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K. Appert, B. Balet, R. Gruber, F. Troyon, T. Tsunematsu*
and J. Vaclavik

EURATOM-Switzerland Fusion Association,
Centre de Recherches en Physique des Plasmas,
Ecole Polytechnique Fédérale de Lausanne,
CH-1007 Lausanne, Switzerland

*Present address: Japan Atomic Energy Research Institute,
Tokai-Mura, Naka-Gun, Japan

ABSTRACT

The heating of toroidal plasmas by resonant absorption of Alfven waves is considered in the framework of ideal MHD. A theory is developed which closely parallels that of ideal MHD stability. Computations are performed using the numerical methods known from stability theory. It is shown that the overall picture of Alfven wave heating in a torus with circular cross-section does not fundamentally differ from that in a cylinder. This type of plasma may be efficiently heated internally when a surface quasi-mode is excited. In contrast, plasmas with elongated cross-sections are shown to suffer from edge heating due to substantial linear mode-coupling. Alfven wave heating of two specific tokamaks, TCA and JET, is discussed.

1. INTRODUCTION

Radio-frequency plasma heating by resonant absorption of Alfven waves has been proposed as early as 1965 by Dolgopolov and Stepanov [1]. It seems ,however, that this first proposal was ignored by the radio-frequency community. It was only in 1973 - 74 when the papers by Jankovich [2], Grossmann and Tataronis [3] and Hasegawa and Chen [4] appeared that experiments started to be planned.

Until now, this heating scheme has been tested experimentally on pinch- and stellarator-like devices [5-10] and it has been found that energy may easily be deposited in the plasma by this method. In some stellarator experiments [9,10], however, there has been an indication of enhanced transport due to the RF heating. At present, it is not yet clear whether this enhanced transport was just due to a bad choice of parameters or whether it is an unavoidable side-effect of this heating scheme. To answer this question many more experiments need to be performed, especially on Tokamaks.

The position of the theoretical development of Alfven wave heating is quite similar to that of the experiments. There is general agreement on the basic physical picture but disagreement concerning the choice of parameters required to optimize the heating scheme. It is agreed that for efficient heating, the energy cannot be directly coupled from the antenna to the shear Alfven wave but that a compressional collective mode has to be excited first. This collective mode, whose frequency must lie in the Alfven continuum, may be the remnant of a surface wave mode [4,11,12] or of a fast magnetosonic cavity mode [13]. At present, the respective merit of these two excitation methods [11,13] is unclear. So also is the physics of the shear Alfven waves, which appears to be sensitive to minor effects [14] such as those due to finite values of ω/ω_{ci} [15,16], or Larmor radius [17,18] and effects due to the geometry like toroidicity [19], helicity or elongation.

In this paper we attempt to assess the effects of toroidicity and elongation (or ellipticity) in axisymmetric equilibria in the framework of ideal MHD. Apart from quantitative changes with respect to cylindrical geometry, toroidicity and ellipticity bring qualitative changes due to the coupling they induce between waves with different poloidal wave numbers. It should be noted that these waves have their resonant surface at different minor radii. It may happen, therefore, that energy which was intended to be absorbed on a surface halfway inside the plasma is coupled into a surface near the plasma edge. It has, in fact, frequently been conjectured that it may be impossible to heat the interior of toroidal plasmas with Alfven waves. The extent to which the effects due to toroidicity and ellipticity modify the favorable results obtained with cylindrical models [12,15] therefore needs to be investigated. A preliminary answer to the question concerning the effect of toroidicity has been given in Ref. 19.

The plan of the paper is as follows. In Section 2 we derive the basic equations and express them in a form which can be handled by a numerical method used in MHD stability calculations. Section 3 contains a discussion of the properties of the computational model of resonant absorption. In Section 4, toroidal and elongation effects of Alfven wave heating are systematically investigated. Using this general background, in Section 5 we investigate the heating possibilities in two specific tokamaks, TCA and JET. Finally, the conclusions of the present study are drawn in Section 6.

Two appendices are also included. Appendix A contains information on the equilibria and antennae used in the computations. The dimensionless units and the coordinate systems are also defined there. Appendix B contains the Green's function formalism used to obtain the solution for the vacuum magnetic field used in the main text.

2. THEORY

2.1 Basic Equations

We consider the excitation of small amplitude waves in a perfectly conducting toroidal plasma. Specifically, we imagine the plasma to be surrounded by a vacuum region (V_I), an infinitely thin current-carrying antenna, then by a second vacuum region (V_{II}) and finally by a perfectly conducting shell, see Fig. 1.

Given a time-varying current density

$$\vec{j} = \delta(A)\, \nabla A \times \nabla \beta \qquad (1)$$

on an antenna surface defined by

$$A(\vec{x}) = 0, \qquad (2)$$

where $\beta(\vec{x},t)$ is an arbitrarily chosen potential, we wish to determine the time-varying magnetic field $\vec{B}_V(\vec{x},t)$ in the vacuum region as well as the plasma displacement, $\vec{\xi}(\vec{x},t)$. We would also like to determine the power emitted by the antenna,

$$P = -\int \vec{j} \cdot \vec{E}\, d^3x \,. \qquad (3)$$

Here $\vec{E}$ is the electric field at the antenna. Finally, it is important to know where in the plasma the energy is deposited.

In this problem the plasma motion is governed by the well-known linearized ideal MHD equations [20].

$$\rho_0 \, \partial^2 \vec{\xi} / \partial t^2 = \vec{F}(\vec{\xi}) \equiv - \nabla p - \vec{B} \times \mathrm{rot}\, \vec{B}_0 - \vec{B}_0 \times \mathrm{rot}\, \vec{B} \tag{4}$$

where p and $\vec{B}$ are the perturbed pressure and magnetic field,

$$p = - \vec{\xi} \cdot \nabla p_0 - \gamma p_0 \, \mathrm{div}\, \vec{\xi} \, , \qquad \vec{B} = \mathrm{rot}\left[\vec{\xi} \times \vec{B}_0 \right] , \tag{5}$$

respectively. The equilibrium is described by the mass density, ρ_0, the pressure, p_0, and the magnetic field, $\vec{B}_0$. The field equations in the vacuum regions are simply

$$\mathrm{div}\, \vec{B}_v = 0 \, , \qquad \mathrm{rot}\, \vec{B}_v = 0 \, . \tag{6}$$

Equations (4)-(6) must be supplemented by the matching conditions on the plasma-vacuum interface and on the antenna, and by the boundary conditions at the perfectly conducting shell.

The two matching conditions on the plasma-vacuum interface are [20]

$$p + \vec{B}_0 \cdot \vec{B} = \vec{B}_0 \cdot \vec{B}_v \, , \tag{7}$$

$$\vec{n}_p \cdot \vec{B}_v = (\vec{B}_0 \cdot \nabla) (\vec{n}_p \cdot \vec{\xi}) , \tag{8}$$

if we assume, for simplicity, that there is no equilibrium surface current flowing along the interface. Eq. (7) represents the first-order pressure balance while Eq. (8) follows from the continuity of the tangential component of the electric field, $\vec{E} - \vec{B}_0 \times \partial \vec{\xi} / \partial t$, in the coordinate system of the moving plasma. (For details see Kadomtsev [20]).

In Eq. (8) we have denoted by $\vec{n}_p$ the outer normal unit vector on the unperturbed plasma surface. We will use the same notation for the outer normal unit vectors on the antenna and on the shell, $\vec{n}_a$ and $\vec{n}_s$, respectively.

The matching conditions on the antenna may be obtained from Ampère's law using Eq. (1) and from div $\vec{B}_V = 0$:

$$\vec{n}_a \times [\![\vec{B}_V]\!]_a = \vec{n}_a \times \nabla\beta , \tag{9}$$

$$\vec{n}_a \cdot [\![\vec{B}_V]\!] = 0. \tag{10}$$

The double bracket indicates a jump across the antenna when passing from the inside to the outside.

Finally, the boundary condition at the conducting shell follows form $\vec{n}_s \times \vec{E} = 0$ and is

$$\vec{n}_s \cdot \vec{B}_V = 0 . \tag{11}$$

The power emitted by the antenna may be obtained by insertion of Eq. (1) into (3). After a partial integration, the use of Faraday's law and the evaluation of the Dirac delta function $\delta(A)$ it becomes

$$P = - \int_a \beta \, \dot{\vec{B}}_V \cdot d\vec{\sigma}_a . \tag{12}$$

The integral in Eq. (12) is a surface integral along the antenna. We will use the corresponding notation for integrals along the plasma-vacuum interface and along the shell, $\int_p d\vec{\sigma}_p$ and $\int_s d\vec{\sigma}_s$, respectively. Note that $\vec{n}_a \, d\sigma_a = d\vec{\sigma}_a$.

The equations (4) through (11) form a closed set. For a given antenna current potential, β, one can, in principle, determine $\vec{\xi}(\vec{x},t)$, $\vec{B}_v(\vec{x},t)$ and $P(t)$. In practice, this is a formidable task because the right hand side of Eq. (4) is a complicated operator in general toroidal geometry. Even the vacuum problem, Eq. (6), is not easy to solve.

