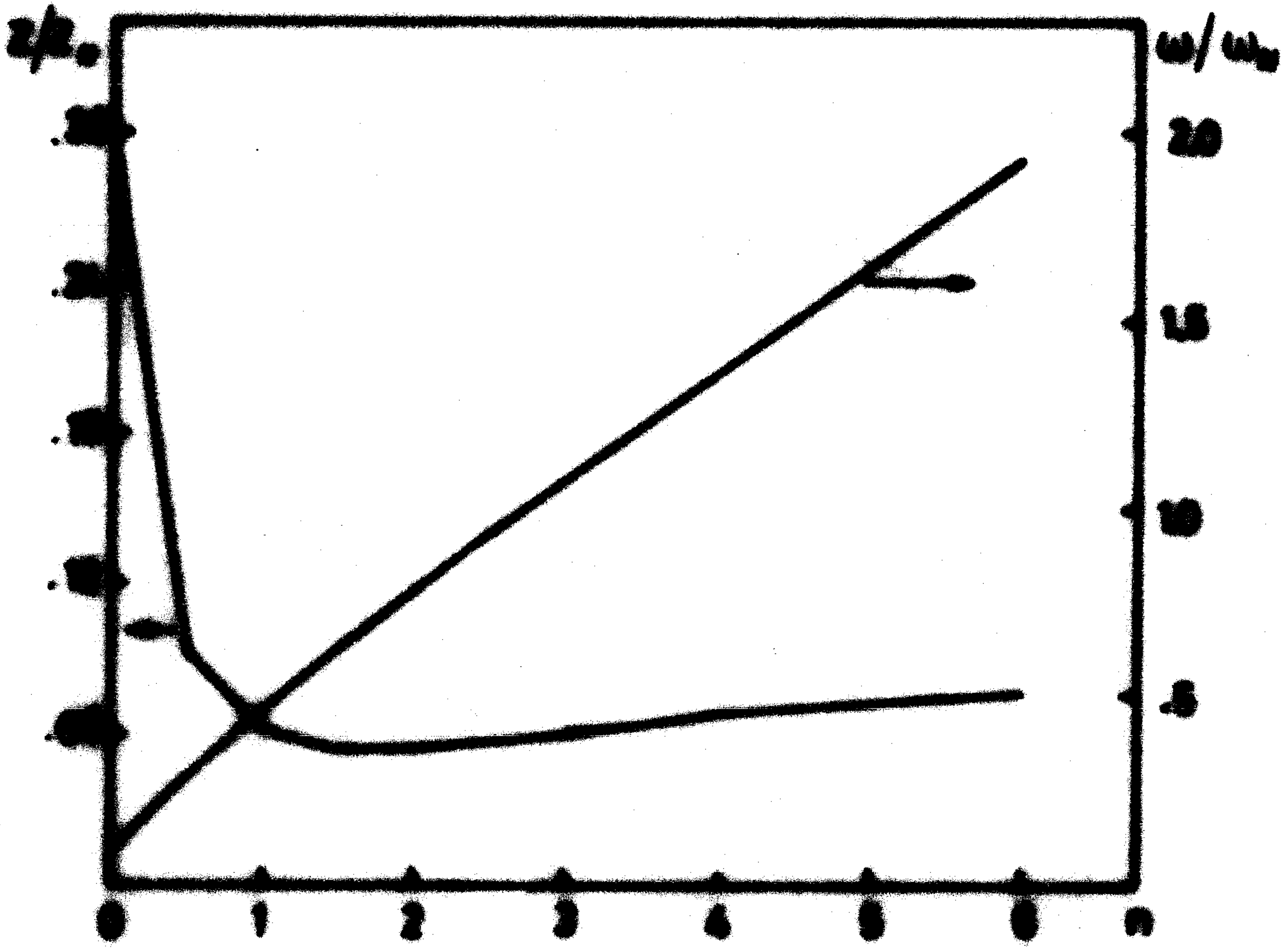


FIG. 3

