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ABSTRACT

Passive feedback stabilization of levitated coils is often considered in connection with the design of internal ring devices (Multipole or Surmac). A novel method using short circuited superconducting coils as stabilizers has been proposed. We present here a numerical calculation of the stability of one levitated coil in the field of two other levitating coils. The stability is provided by a set of superconducting short circuited loops placed around the floating ring. Stable configuration does exist. We also found that for a given magnetic configuration, there exists a minimum current in the levitated coil below which it is unstable.

I INTRODUCTION

In this work, we have studied numerically the stability of one levitated superconducting coil surrounded by short circuited superconducting loops. The novel idea {1} of using short circuited superconducting loops as feedback system of levitated coils presents many advantages if applied to the design of Multipole or Surmac (Surface Magnetic Confinement {2}) plasma devices. Let us just recall that Multipole/Surmac devices are internal ring confinement machines. The advantages of levitated superconducting Multipole/Surmac are the steady state character of the vacuum magnetic field configuration and the elimination of the plasma loss on mechanical support of the internal coils.

In the past three plasma devices with one single superconducting levitated ring have been built and operated successfully: they were the Princeton Plasma Physcis Laboratory FM-1 {3}, the Lawrence Livermore Laboratory (LLL) Levitron {4} and the Culham Laboratory Levitron {5}.

In these three devices, the magnetic field configuration destabilizes the floating coil both in the slide and the tilt displacements. In the three cases, active feedback control of the ring position was used. An optical system detects any deviation of the coil from its equilibrium position: the error signal was fed to a current amplifier which drives feedback coils. Restoring forces and torques, proportional to the displacement and the velocity, would then bring back the ring to its equilibrium position {6,7,8}. In case of complete loss of control of the levitated coil, the Princeton FM-1 machine relied on a mechanical catcher system {3}, whereas in the LLL-Levitron and the Culham-Levitron damping plates or a cage at low temperature were used to slow it down {4, 7, 9, 10, 11}.

Although File et al. {12} have successfully demonstrated the levitation and stabilization of two coils using active feedback, the use of this method does not seem practical for a high order Multipole/Surmac such as an Octopole (four levitated coils) or Dodecapole (six levitated coils). The coupling of the various degrees of freedom of the rings, the complexity of the optical detection used to monitor their position, and the required electronics would favor a passive feedback system. By passive feedback system, we mean a system in which no detection nor externally-controlled restoring forces are required.

It is well known that permanent magnets or superconducting coils can be levitated when surrounded by superconducting walls. As a back-

ground, let us recall that pulsed Multipole/Surmac devices in which the internal coils are stabilized by conducting walls have been built in the past. The four rings of the Wisconsin Octopole are indeed stabilized by current induced in the thick conducting walls. Wall stabilization is also used in the Hammel's Quadrupole and UCLA's Dodecapole to maintain the rings in a force free configuration. The damping plates of the LLL Levitron also provides stability of the ring but only for motion at frequency greater than 10 Hz. This frequency limit is due to the resistive decay of the eddy currents in the plate {4}. The Culham Levitron cage is also used to slow down the rate of ring fall by eddy currents {7}.

Transposing the concept of wall stabilization to a steady state Multipole/Surmac device is not practicable for the following reasons. First, surrounding the device by a superconducting wall would seriously limit the accessibility to the plasma. Moreover, aside from all the cryogenic problems associated with the cooling of the walls, Type II superconductors are not available in large sheets. To overcome these difficulties, it has been proposed to replace the superconducting walls by superconducting loops placed around the levitated coils {1}, {13}.

The present report describes a numerical study of one levitated coil in the magnetic field of two levitating coils. The paper is organized as follows. In section II, the coil arrangement will be described. We shall also give an intuitive description of the mechanism of the passive feedback. A normal mode analysis of the motion of the levi-

taded ring is then presented in Section III. Numerical results will be presented.

II THE MAGNETIC CONFIGURATION

We consider one levitated ring (No 1) in the field of two symmetrically located coils (No 2 and 3) (Fig. 1). The three coils, after being energized at the desired current, operate in the persistent mode. As passive feedback system, a set of twelve short circuited coils are positioned around coil 1 (cf. Fig. 1). These coils are called bumper coils and we thus have eight horizontal bumpers (four above and four below coil 1) and four vertical coils. All the physical parameters of the system (inner, outer radii, angular width, height and spacing of bumpers) are variable. For simplicity, we consider all the coils and bumpers as single turn. However, to be able to compute their self-inductance, we consider the radius of the superconducting wire as finite.