For axisymmetric systems, however, efficient numerical methods have been developed for the purpose of ideal MHD stability calculations. In these studies the eigenvalue problem originating from Eq. (4),

$$-\omega^2 \rho_0 \vec{\xi} = \vec{F}(\vec{\xi}), \qquad (13)$$

supplemented by Eq. (5) through (8) and (11) is treated by the finite element method. The same method can be directly applied to our heating problem. In fact, we can even envisage the use of complete sections of our stability code ERATO [21] for the new Alfven heating code. All that is required is to reformulate the vacuum problem [22] including the additional matching conditions on the antenna, Eq. (9) and (10), and to calculate the power, Eq. (12).

Details of the axisymmmetric equilibria and antennae used in the calculations presented in this paper are given in Appendix A.

2.2 Weak Variational Form of the Equation of Motion

The link between the ERATO stability calculations and the heating problem is most obvious in Galerkin's weak variational form of Eq. (4) through (11). We will therefore derive it here in terms of $\vec{\xi}$ and the magnetic potential Φ defined by

$$\vec{B}_v = \nabla \Phi . \tag{14}$$

Let $\vec{\eta}(\vec{x})$ be an arbitrary test function associated with $\vec{\xi}(\vec{x},t)$. Following Kadomtsev [20] we can then show that

$$\int_V \rho_0 \vec{\eta} \cdot \frac{\partial^2 \vec{\xi}}{\partial t^2} d^3x - \int_V \{ p \, \mathrm{div} \vec{\eta} + \vec{B} \cdot [\vec{\eta} \times \mathrm{rot} \vec{B}_0] - \vec{B} \cdot \mathrm{rot} [\vec{\eta} \times \vec{B}_0] \} d^3x = \int_{\delta V} (p + \vec{B} \cdot \vec{B}_0) \vec{\eta} \cdot d\vec{\sigma} . \tag{15}$$

Here V may be any plasma volume limited by a magnetic surface δV. The surface element is denoted by $d\vec{\sigma}$, where $\vec{B}_0 \cdot d\vec{\sigma} = 0$ has been used in the derivations of Eq. (15).

If we specialize Eq. (15) to the entire plasma volume, V_p, the right hand side, denoted by C, may be expressed in terms of the vacuum field. Using the pressure balance relation (7) we obtain after a partial integration

$$C = - \int_p (\vec{B}_0 \cdot \vec{B}_v) \vec{\eta} \cdot d\vec{\sigma}_p = \int_p \Phi (\vec{B}_0 \cdot \nabla) \vec{\eta} \cdot d\vec{\sigma}_p . \tag{16}$$

In the stability calculations, the expression (16) is the starting point for the numerical determination of the vacuum contribution [22].

In those calculations a Green's function method is used to express the potential Φ on the plasma surface as a functional of its normal derivative $\partial\Phi/\partial n_p$.

For the present case, Φ has two sources:

$$\Phi(\vec{x}_p) = \int_p Q(\vec{x}_p, \vec{x}_p{}') \, \nabla' \Phi(\vec{x}_p{}') \cdot d\vec{\sigma}_p{}' + \Phi_E(\vec{x}_p) \, . \qquad (17)$$

The first term results from $\partial\Phi/\partial n_p$ on the plasma surface or rather from the displacement of the plasma surface, given by Eq. (8). This term is identical to the one in the stability calculations and may be regarded as the potential Φ resulting from an internal source. The quantity Φ_E is then the potential associated with an external source, namely the current imposed on the antenna. Note that $\vec{n}_p \cdot \nabla\Phi_E(x_p) = 0$. The explicit expressions for Q and Φ_E are derived in Appendix B, Eq. (B20) and (B22) respectively.

With the help of Eq. (8) and (17), the vacuum contribution to the weak form (15) (with $V = V_p$) may finally be written as

$$\begin{aligned} C = \int_p \int_p & Q(\vec{x}_p, \vec{x}_p{}') (\vec{B}_0 \cdot \nabla) \vec{\xi}(\vec{x}_p{}', t) \cdot d\vec{\sigma}_p{}' \, (\vec{B}_0 \cdot \nabla) \vec{\eta}(\vec{x}_p) \cdot d\vec{\sigma}_p \\ & + \int_p \Phi_E(\vec{x}_p) (\vec{B}_0 \cdot \nabla) \vec{\eta}(\vec{x}_p) \cdot d\vec{\sigma}_p \, . \end{aligned} \qquad (18)$$

2.3 Discretization

The weak form (15) with the contribution (18) corresponds to the strong variational form used as a starting point for the discretization of the stability problem [21]. The discretization procedure is described in detail in Ref. 21 and will therefore not be repeated here. The result is merely cited. Assume that the approximation to $\vec{\xi}(\vec{x},t)$ is defined by n discrete values $\xi_i(t)$. In addition, let η_i and $\Phi_i(t)$ denote the discrete values corresponding to $\vec{\eta}(\vec{x})$ and $\Phi_E(\vec{x}_p,t)$. The discretized version of the weak form is then

$$\eta_i B_{ij} \ddot{\xi}_j = \eta_i A_{ij} \xi_j + \eta_i C_{ij} \Phi_j \, , \tag{19}$$

where η_i have arbitrary values. The matrices $\vec{\vec{A}},\vec{\vec{B}}$ and $\vec{\vec{C}}$ are the discrete forms of the bilinear integral operators in Eqs. (15) and (18). $\vec{\vec{A}}$ and $\vec{\vec{B}}$ are available in ERATO [21].

Using the arbitrariness of η_i we find

$$B_{ij} \ddot{\xi}_j = A_{ij} \xi_j + C_{ij} \Phi_j \tag{20}$$

which is the discrete form of Eq. (4) including the solution of Eq. (6) and all the boundary conditions (7) through (11). Eq. (20) allows one to determine numerically the plasma response, $\vec{\xi}(\vec{x},t)$ to an external driving current associated with the term $C_{ij}\Phi_j$.

2.4 Energy and Power

Once the wave structure $\vec{\xi}$ is known, there remains the problem of determining the absorbed power. Moreover, we would like to obtain a complete picture of energy conservation in the system. This picture will enable us to check the computer code, and also provide us with some physical insights.

There is an obvious way to obtain energy conservation laws from a weak variational form [20]. If we put $\vec{\eta} = \partial\vec{\xi}/\partial t$ in Eq. (15) we find

$$\frac{d}{dt}\left[K(V)+W(V)\right] = -\int_{\delta V} \Pi \, \dot{\vec{\xi}} \cdot d\vec{\sigma}. \tag{21}$$

Here K and W are the kinetic and potential energy of the plasma contained in a volume V limited by an arbitrary magnetic surface δV. The expression on the right hand side is the work done by the plasma against the total pressure, $\Pi = p + \vec{B} \cdot \vec{B}_0$. Alternatively, this expression can be interpreted as an energy flux through the magnetic surface δV and can be used to determine the radial energy deposition profile.

If we specialize Eq. (21) to the entire plasma volume we obtain

$$\dot{K}_p + \dot{W}_p = \int_p \Phi \, \nabla\dot{\Phi} \cdot d\vec{\sigma}_p\ , \tag{22}$$

which corresponds to the weak form (15) combined with (16). An alternative form of Eq. (22) is

$$\dot{K}_p + \dot{W}_p + \dot{W}_v^I = \int_p \Phi_E \, \nabla\dot{\Phi} \cdot d\vec{\sigma}_p\ , \tag{23}$$

where $\dot{W}_v{}^I = \int (\Phi_E - \Phi) \; \nabla\dot{\Phi} \cdot d\vec{\sigma}_p$ is the change of the magnetic field energy in the vacuum due to the internal source $\partial\Phi/\partial\vec{n}_p$ on the plasma boundary.

The numerical counterpart of Eq. (23) is obtained from (19) by choosing $\eta_i = \xi_i$ and can be written as

$$\frac{d}{dt}\left(\frac{1}{2}\dot{\xi}_i B_{ij}\dot{\xi}_j - \frac{1}{2}\xi_i A_{ij}\xi_j\right) = \dot{\xi}_i C_{ij} \Phi_j . \tag{24}$$

The term $-\frac{1}{2}\xi_i A_{ij}\xi_i$ includes therefore $W_p + W_v{}^I$.

Finally, we may write Eq. (22) in the form

$$\dot{K}_p + \dot{W}_p + \frac{d}{dt}\frac{1}{2}\int_{V_I+V_{II}} (\nabla\Phi)^2 d^3x = -\int_a [\![\Phi]\!] \nabla\dot{\Phi}\cdot d\vec{\sigma}_a . \tag{25}$$

In contrast to Eq. (23), the total vacuum magnetic energy is included on the left hand side of the above equation. The right hand side must therefore be identified with the power delivered by the antenna,

$$P \equiv -\int_a [\![\Phi]\!] \nabla\dot{\Phi}\cdot d\vec{\sigma}_a . \tag{26}$$

With the use of Eqs. (9) and (14) it can easily be shown that the relationship (26) is identical to (12) as required.