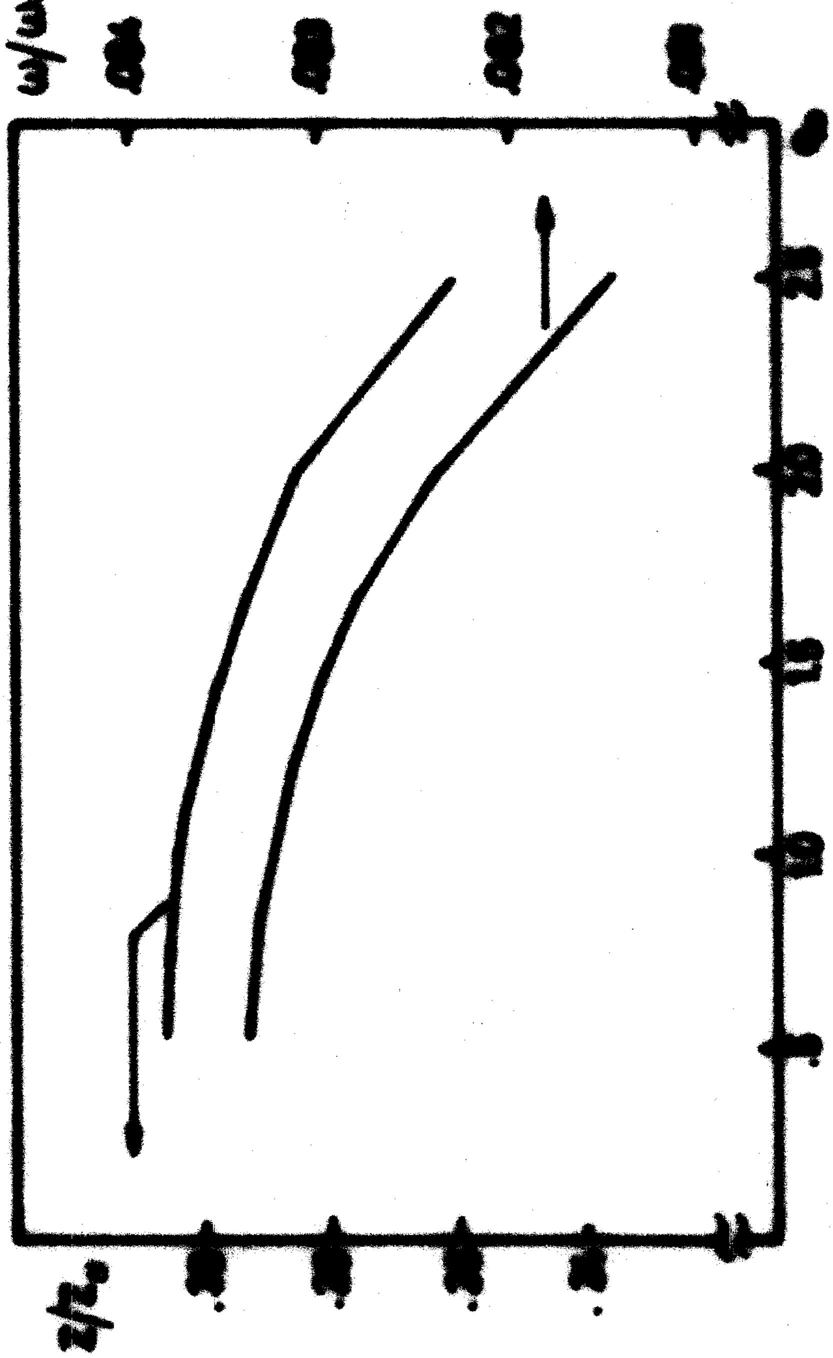


Fig. 4