Let us describe now the cooldown process and the physical mechanism of the passive feedback system. First, the three coils 1, 2 and 3 are cooled down below their critical temperature T_c, energized and then switched to the persistent mode. During all this phase, the bumper coils are maintained above T_c. It is only after the required currents flow in coils 1, 2 and 3 that they become superconducting.

This procedure is essential to obtain the desired vacuum magnetic field configuration without field error, a condition which is absolutely necessary in the case of a Multipole/Surmac device. To understand this point, let us assume that the bumpers are cooled down below T_c simultaneously as the three other coils. Since they are short circuited coils, they trap the flux which links them at the moment where the normal to superconducting state transition occurs: this flux is equal to zero. When currents flow in coils 1 to 3, some flux linkage will be established and, as a consequence, currents will also flow in the bumper coils in order to keep the total flux across them equal to zero. These currents would then effect the vacuum field configuration created by coils 1, 2 and 3.

In the configuration of figure 1, the only unstable degrees of freedom of coil 1 are sliding displacements {6}. When coil 1 slides, the mutual inductance between it and the bumpers changes. As a result, current flows in the bumpers, which in turn creates a feedback magnetic field which counteracts the change of flux due to coil 1 displacement. The interaction of the current in coil 1 and the feedback field created by the horizontal bumper then produces a restoring force on coil 1.

From the physical picture of the origin of the restoring force, we see that two physical quantities are of importance: the mutual inductance between the levitated coil and the bumper self-inductance. The higher the rate of change of the mutual inductance for a displacement of the

levitated coil, the stronger the restoring force. Since the feedback magnetic field depends on the current flowing in the bumpers, it is advantageous to maximize this current for a given displacement of the levitated coil: this could be done by lowering the self-inductance of the bumper. The design presented in figure 1 is an attempt to maximize the flux linkage between the levitated coil and the bumpers, while preserving the symmetry and accessibility to the system. The stability of the levitated ring is then studied in function of the various bumper coil parameters.

III STABILITY OF THE LEVITATED COIL

The method {14} used to study the stability parallels a similar treatment by Fried {15} for the case of parallel wires.
The analysis is based on energy principle: if a stable equilibrium position exists, then it must correspond to a minimum energy state. When displaced from this minimum energy position, the levitated coil will oscillate. Due to dissipation, the oscillation will stop. It is easy to show that there exists an equilibrium position. If the current in coils 2 and 3 is identical and if we neglect gravity, then the equilibrium position for coil (1) is the midplane between the two levitating coils.

The energy of the system is given by

$$W = \frac{1}{2} \left[\Phi_i \; X_{ij} \; \Phi_j \right] \tag{1}$$

where Φ_i is the flux linking coil i. X_{ij} are the matrix elements of the inverse matrix $\underline{\underline{X}}$ of the mutual inductance matrix $\underline{\underline{M}}$:

$$\underline{\underline{M}}\,\underline{\underline{X}} = \underline{\underline{X}}\,\underline{\underline{M}} = \text{Identity matrix} \tag{2}$$

or

$$\sum_j X_{ij}\, M_{ik} = \sum_j M_{ij}\, X_{jk} = \delta_{ik} \tag{3}$$

Let us recall that M_{ij} can be computed for arbitrary shape coils by Neuman's formula

$$M_{ij} = \frac{\mu o}{4\pi} \oint_{C_i} \oint_{C_j} \frac{dl_i\, dl_j}{r} \tag{4}$$

The energy W is expressed in term of flux Φ_i instead of the current I_i since we are considering a system in which flux is conserved instead of current.