The explicit evaluation of the power, P, is achieved by the same Green's function method as was used to derive relation (17). We show in Appendix B, Eq. (B23), that

$$\vec{n}_a\cdot\nabla\Phi(\vec{x}_a) = \int_p Z(\vec{x}_a,\vec{x}_p)(\vec{B}_0\cdot\nabla)\vec{\xi}(\vec{x}_p,t)\cdot d\vec{\sigma}_p + \psi_E(\vec{x}_a) , \tag{27}$$

By analogy with Eq. (17), we may call the first term on the right hand side the "internal" part of $\vec{n}_a \cdot \nabla\Phi(\vec{x}_a)$ and $\psi_E(\vec{x}_a)$ the "external"part, since $\psi_E(\vec{x}_a)$ is identically zero when the antenna current vanishes. The kernel $Z(\vec{x}_a, \vec{x}_p)$ and the potential ψ_E are defined by identification of Eq. (27) with Eq. (B23).

By comparison of Eqs. (23) and (26), using (27), we find that

$$\int_P \Phi_E \, \nabla \dot{\Phi} \cdot d\vec{\sigma}_P = P + \int_a [\![\Phi]\!] \, \psi_E(\vec{x}_a) \, d\sigma_a \, , \qquad (28)$$

a relation which is useful for checking the computer code.

2.5 Steady State Solution

With Eqs. (20) and (26) we apparently now have all that is required for the treatment of the Alfvén wave heating problem. One approach is to impose a current on the antenna, $\propto \sin \omega t$, having a frequency in the Alfven range and then solve the system of ordinary differential equations in time by using, say, a Runge-Kutta method. At each timestep we could evaluate the power at the antenna, using Eq. (26), and we would find that energy flows at a constant rate from the antenna to the resonant surfaces where it would accumulate in ever diminishing thin layers [19].

This approach provides one with a good physical insight into the resonant absorption, but is a very computer time consuming procedure; for the two-dimensional problem at hand it is, moreover, a prohibitively expensive procedure. Since it is not of fundamental importance we avoid computing the complete time evolution, but ask merely for the constant power input from the antenna and where it is deposited in the plasma.

Here we can make use of an important property of resonant absorption: the flux of energy into a resonant layer is independent of whether or not it is absorbed there [19]. The inclusion of an arbitrary, but sufficiently small, dissipative term in Eq. (20) therefore does not affect the power calculated. The dissipative system tends towards a stationary state having a temporal dependence exp(i ω t). For our present calculations, it is sufficient only to know this state.

The equation to be solved then becomes

$$(-\omega^2 + 2i\nu\omega) B_{ij} \xi_j = A_{ij} \xi_j + C_{ij} \Phi_j \quad , \tag{29}$$

which is a linear algebraic equation for the complex ξ_i. Here ω denotes the pump frequency and ν is the artificial damping rate. Using the complex solution for ξ_i we obtain the time-averaged complex power at the antenna

$$\mathcal{P} = -\frac{1}{2} \int_a [\![\Phi]\!] \, \nabla \dot{\Phi}^* \cdot d\vec{\sigma}_a \, . \tag{30}$$

The star indicated a complex conjugate. The real and imaginary parts of P in Eq. (30) represent the resistive and the reactive powers, respectively.

We will express results in terms of

$$\bar{p} = \mathrm{Re}\, \mathcal{P} / 2\pi R \, , \tag{31}$$

which is the resistive power due to a unit current (see Appendix A) per unit length of the torus (major radius R), and in terms of

$$Q(\omega) = \mathrm{Im}\, P / \mathrm{Re}\, P \quad , \tag{32}$$

which is an important quantity for the experimentalist. Note that Q is not the cavity Q but simply the ratio of reactive to resistive loading impedance of the antenna as a function of the pump frequency. We also calculate the resistive part of the energy flux through a magnetic surface δV:

$$S = - \frac{\omega}{2\pi R} \, \mathrm{Im} \int_{\delta V} \Pi \, \vec{\xi}^{\,*} \cdot d\vec{\sigma} \tag{33}$$

For a check of the computer code, the radial derivative of S perpendicular to δV, may be compared to the energy absorbed on a given magnetic surface,

$$D = \frac{\nu}{\pi R} \int_{\delta V} \rho_0 \, |\dot{\vec{\xi}}|^2 \, d\sigma \ . \tag{34}$$

3. PROPERTIES OF THE COMPUTATIONAL MODEL OF RESONANCE ABSORPTION

Although the present investigation concentrates on the physics of resonant absorption of Alfven waves, it is also necessary to discuss in some detail the properties of the computational model that is used. A comprehensive description of the "code physics" greatly facilitates the comprehension of the "real physics" to be presented in following sections.

Resonant absorption in a cold plasma is, in general, connected with the existence of a continuous spectrum. In particular, it is known that the ideal MHD eigenvalue problem for axisymmetric plasmas, eq.(13), leads to a continuous spectrum of shear Alfven waves [19,27]. The physical reason for the continuous spectrum is the following. In a diffuse equilibrium, which for simplicity we assume to be cylindrical, the Alfven velocity is constant on a magnetic surface and varies from one surface to the next. The frequency associated with an azimuthal wave number m and a longitudinal wave length 2π / k is

$$\omega_A(\rho) = \left(\frac{m}{\rho} B_{0\theta} + k B_{0z}\right) / \rho_0^{1/2} \qquad (35)$$

Since the mass density ρ_0 and field B_0 depend on ρ, the cylindrical radial coordinate, ω_A also depends on ρ. For tokamak equilibria and for most choices of m and k of interest for heating, ω_A is a monotonically increasing function of ρ. A wave motion having specific values of ω, m and k must therefore be confined to a narrow region around the so-called resonant surface, ρ_s, where $\omega = \omega_A(\rho_s)$. Conversely, a wave must exist for each ρ_s within the plasma whose frequency is given by $\omega = \omega_A(\rho_s)$. Shear Alfven waves therefore have a continuous spectrum.

We wish to determine how this continuous spectrum changes if the eigenvalues problem, Eq. (13), is discretized in space. Obviously the spatial discretization must lead to an approximation such that the continuous spectrum becomes discrete. In fact, the number of discrete frequencies in the "continuum" is related to the number of spatial grid points. If at some radial position ρ the width of the grid is $\Delta\rho$, one can easily determine that the spacing, $\Delta\omega$, between adjacent frequencies of the "continuum" is

$$\Delta\omega = (d\omega_A / d\rho)\Delta\rho . \qquad (36)$$

It should be noted that we shall consider not the system described by Eq. (20), but rather a "dissipative" system, Eq. (29). The damping coefficient is for all modes the same and equal to ν. The numerical problem therefore resembles a system of coupled oscillators with eigenfrequencies having different real parts but a common damping coefficient. Imagine now that such a system is to be excited with a given pump frequency ω, such that $\omega_i < \omega < \omega_{i+1}$, where ω_i and ω_{i+1} denote two adjacent eigenfrequencies of the system.

Let us first discuss the case where $\nu \ll \omega_{i+1}-\omega_i = \Delta\omega$. If ω is chosen close enough to the eigenfrequency ω_i, i.e. $\omega-\omega_i \ll \nu$, the oscillator which has an eigenfrequency ω_i is resonantly excited and the dissipated energy is proportional to $1/\nu$. If on the other hand ω lies midway between ω_i and ω_{i+1}, none of the oscillators is in resonance and only a small amount of energy proportional to ν is dissipated. These two situations obviously have nothing in common with the excitation of a continuous spectrum. On the contrary, the eigenfrequencies of the system behave as discrete eigenfrequencies.

However, the situation is quite different in the opposite limit for which $\nu \gg \Delta\omega$. In this case the resonances at adjacent eigenfrequencies overlap and many oscillators respond simultaneously to the pump: a "continuum" is excited and "resonant absorption" takes place. For ν within certain limits, the amount of dissipated energy is independent of ν.

The above comments may be illustrated by a calculation concerning TCA, the Lausanne tokamak. At this stage we do not try to model the experimental equilibrium as precisely as possible by using a numerical equilibrium code, but are content with the analytical Solov'ev equilibrium described by Eq. (A1). In dimensionless units the only free parameters of this equilibrium are the ellipticity, ε, the inverse aspect ratio, a/R, and the safety factor on axis, q_0 (see Appendix A). With $\varepsilon = 0$, $a/R = .275$ and $q_0 = 1$ we obtain an almost circular plasma with a flat q profile where q assumes the value of 1.4 on the boundary. The plasma β has a value of 6.4% on the axis. For the antenna and the conducting shell we also choose an almost circular cross-section, their radii being assigned in units of plasma radii, values of $\alpha_a = 1.6$ and $\alpha_s = 2.0$, respectively (see Eq. (A3)). The excitation current, as described by Eq. (A6), is of single helicity, $n = 2$, $m = 1$.

Results of computations that made use of the modified ERATO code are now described. For the first set of calculations we used a grid with 42 equal intervals in s and 15 in χ $(0 < \omega < \pi)$, where s and χ are the radial and azimuthal coordinates, respectively (see Appendix A).