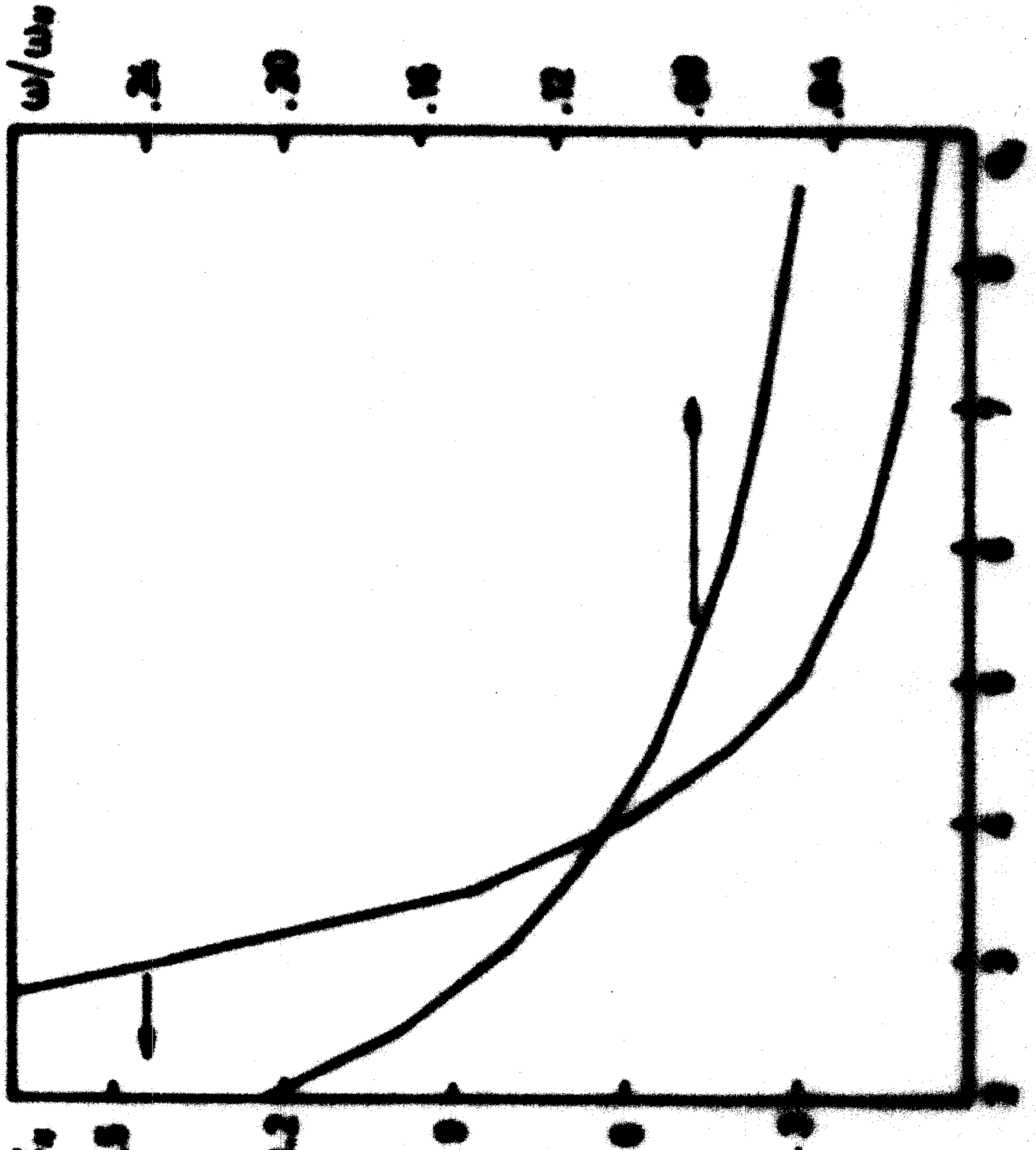


рис. 5