From the spectral version of ERATO two adjacent eigenvalues were found, $\omega = 1.0292$ and $\omega = 1.0413$. These eigenvalues are associated with eigenfunctions peaking around $s = 0.5$. The heating version of ERATO was then used to calculate the absorbed power, $\bar{p}$, defined by Eq. (31), as a function of the artificial damping rate. Calculations were made once with a pump frequency equal to the eigenfrequency $\omega = 1.0292$ and again with a frequency $\omega = 1.0333$ lying between the two eigenfrequencies. The results are displayed in Fig. 2.

For $\nu \ll \Delta\omega \approx .012$, we indeed find that $\bar{p} \propto 1/\nu$ if an eigenmode is excited (curve a). We also find, as expected, that $\bar{p} \propto \nu$ for an excitation frequency between the two eigenfrequencies (curve b). The astonishing fact is that both frequencies lead to the same absorbed power for values of ν as small as $\nu \approx \Delta\omega$. This implies that the simultaneous excitation of two modes is sufficient to simulate a continuum. It is also pleasing to note that the value obtained for $\bar{p}$ is insensitive to ν for $\nu > \Delta\omega$. This behaviour is typical of resonant absorption.

We may further confirm our ideas by studying the effect of different grid spacing. A grid consisting of 80 equal intervals implies a frequency gap of $\Delta\omega = .006$ at the resonant surface of the pump frequency, $\omega = 1.0333$. The results given by curve c indicate that a lower value of ν, which yields the required solution, may be chosen, corresponding to the lower value of $\Delta\omega$. These calculations have been made with 29 azimuthal intervals. A mesh size of 80 x 29 is the maximum that can be handled by our computer.

Much can still be gained by a judicious choice for the radial mesh spacing. As long as only a few resonant surfaces contribute to the total resonant absorption we may opt for a mesh having unequal spacing, concentrating mesh points around the resonant surfaces. The benefit resulting from this modification is demonstrated by curve d. Here a 42 x 15-mesh has been used as was for cases a and b. The radial mesh points have been concentrated around the resonant surface, s = 0.5, implying a local frequency gap of $\Delta\omega$ = .003. The result, which shows an insensitivity on the artificial damping rate for ν as small as 3×10^{-4}, seems to contradict the simple conclusions we have made above. The contradiction is , however, only apparent. In case d, the pump frequency coincides, to 3 significant figures, with an eigenfrequency. For this reason the power, $\bar{p}$, at first increases with decreasing ν and only starts to decrease when ν becomes smaller than the mismatch in frequency between the pump and the eigenmode.

From Fig. 2 we conclude that the correct results is obtained if the artificial damping rate, ν, is not smaller than the frequency spacing at the resonant surface. There is, unfortunately, an inherent problem for toroidal equilibria or for equilibria with non-circular cross-section. In general, a pump excites several resonant surfaces on which different amounts of energy are absorbed. Manipulation of the local spacing of mesh points can then become a difficult and un-rewarding task. We have therefore used a standard mesh consisting of 60 x 25 irregular intervals for most of our calculations and checked the results occasionally by changing the value of ν.

The results of such a check concerning a numerical equilibrium of TCA is displayed in Fig. 3 and 4. Fig. 3 shows the mode structure, (ξ_s, ξ_χ), in the poloidal plane ϕ = 0 excited by two superposed helical antennae (m = 1, n = 2). The pump frequency is ω = .94. Figs. 3(a) and (b) were obtained with ν = .008 and ν = .074, respectively. In Fig. 4 the corresponding resistive energy flux S, defined in Eq. (33), is shown. As expected, the principle effect of a high value of ν is to smear out the main resonance (m = 1, n = 2) at the s = .5 surface. The overall resonant absorption power, $\bar{p}$ = S(s=1), is quite insensitive to ν, the difference between the two cases considered here being of the order of 3%. The reactive power, Im P, differs by 15%. In view of the complicated mode structures displayed in Fig. 3, these results are pleasing. It should be noted, however, that only a few check runs might not always be sufficient. For example, in a study of TCA, pronounced peaks in the resistive power as a function of the pump frequency were found, which were eventually discovered to be of numerical origin.

At this point, the origin of the modes appearing near the plasma surface and the poor excitation of the mode (m = -1, n = 2) are not discussed. These are due to toroidal effects and will be discussed later.

The results of this section show that ν has to meet certain conditions determined by the spatial discretization. Check runs provide confidence in the numerical scheme. It should be expected, however, that for very complicated mode structures a substantial number of grid points are necessary to avoid features of purely numerical origin.

4. GEOMETRICAL EFFECTS ON ALFVEN WAVE HEATING

4.1 Toroidicity

We now turn to a systematic study of toroidal effects. In a torus, the toroidal field B_T varies, to lowest approximation, as $[1 + (s/R)\cos\chi]^{-1}$ on a magnetic surface having a radial coordinate, s. The angular dependence causes a linear coupling between modes $\propto \exp(im\chi)$ and modes $\propto \exp[i(m+1)\chi]$. In its simplest form this coupling is illustrated in Fig. 5. The equilibrium and antenna parameters are the same as for Fig. 2. For Fig. 5, the dominant plasma response, i.e. the poloidal displacement

$$\xi_\chi(s,\chi,\phi) = \xi_\chi^s(s,\chi)\cos n\phi + \xi_\chi^a(s,\chi)\sin n\phi \tag{37}$$

has been Fourier-analysed. Here $\xi_\chi{}^s$ and $\xi_\chi{}^a$ are a symmetric and an anti-symmetric function of χ, respectively. The separation into $\xi_\chi{}^s$ and $\xi_\chi{}^a$ is required by ERATO in order to exploit the "up-down-symmetry". A displacement of this form may be decomposed into

$$\xi_\chi(s,\chi,\phi) = \sum_{m=-\infty}^{+\infty} \xi_\chi^{m,n}(s)\cos(m\chi + n\phi), \tag{38}$$

where

$$\xi_\chi^{m,n} = \begin{cases} \frac{1}{\pi}\int_0^\pi (\xi_\chi^s \cos m\chi - \xi_\chi^a \sin m\chi)\,d\chi & m \neq 0 \\ \frac{1}{2\pi}\int_0^\pi \xi_\chi^s \, d\chi & m = 0 \end{cases} \tag{39}$$

In Fig. 5, the real parts of the dominant Fourier components, $\mathrm{Re}\xi_\chi{}^{m,2}$, are shown near the resonant surface of the (n = 2, m = 1) mode, excited with a frequency ω = .95 and using a damping rate of ν = .02. The m = 1 component is dominant as is expected. There is, however, a contribution of 30% from the m = 2 component. The other neighbour of m = 1, namely m = 0, is represented with 18%. The m = 3 component appears with an 8% contribution. These fractions are high in the view of the fact that the inverse aspect ratio of the considered resonant surface at s = 0.37 is rather small; its value is approximately sa/R = 0.1. It could therefore be expected that surfaces with greater aspect ratios, i.e. surfaces near to the plasma edge, would be composed of even more complicated mixtures of poloidal components.

Before we study such complicated structures, let us ensure that the simple idea of toroidal coupling concerning small inverse aspect ratios is correct. It is to be expected that the contributions from the nearest neighbours of m = 1, i.e. the components m = 0 and m = 2, should linearly disappear, as a/R tends to zero. In addition, more distant components should depend on higher powers of a/R, specifically, $\xi_{\chi}{}^{m,2} \propto (a/R)^{|m-1|}$.

These ideas are tested on a family of equilibria which contains the equilibrium used for Fig. 5. If we wish to study the influence of only the inverse aspect ratio, a/R, on the resonant Alfven waves, a family of toroidal equilibria must be chosen which have identical toroidal current, i.e., apart from a/R effects, identical magnetic field pitch. For the same reason, the pitch of the antenna must be constant.

For the calculations of Fig. 5, values of q_0 = 1, n = 2 and a/R = 0.275 were chosen. This equilibrium is therefore a member of the family of equilibria defined by na/R = 0.55 and nq_0 = 2. Since we intend to study the limit $a/R = 0.55/n \rightarrow 0$, we choose $n > 2$. This is achieved by adding, for each integral increase in n, a new section to the torus, allowing each time one more wavelength to fit into the torus.

Figure 6 shows the dependence on a/R of three quantities. The first quantity,

$$\lambda_m = \max_s Re\, \xi_\chi^{m,2} / \max_s Re\, \xi_\chi^{1,2} \,, \tag{40}$$

is shown for different m numbers. The values of λ_m for $a/R = .275$ are simply the ratios between the maxima of the components $m \neq 1$ and the component $m = 1$ in Fig. 5. The second quantity, s_r, is the radial coordinate of the resonant surface. The last quantity, p, is the resistive power per unit length of the torus, defined by Eq. (31), delivered by the antenna.

From Fig. 6 it can be seen that λ_0 and λ_2 depend linearly on the inverse aspect ratio for $a/R < 0.2$. It is also observed that λ_3 is proportional to $(a/R)^2$ in this range. These findings are consistent with the simple picture of toroidal coupling. For $a/R > 0.2$, however, these dependencies become weaker, consistent with the fact that the resonant surface is pushed towards the centre for increasing values of a/R. The inverse aspect ratio of the resonant surface, $s_r a/R$, therefore does not grow linearly with a/R but has a weaker dependence. The quadratic dependence of the resistive power on the inverse aspect ratio appears to be related to the change in the position of the resonant surface. We can, in fact, recover approximately the value of p obtained for the cylindrical equilibrium if the pump frequency ω is increased in such a manner that s_r regains its value obtained in the cylindrical model.

We now turn our attention to a complicated but by no means unusual mode structure which has been calculated for a Solov'ev equilibrium with a/R = .333 and circular cross-section. The minor radii of the antenna and the shell are given by α_a = 1.1 and α_s = 1.2, respectively. The antenna current is of single helicity m = 1, n = 6.

The appearance of a large number of resonant surfaces, due to the plasma toroidicity, can be understood from an inspection of the Alfven spectrum in the cylindrical limit. This limit is obtained from Eq. (35) when the cylindrical radial coordinate ρ is replaced by s, and B_{0z}, $B_{0\theta}$ and k are replaced by B_T, $\langle B_{0\chi}\rangle$ and n/R, respectively. We then have

$$\omega_A(s) = \left(B_T / R\, \varrho_0^{1/2}\right)\left(n + m/q(s)\right) . \qquad (41)$$

Equation (41) takes the particularly simple form, $\omega_A R/a = n + m$, on the axis, since in our units $B_T(0) = \rho_\theta(0) = q(0) = a = 1$.

In Fig. 7 the Alfven frequencies, defined by Eq. (41), are plotted for m numbers ranging from 2 down to -5. The pump frequency, ω = 7.4 a/R, is indicated by a horizontal line. The points of intersection with the Alfven frequencies indicate that at least seven Alfven modes of different helicity can, in principle, be resonant with the pump. To determine whether they are coupled to the pump or not, the dominant Fourier components, $|\xi_\chi{}^{m,6}|$, of the actual plasma response have been calculated. A plot of these components shows that all the modes are coupled to the pump, which raises the question of whether or not the heating scheme is spoiled by their presence. A partial answer to this question is obtained from the plot of the resistive part of the energy flux, S. At least for this equilibrium, and with the excitation parameters chosen, the energy flux curve strongly resembles that which is obtained using cylindrical geometry. The heating scheme is therefore not spoiled by the influence of toroidicity.

Some additional remarks concerning Fig. 7 are worth noting. As we have seen in Fig. 5, the dominant Fourier component is accompanied by components of other m numbers. In the same way all the dominant components in Fig. 7 are accompanied by other components. For this reason each mode $m \neq 1$ has a certain content of the $m = 1$ mode, and is therefore accessible to the antenna which imposes $m = 1$. Figure 7 shows that all the resonant surfaces are shifted towards the centre with respect to their position in the cylindrical model. The most striking toroidal effect, however, is the fact that the resonant surfaces corresponding to $m = 0$ through $m = -5$ in the cylindrical model, are not dominated by their intrinsic angular dependence but by a wave number decreased by one, i.e. $m = -1$ through -6. Finally, we note from Fig. 7 that if, for a particular equilibrium, surfaces with higher m numbers should pose a problem their number may be limited by choosing an antenna with smaller values of n and m. This last remark is especially valid for an antenna intended to be used in a diagnostic application for the measurement of the poloidal magnetic field and the safety factor [29].

In studies of Alfven wave heating using a cylindrical model [12,28], it has been found that most efficient coupling to surfaces deep in the plasma can be achieved when a surface quasi-mode (kink-like mode) is excited. It remains to be determined if this is also true for toroidal geometry. Some preliminary results which show that this is indeed the case have been given elsewhere [19]. A much clearer indication is given by the analysis of the results presented in Fig. 8.

In Fig. 8, contours of the absorbed power, p, and Q factor in the plane of varying n and ω are presented. The plots on the left side were calculated for the Solov'ev equilibrium used for Fig. 2. The same minor radii of antenna and shell are used. The poloidal wavenumber of the antenna is m = 1. The plots on the right side have been obtained with a cylindrical model having the same magnetic field pitch. The wavenumber k in the cylindrical model is given by k = n/R. For each value of n, or k, the pump frequency has been scaled by the corresponding Alfven frequency on the axis, $\omega_A(0) = (n + m)a/R$. The choice of the frequency coordinate $(\omega - \omega_A(0))/\omega_A(0)$ has the advantage that excitation parameters ω and k leading to the same resonant surface, ρ_s, in cylindrical geometry lie on approximately straight vertical lines. Moreover, these lines are approximately equidistant in $\log(\omega-\omega_A(0))/\omega_A(0)$ for equidistant resonant surfaces. The frequency coordinate, therefore, can be directly interpreted as the radial coordinate of the excited resonant surface. This behaviour remains more or less true even for the dominantly excited resonant surface in the toroidal case.

The most striking feature of Fig. 8 is the fact that there is no qualitative difference between the two models. The broad maximum of p, which we interpret as the resonance with the surface quasi-mode, has approximately the same position in the (n,ω)-plane and is oriented in the same direction. There is , however, a 50% discrepancy in height. This difference manifests itself also in the Q values which, on the average, are a factor 2 greater for the torus compared to the cylinder.

To investigate this discrepancy, an attempt was made to simulate the torus more accurately by other cylindrical models. It was found that the currentless cylinder gives the height of the resonance exactly, but fails to predict correctly the coordinates n and ω of the maximum. In other words, it was found that in a family of Solov'ev equilibria, the maxima of $\bar{p}$ is independent of a/R, if $q_0 = 1$ for each member of the family. In this case the limit $a/R \to 0$ leads to the currentless cylinder. The constancy of $\bar{p}_{max}$ has been verified in the range $0.15 < a/R < 0.333$; a variation of at most 10% has been found. The failure to predict the value of ω, or alternatively ρ_s, is understandable in view of the results of our cylindrical studies [28] which showed that the curvature of the magnetic field lines in current -carrying cylindrical plasmas has a strong influence on the position of the resonance with the surface quasi-mode.

The results of this section appear to indicate that toroidal effects do not play an important role in Alfven wave heating. This is, however, a premature conclusion. It should be remembered that idealized antennae of single helicity have been used for all the calculations presented in this section, and that experimentally it is difficult, if not impossible, to construct such antennae. A problem related to antenna design has been encountered in the application of our code to TCA, the discussion of which is given in Section 5.1.

4.2 Ellipticity

By using the Solov'ev equilibrium, Eq. (A1), for different ellipticity parameters, ε, we can perform a systematic study of the influence of ellipticity on Alfven wave heating. There are two main differencies between toroidal and elliptic coupling. The elliptic geometry implies that the equilibrium quantities vary according to $1 - \varepsilon\cos 2\chi$ in the poloidal direction. In contrast to toroidal coupling, ellipticity therefore couples modes $\propto \exp[i(m\pm 2)\chi]$ to the basic mode, $\exp(im\chi)$. An even more important difference lies in the magnitude of the "small" parameter. Whereas a/R in the toroidal case is seldom greater than 0.4, plasmas with an elongation of 2, i.e. $\varepsilon = 1$, can easily be envisaged.

In order to separate elliptical and toroidal effects, a large aspect ratio torus has been considered for our study, a/R = 0.0055, together with a (m = 1, n = 100) excitation. A value for the safety factor on axis of 0.02 has been chosen yielding $nq_0 = 2$ as was the case for the investigation of toroidal effects, the results of which were presented in Fig. 6. The antenna and the shell are also elliptic with the same ellipticity as the plasma. Their relative dimensions are given by $\alpha_a = 1.6$ and $\alpha_s = 2.0$.

The average values of the Fourier components of the plasma response, defined by

$$\langle |\xi_\chi^{m,n}| \rangle \equiv \int_0^1 |\xi_\chi^{m,n}| \, ds, \tag{42}$$

have been calculated for this configuration. In Fig. 9 are plotted the ratios, $x^{m,n}$, defined by

$$x^{m,n} \equiv \langle |\xi_\chi^{m,n}| \rangle / \langle |\xi_\chi^{1,n}| \rangle . \tag{43}$$

The introduction of the average, Eq. (42), is necessary since it was not possible to distinguish any simple dependency of the local $\xi_\chi^{m,n}(s)$ when the ellipticity was varied within reasonable bounds, say $0<\varepsilon<.6$. The ellipticity appears to be very efficient mode-coupling mechanism as can be seen from Fig. 9. These results also show that the coupling is indeed from m to m ± 2.

Due to this efficient coupling, ellipticity may have a detrimental effect on the heating scheme. In Fig. 10, the radial profile of the resistive energy flux, S, is plotted for different values of ε. This graph shows evidence of an edge heating problem. We see that for a modest ellipticity of $\varepsilon = 0.25$, 7% of the energy is deposited in the immediate neighbourhood of the plasma edge. For an ellipticity of the order of 0.5, which is approximately that of JET, 25% of the energy is deposited near the edge. For even higher ellipticities, more than 50% of the energy goes to the plasma edge.

There is a rather clear conclusion to this study concerning the influence of elongation on Alfven wave heating: circular plasmas are the best. Plasmas of JET-like shapes are marginal. Detailed optimization studies for such plasmas are necessary.

In this section, toroidicity has been studied in detail and ellipticity to some extent. The effects due to triangularity, rectangularity, etc. could also be investigated. All these more complex geometrical features would certainly involve new types of coupling. We have, however, already seen that for the case of ellipticity it was difficult to distinguish clear features. It is therefore more appropriate to consider now particular applications of the heating scheme rather than to discuss in general more complex geometries.

5. APPLICATIONS

5.1 TCA (Tokamak Chauffage Alfven, Lausanne)

To model this particular tokamak, we consider a numerical equilibrium which is characterized by a fairly circular, slightly triangular cross-section, as in Fig. 3, with an inverse aspect ratio of $a/R = .29$. The safety factor assumes the value of 1.16 on the axis and increases to 3.36 on the boundary, with $\beta = 5 \times 10^{-3}$ and $\beta_{pol} = 0.48$. A double-helical antenna with $n = 2$ and $m = \pm 1$ is considered. The antenna and shell radii are given by $\alpha_a = 1.18$ and $\alpha_s = 1.59$, respectively.

The mode structures presented in Fig. 3 and 4 have been obtained for this equilibrium with a pump frequency of $\omega = .94$. The innermost surface ($s = .5$) which is strongly excited is the ($n = 2$, $m= 1$) surface. The next surface ($s = .77$) is that corresponding to ($n = 2$, $m = 0$) in the cylindrical limit. However, as has been concluded from Fig. 7, the resonant surface does not exhibit its intrinsic angular dependence, but rather a dependence $m = -1$. Since it is therefore directly excited by the antenna, the question arises as to why its excitation is so poor. Near to the plasma edge, surfaces with negative m-numbers are excited. Due to this excitation, substantial edge heating is expected.

In Fig. 11, we demonstrate that the edge heating is connected with the excitation of the (n = 2, m = -1) mode. Shown in this figure are the radial profiles of the resistive energy flux resulting from four different antenna structures, S_{+1}, S_{-1}, $S_{\pm 1}{}^{io}$ and $S_{\pm 1}{}^{tb}$. Also shown is the sum of the first two fluxes, S_{+1} + S_{-1}. The four structures are the following: S_{+1} is the flux obtained with an antenna of single (n = 2, m = 1) helicity, S_{-1} with an antenna of single (n = 2, m =-1) helicity. $S_{\pm 1}{}^{io}$ is obtained with two superposed helices which are phased in such a manner that the maximum poloidal currents lie inside and outside of the torus (I^s = 2, I^a = 0 in Eq. (6)), i.e. the plasma is squeezed between the inside and the outside of the torus. Finally $S_{\pm 1}{}^{tb}$ is obtained with two helices (I^s = 0, I^a = 2) phased in such a manner that the plasma is squeezed between the top and the bottom of the torus. Without toroidal effects these two double-helices should excite the plasma in exactly the same way and the resulting flux should be just the sum of the two fluxes S_{-1} and S_{+1}. Fig. 11 shows, however, that this is not the case. We see that the flux $S_{\pm 1}{}^{io}$ is dominantly absorbed on the (n = 2, m = 0) surface, whereas $S_{\pm 1}{}^{tb}$ is dominantly absorbed on the (n = 2, m = 1) surface. The heating near the plasma edge is, however, present for both these antenna configurations. As can be seen from the plot of S_{-1}, the edge heating is clearly connected with the excitation of the (n = 2, m =-1) mode.

This finding seems to contradict our previous conclusion that toroidal effects do not influence Alfven wave heating significantly. It should be noted, however, that an antenna of single helicity as considered in Section 4.1 excites basically one mode. Since we aim at placing the dominant resonant surface as near as possible to the axis, the inverse aspect ratio of this surface is small and hence toroidal effects are small. In contrast an antenna of double helicity excites in general two modes, one of which is inevitably located somewhat nearer to the edge where toroidal effects are more important. We therefore conclude from the above discussion that an optimal antenna for TCA would be one of single helicity.

5.2 JET

For the study concerning JET, a numerical equilibrium with the following characteristics is considered: inverse aspect ratio 0.4, ellipticity 0.68, slight triangularitiy (see Fig. 12), q_0 = 1.37 on the axis and q = 6 on the boundary, β = 1% and β_{pol} = .47. Antenna and shell dimensions are given by α_a = 1.1 and α_s = 1.2, respectively. The antenna currents are of single helicity m = 1, $1 \leq n \leq 10$.

The plasma response for an n = 4 excitation is shown in Fig. 12. One can distinguish the innermost surface as that corresponding to the mode excited primarily by the antenna, namely n = 4, m = 1. Further out there exists a multitude of modes, as was also observed in Fig. 7. Under the influence of ellipticity and triangularity, however, they have lost their unique character. Unlike the situation in Fig. 7, it is therefore impossible for the present example to provide a clear classification of these modes.

Due to the fine structure exhibited in Fig. 12, people with some numerical experience may suggest that the mesh used for this calculation (60x25) is too coarse to describe correctly the depicted plasma response. Check runs with different artificial damping rates, ν, have indeed revealed the limitations of the mesh. By varying the mesh size up to (80x29), as well as the distribution of mesh points, we estimate that at least three times more radial points would be needed to obtain reliable results. On the other hand, we gained the impression that the code could nevertheless be used for a correct assessment of Alfven wave heating on JET.

Forty computer runs were made in order to obtain information for a contour plot as was constructed for Fig. 8. Knowing the energy deposition profile, D(s) defined by Eq. (34), the mean "radius" $\langle s \rangle$ at which the energy is absorbed was calculated, i.e.

$$\langle s \rangle = \int D s \, ds \Big/ \int D \, ds \, . \tag{44}$$

In all the runs, $\langle s \rangle$ was never smaller than 0.47, the best penetration being obtained with an (n = 2) antenna. For this run, the basic (n = 2, m = 1) surface appeared at s = .23. The fact that $\langle s \rangle$ = .47 demonstrates that there is substantial heating near the edge even for this case. Overall, the results are rather disappointing. They can, however, easily be explained by the coupling of the antenna to modes near the surface due to the ellipticity and the triangularity of the JET equilibrium. It seems therefore to be difficult to heat JET with Alfven waves. If, however, for some reason the cross-section of JET would become more circular, Alfven wave heating should be reconsidered.

6. CONCLUSIONS

In the framework of ideal MHD, a theory of Alfven wave heating has been formulated which closely parallels the δW-formulation of ideal MHD stability theory. In this way, the toroidal stability codes, which have involved a tremendous effort over the last decade, can be simply modified for the purpose of Alfven wave heating studies. The main modification consists of the introduction of a driving antenna in the vacuum adjacent to the plasma. We have shown that in ideal MHD the problem of Alfven wave heating can be reduced to the problem of solving a set of linear algebric equations for the discretized plasma displacement, Eq. (29).

The unavoidable limitation in grid size set by finite computer memory has led us to discuss the numerical scheme in some detail. To obtain confidence in the numerical results, we have undertaken an investigation of the properties of the discretized continuous spectra which play an essential role in our calculations. Physical arguments have been advanced to explain these properties.

The results of a systematic study of toroidal effects on Alfven wave heating have then been presented. We have found that although the mode structures may become very complicated in a toroidal plasma, the power emitted by an antenna of single helicity may still be absorbed in the interior of the plasma. This is contrary to some pessimistic conjectures which have been made in the past. The best heating efficiency is obtained when the pump excites resonantly a surface quasi-mode (kink-like mode), a feature of Alfven wave heating which is well known from slab and cylindrical models. It has been shown that, even quantitatively, quite correct results can be obtained from a cylindrical model. In an application to the Lausanne tokamak,TCA, where an antenna of double helicity is used, a toroidal effect on the power deposition profile, namely a certain amount of edge heating, has been found. This phenomenon has been explained by toroidal coupling of the second antenna-excited mode to modes near the plasma edge. It has been concluded that antennae of single helicity should be used for Alfven wave heating.

A short investigation of the influence of ellipticity has yielded the clear result that elliptic coupling of the basically excited mode to modes near the plasma edge is important and has disastrous consequences. We have found that an ellipticity of 0.5 is sufficient to couple 25% of the energy to regions near the plasma edge. This result has been confirmed in an application of our code to JET using a numerical equilibrium of elongated cross-section. It was found to be impossible to deposit the bulk of the energy in the inner part of the plasma cross-section.

The present investigation, together with the corresponding studies for cylindrical geometry [12,28], provide us with a clear picture of the geometrical effects on Alfven wave heating in tokamaks. It is therefore, in our opinion, reasonable to base the next step of the investigation, the kinetic theory [17,30,31], on cylindrical rather than on toroidal geometry.

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APPENDIX A Equilibria, Antennae and ERATO coordinates

For the studies of the fundamental dependencies on toroidicity and elongation we use the simple axisymmetric equilibrium of Solov'ev [23,24] which can be defined analytically by the poloidal flux function

$$\Psi(r,z) = \frac{(1+\varepsilon)\,B_{T0}}{2q_0 R^2}\left[\frac{r^2 z^2}{(1+\varepsilon)^2} + \frac{(r^2-R^2)^2}{4}\right]. \qquad \text{(A1)}$$

Here r and z denote the radial coordinate measured form the main axis of the torus and the vertical coordinate measured from the midplane, respectively. B_{T0} is the value of the toroidal magnetic field on the magnetic axis, q_0 is the safety factor on the axis, R is the radial position of the axis, a is a length characterizing the plasma width and ε is a dimensionless parameter which measures the ellipticity of the cross-section.

The main characteristic features of the equilibrium are the profiles of the toroidal magnetic field and of the toroidal current, which are as $B_T \propto 1/r$ and $j_T \propto r$ respectively. Further details can be found elsewhere [21,24].

In the numerical calculations it is convenient to use dimensionless quantities. The units of length, time and magnetic field are chosen to be a, a/c_{A0} and B_{T0} respectively; the Alfven velocity, c_{A0}, is the value on the magnetic axis. Using these units, Solov'ev's equilibrium is determined by 3 parameters: inverse aspect ratio, a/R, ellipticity, ε, and safety factor, q_0.

The flat and rather unrealistic current profiles inherent in

Solo'ev's equilibrium necessitates the use of a numerical equilibrium code [25] for studies concerning specific machines such as TCA and JET. In this case we measure a/R in the plane which includes by the magnetic axis.

The mass density, normalized to the density on the axis, has the profile

$$\rho_0 = 1 - 0.9\, \Psi / \Psi_{surface} \,. \tag{A2}$$

In the present work, density profiles different from Eq. (A2) have not been considered since their influence on Alfven wave heating may be adequately investigated using cylindrical models.

For the definition of the antenna and the conducting shell geometry, a coordinate system (ρ,θ,ϕ) is used, where ρ denotes the distance from the magnetic axis, θ is the poloidal angle and ϕ is the toroidal angle (see Fig. 1). In this coordinate system a general antenna geometry would be defined by $A(\rho,\theta)$ = constant. We make, however, a special choice for the antenna as well as for the shell. We define the antenna and the shell geometry in relation to the plasma surface, $\rho_p(\theta)$;

$$\begin{aligned} \text{antenna:} \quad & 0 = \rho - \alpha_a\, \rho_p(\theta) \equiv A(\rho,\theta)\,, \\ \text{shell:} \quad & 0 = \rho - \alpha_s\, \rho_p(\theta)\,. \end{aligned} \tag{A3}$$

The quantities α_a, α_s are input parameters subject to the requirement that $1 < \alpha_a < \alpha_s$.

Having chosen the antenna geometry we may now prescribe the antenna current by defining the potential β. The choice of β is not entirely free since the axisymmetric nature of the problem has been used in the ERATO code. The most general form for β, compatible with ERATO is

$$\beta(\rho,\theta,\phi) = b_s(\rho,\theta)\cos n\phi + b_a(\rho,\theta)\sin n\phi \tag{A4}$$

where $b_s(\rho,\theta)$ and $b_a(\rho,\theta)$ are a symmetric and an antisymmetric function of θ, respectively. We do not, however, use the freedom offered by Eq. (A4), but restrict both b_s and b_a to be independent of ρ and harmonic functions of θ:

$$\beta_0(\theta,\phi) = -\frac{a B_{T0}}{2}\left[I^s \cos m\theta \cos n\phi + I^a \sin m\theta \sin n\phi\right]. \tag{A5}$$

In order to understand the physical significance of I^s and I^a we first write down the toroidal component of the surface current density, $\nabla A \times \nabla \beta_0$, defined by Eq. (1).

$$J_\phi = \frac{m}{\rho}\,\frac{\partial A/\partial \rho}{|\nabla A|}\,\frac{a B_{T0}}{4}\left[(I^s - I^a)\sin(m\theta + n\phi) + (I^s + I^a)\sin(m\theta - n\phi)\right]. \tag{A6}$$

Here we interpret J_ϕ as being harmonically distributed surface currents flowing in two sets of helical conducting sheets. The current per sheet or "wire", I, may we obtained from an integral the short way around the torus:

$$I = \frac{1}{2m}\oint |J_\phi|\, d\ell \,. \tag{A7}$$

With

$$d\ell = |\nabla A| \left(\frac{\partial A}{\partial \rho}\right)^{-1} \rho \, d\theta, \tag{A8}$$

one easily finds that

$$I_{\pm} = \frac{a B_{T0}}{2} \left(I^{s} \mp I^{a} \right). \tag{A9}$$

In the dimensionless units used for the numerical calculation, aB_{T0} is just the unit of current. We will use either $I^s = I^a = 1$ which corresponds to a unit current per wire in a (m,n)-helical antenna or we use $I^s = 0$, $I^a = 2$ corresponding to the superposition of an (m,n)- and a (-m,n)-helical antenna with the same unit current per wire.

In ERATO [21], the plasma equation of motion, Eq. (13), is not written in the orthogonal coordinate system (ρ,θ,ϕ) which has been used above in the definition of the antenna. For numerical and physical reasons a non-orthogonal coordinate system (s,χ,ϕ) has been chosen. The radial coordinate, s, is defined by

$$s = \left(\Psi / \Psi_{surface} \right)^{1/2} \tag{A10}$$

The coordinate s labels the magnetic surfaces and becomes the radial cylindrical coordinate ρ when the inverse aspect ratio a/R of the torus approaches zero.

The coordinate χ, corresponding to the poloidal angle, θ, is chosen in such a way that it engenders a simple form of the operator

$B_0 \cdot \nabla$, the gradient along the equilibrium field;

$$\vec{B}_0 \cdot \nabla \propto \partial/\partial\chi + q(s)\,\partial/\partial\phi\ , \qquad \text{(A11)}$$

where q(s) is the safety factor. The coordinate χ ranges from $-\pi$ to π, $\chi = 0$ being defined by the outward direction perpendicular to the axis of symmetry of the torus. In the large aspect ratio limit, χ also tends towards the corresponding cylindrical coordinate, θ.

The coordinate ϕ is the usual toroidal angle which is an ignorable coordinate for axisymmetric equilibria as considered in ERATO. ERATO treats only single toroidal Fourier modes, $\exp(in\phi)$.

Finally, it is assumed in ERATO that the equilibrium has an additional symmetry plane defined by the magnetic axis: the "up-down symmetry". Therefore, only the upper poloidal plane has to be treated numerically, the displacement vectors ξ obeying certain symmetry conditions across the symmetry plane.

APPENDIX B The vacuum field

The magnetic potential, Φ, defined by Eq. (14), obeys Laplace's equation,

$$\Delta \Phi = 0, \tag{B1}$$

which follows from Eq. (6). The matching and boundary conditions (8) through (11) written in terms of the potential are

$$\vec{n}_p \cdot \nabla \Phi(\vec{x}_p) = \left(\vec{B}_0 \cdot \nabla\right)\left(\vec{n}_p \cdot \vec{\xi}\right)\Big|_{\vec{x}_p} \equiv \Xi_p, \tag{B2}$$

$$[\![\Phi]\!]_a = \beta(\vec{x}_a) \equiv \beta_a, \tag{B3}$$

$$[\![\vec{n}_a \cdot \nabla \Phi]\!]_a = 0 \tag{B4}$$

and

$$\vec{n}_s \cdot \nabla \Phi(\vec{x}_s) = 0. \tag{B5}$$

Here Ξ_p is the "internal" source of Φ and β_a the "external" source.

We do not need the solution Φ throughout the entire vacuum region, but only certain relations between Φ and $n \cdot \nabla\Phi$ on the plasma boundary and on the antenna, as in Eq. (17) and (27). Harmonic functions can in fact be expressed in terms of their values on the boundary, δV, of a domain, V, in which they are regular [26]. In particular, for a point x lying on the boundary one can prove with the help of Green's formula that

$$\Phi(\vec{x}) = -\frac{1}{2\pi}\int_{\delta V} [\,\Phi(\vec{x}')\,\nabla' G(\vec{x},\vec{x}') - G(\vec{x},\vec{x}')\,\nabla'\Phi(\vec{x}')]\cdot d\vec{\sigma}' \qquad \text{(B6)}$$

where $d\vec{\sigma}'$ is the outer normal and

$$G(\vec{x},\vec{x}') = 1/|\vec{x}-\vec{x}'| \,. \qquad \text{(B7)}$$

On applying Eq. (B6) to the vacuum region V_I between the plasma and the antenna (see Fig. 1), one obtains an equation for $\Phi(x_p)$ on the plasma surface and a second equation for $\Phi(x_{a-})$ on the inside of the antenna. Likewise, on applying Eq. (B6) to region V_{II} one obtains $\Phi(x_{a+})$, the potential on the outside of the antenna, and $\Phi(x_s)$ on the shell.After elimination of $\Phi(x_{a+})$ in favour of $\Phi(x_a) \equiv \Phi(x_{a-})$ using Eq. (B3) and (B4), a system of 4 integral equations is obtained for the functions $\Phi_p \equiv \Phi(x_p)$, $\Phi_a \equiv \Phi(x_a)$, $\Xi_a \equiv n_a \cdot \nabla\Phi(x_a)$ and $\Phi_s \equiv \Phi(x_s)$:

$$(D_{pp}-2\hat{I})\Phi_p - (D_{pa}-2\hat{I})\Phi_a + E_{pa}\,\Xi_a = E_{pp}\,\Xi_p\,, \qquad \text{(B8)}$$

$$D_{ap}\Phi_p - D_{aa}\Phi_a + E_{aa}\,\Xi_a = E_{ap}\,\Xi_p\,, \qquad \text{(B9)}$$

$$(D_{aa}-2\hat{I})\Phi_a - E_{aa}\,\Xi_a - (D_{as}-2\hat{I})\Phi_s = (2\hat{I}-D_{aa})\beta_a\,, \qquad \text{(B10)}$$

$$D_{sa}\Phi_a - E_{sa}\,\Xi_a - D_{ss}\,\Phi_s = -D_{sa}\,\beta_a\,. \qquad \text{(B11)}$$

Here, $\hat{I}$ is the unit operator, and the integral operators $D_{\mu\nu}$ and $E_{\mu\nu}$, where μ = p,a,s and ν = p,a,s, are defined by

$$D_{\mu\nu}\,\Phi_\nu = \frac{1}{2\pi}\int_\nu \left[\Phi(\vec{x}_\nu{}') - \Phi(\vec{x}_\nu)\right] \nabla' G(\vec{x}_\mu, \vec{x}_\nu{}') \cdot d\vec{\sigma}_\nu{}' \quad \text{(B12)}$$

and

$$E_{\mu\nu}\,\Xi_\nu = \frac{1}{2\pi}\int_\nu G(\vec{x}_\mu, \vec{x}_\nu{}')\, \nabla'\Phi(\vec{x}_\nu{}') \cdot d\vec{\sigma}_\nu{}' . \quad \text{(B13)}$$

The term proportional to $\Phi(x_\nu)$ in (B12) has been introduced for convenience in the numerical treatment of the integral equations [22]. It may easily be evaluated using the relation

$$\int_\nu \nabla' G(\vec{x}_\mu, \vec{x}_\nu{}') \cdot d\vec{\sigma}_\nu{}' = \begin{cases} 0\,, & \mu \text{ outside } \nu \\ -2\pi, & \mu = \nu \\ -4\pi, & \mu \text{ inside } \nu \end{cases} \quad \text{(B14)}$$

which follows from Gauss' integral theorem [26].

Equations (B8) through (B11) may now be used to calculate Φ_p in terms of Ξ_p and β_a. By adding Eq. (B8) and (B10), and Eq. (B9) and (B11), two combined equations are obtained in which Φ_a and Ξ_a appear in expressions of the form

$$e_{pa} = (D_{aa} - D_{pa})\,\Phi_a - (E_{aa} - E_{pa})\,\Xi_a \quad \text{(B15)}$$

and

$$e_{sa} = (D_{sa} - D_{aa})\,\Phi_a - (E_{sa} - E_{aa})\,\Xi_a . \quad \text{(B16)}$$

Using Green's formula in V_{II} for e_{pa} and V_I for e_{sa}, one can express e_{pa} in terms of Φ_s and β_a and e_{sa} in terms of Φ_p and Ξ_p. Combining the obtained relations then yields

$$(D_{pp}-2\hat{I})\Phi_p-(D_{ps}-2\hat{I})\Phi_s = E_{pp}\Xi_p+(2\hat{I}-D_{pa})\beta_a\,, \qquad \text{(B17)}$$

$$D_{sp}\,\Phi_p - D_{ss}\,\Phi_s = E_{sp}\Xi_p - D_{sa}\,\beta_a\,. \qquad \text{(B18)}$$

Apart from the terms due to the external source, β_a, these integral equations are precisely what has been found for the stability problem [22].

It is easy to solve Eq. (B17) and (B18) numerically. We obtain

$$\Phi_p = Q_{pp}\,\Xi_p + \Phi_E(\vec{x}_p)\,, \qquad \text{(B19)}$$

where

$$Q_{pp} = M_{pp}^{-1}\left[E_{pp}-(D_{ps}-2\hat{I})D_{ss}^{-1}E_{sp}\right], \qquad \text{(B20)}$$

$$M_{pp} = D_{pp}-2\hat{I}-(D_{ps}-2\hat{I})\,D_{ss}^{-1}\,D_{sp} \qquad \text{(B21)}$$

and

$$\Phi_E(\vec{x}_p) = M_{pp}^{-1}\left[(D_{ps}-2\hat{I})D_{ss}^{-1}D_{sa}+2\hat{I}-D_{pa}\right]\beta_a\,. \qquad \text{(B22)}$$

Note that $M_{pp}^{-1}\,M_{pp} = \hat{I}$ and $D_{ss}^{-1}\,D_{ss} = \hat{I}$. Equation (B19), together with (B2), (B20), (B21) and (B22), constitutes the solution of the vacuum field on the boundary, Eq. (17), which is needed for the weak variational form.

On the other hand, for the power, defined by Eq. (26), Ξ_a is required. It is straighforward to eliminate Φ_a from Eq. (B8) and (B9) and to use then (B19), which yields the result

$$\Xi_a = T_{pa}^{-1} (V_{pp} - U_{pp} Q_{pp}) \Xi_p - T_{pa}^{-1} U_{pp} \Phi_E(\vec{x}_p). \qquad (B23)$$

This equation corresponds to Eq. (27) in the main text. Here

$$U_{pp} = D_{pp} - 2\hat{I} - (D_{pa} - 2\hat{I}) D_{aa}^{-1} D_{ap}, \qquad (B24)$$

$$T_{pa} = E_{pa} - (D_{pa} - 2\hat{I}) D_{aa}^{-1} E_{aa}, \qquad (B25)$$

and

$$V_{pp} = E_{pp} - (D_{pa} - 2\hat{I}) D_{aa}^{-1} E_{ap}. \qquad (B26)$$

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FIGURE CAPTIONS

Fig. 1 Toroidal plasma (ρ_p) surrounded by vacuum V_I, antenna (ρ_a), vacuum V_{II} an conducting shell (ρ_s).

Fig. 2 Absorbed power, $\bar{p}$, as a function of the artificial damping rate, ν, for three different grids (a = b,c,d) and two slightly different pump frequencies (a,b = c = d). The best result (d) is obtained with an irregular mesh.

Fig. 3 Plasma displacement in the poloidal plane due to the excitation of a TCA equilibrium by a bi-helical antenna for two different damping rates, ν.

Fig. 4 Resistive energy flux, S, associated with the plasma displacements depicted in Fig. 3, versus radial coordinate, s.

Fig. 5 Real part of the Fourier components, $\xi_\chi{}^{m,2}$, defined by Eq. (39), versus radial coordinate, s, in the vicinity of the resonant surface of the (n = 2, m = 1) mode dominantly excited in a TCA equilibrium.

Fig. 6 The dependence of three quantities on the inverse aspect ratio a/R, for a class of Solov'ev-equilibria. Shown are the relative amplitude of $m \neq 1$ components of λ_m, defined by Eq. (40), at the location, s_r, of the dominatly responding surface, m = 1, as well as the absorbed power, $\bar{p}$.

Fig. 7 Dominant Fourier components, $\xi_\chi^{m,6}$, of the poloidal displacement and the associated resistive energy flux, S, versus radial coordinate,s, in a fat torus of inverse aspect ratio 0.333 excited by a (n = 6, m = 1) antenna. The multitude of resonant surfaces is due to m ≠ 1 modes having an Alfven frequency ω_A, defined by Eq. (41), at the pump frequency ω, as shown by the circles and the broken lines.

Fig. 8 Contours of the absorbed power $\overline{p}$ (top) and Q-factor (bottom), in the plane of frequency, ω, and toroidal wavenumber, n (k = n/R), as obtained from toroidal (left) and cylindrical (right) models. The location of the resonant surfaces, ρ_s, in the cylinder is indicated by broken lines.

Fig. 9 Relative amplitude, $x^{m,n}$, defined by Eq. (43), of the m ≠ 1 modes in a large aspect ratio torus (a/R = .0055) as a function of the ellipticity, ε. The antenna excites dominantly the (n = 100, m = 1) mode.

Fig. 10 Energy flux, S, versus radial coordinate, s, in large aspect ratio tori (a/R = .0055) of different ellipticity, ε. All the fluxes for ε > 0.5 exhibit steep gradients near the plasma edge. For the sake of clarity, the fluxes calculated for ε = 0.5, 1.0 and 1.25 have not been plotted in the vicinity of the plasma edge.

Fig. 11 Energy flux, S_{-1}, S_{+1}, $S_{\pm1}{}^{io}$, and $S_{\pm1}{}^{tb}$ versus radial coordinate, s, obtained form 4 different antennae for a TCA equilibrium (see text for antenna arrangement). The flux shown in the centre diagram is the sum of S_{-1} and S_{+1}.

Fig. 12 Plasma displacement in the poloidal plane of a JET equilibrium due to excitation with an (n = 4, m = 1) antenna.