Defining the flux matrix $\underline{\Phi}$ as :

$$\underline{\Phi} = \begin{Bmatrix} \Phi_1 \\ \\ \Phi_n \end{Bmatrix} \tag{5}$$

we can rewrite equation (1) as a matricial product

$$W = \frac{1}{2}\left({}^t\underline{\Phi}\, \underline{\underline{X}}\, \underline{\Phi} \right) \tag{6}$$

For a given coils configuration, M_{1j}, and therefore W, is a function of the five variables describing the five degrees of freedom of the levitated coil 1. The displacement of the levitated coil from its equilibrium position is described by five variables x_α, three for the position of its center (α: 1, 2, 3) and the two Euler angles (α: 4, 5). Developing $\underline{\underline{X}}$ in Taylor serves of x_α we get an expression for W when coil 1 is displaced.

$$W = \frac{1}{2}\left[{}^t\underline{\Phi} \left(\underline{\underline{X}}\ (x_\alpha = 0) + \frac{1}{2} \left. \frac{\partial^2 \underline{\underline{X}}}{\partial x_\alpha \partial x_\beta} \right|_{x_\alpha = x_\beta = 0} x_\alpha x_\beta \right) \underline{\Phi} \right] \tag{7}$$

The force or torque F_α exerted on coil (1) is then given by

$$F_\alpha = -\frac{\partial W}{\partial x_\alpha}$$

$$F_\alpha = -\frac{1}{4}\left[{}^t\underline{\Phi} \cdot \left. \frac{\partial^2 \underline{\underline{X}}}{\partial x_\alpha \partial x_\beta} \right|_{x_\alpha = x_\beta = 0} \cdot \underline{\Phi}\, x_\beta \right] \tag{8}$$

Defining a matrix $\underline{\underline{A}}$ by

$$\underline{\underline{A}} = A_{\alpha\beta} = -\frac{1}{4}\left[{}^t\underline{\Phi} \cdot \left. \frac{\partial^2 \underline{\underline{X}}}{\partial x_\alpha \partial x_\beta} \right|_{x_\alpha = x_\beta = 0} \cdot \underline{\Phi} \right] \tag{9}$$

F_α can be written as

$$F_\alpha = A_{\alpha\beta} \; x_\beta \tag{10}$$

Equation (9) combines with Newton's law to yield

$$m_\alpha \frac{d^2 x_\alpha}{dt^2} = A_{\alpha\beta} \; x_\beta \tag{11}$$

where m_α is equal to coil (1) mass m (α: 1, 2, 3) or to its moment of inertia ($m/2R^2$) with respect to one diameter (α: 4, 5).

Decomposing the solution of equation (11) in Fourier component $\exp(i\omega t)$, we immediately see that $-\omega^2$ is proportional to an eigenvalue λ_α of $A_{\alpha\beta}$. If λ_α is negative, then the corresponding displacement x_α is stable. Conversely, λ_α positive means that the displacement x_α is unstable.

We have written a numerical code which first computes $\underline{\underline{A}}$ and then solves for its eigenvalue λ_α. The program computes M_{ij} directly from the Neumann's formula (4). The various derivative $\partial^2 \underline{\underline{X}}/\partial x_\alpha \partial x_\beta$ was computed from derivative of $\underline{\underline{M}}$ using the following relations

$$\frac{\partial^2 \underline{\underline{X}}}{\partial x_\beta \partial x_\alpha} = - \frac{\partial \underline{\underline{X}}}{\partial x_\beta} \frac{\partial \underline{\underline{M}}}{\partial x_\alpha} \underline{\underline{X}} - \underline{\underline{X}} \frac{\partial^2 \underline{\underline{M}}}{\partial x_\beta \partial x_\alpha} \underline{\underline{X}} - \underline{\underline{X}} \frac{\partial \underline{\underline{M}}}{\partial x_\alpha} \frac{\partial \underline{\underline{X}}}{\partial x_\beta} \tag{12}$$

$$\frac{\partial \underline{\underline{X}}}{\partial x_\alpha} = - \underline{\underline{X}} \; \frac{\partial \underline{\underline{M}}}{\partial x_\alpha} \; \underline{\underline{X}} \tag{13}$$

The calculation has been made for the configuration of figure 1. Typical size of coils 1, 2, 3, horizontal and vertical bumpers are shown in table 1. These values are comparable to those of an experimental device of our laboratory.

Since only the sliding displacements are unstable, the discussion will be concerned mainly with their behavior in the presence of stabilizing coils. Figure 2 shows the dependence of ω^2 corresponding to this mode as a function of the current I_1 in the levitated coil for a given stabilizing bumper coils configuration. For low value of I_1, the sliding motion is still unstable. However, above a certain critical value I_c, ω^2 becomes positive indicating that the sliding motion is indeed stabilized by the passive feedback. The interpretation is the following. The destabilizing force due to coils 2 and 3 is proportional to I_1 whereas the restoring force is proportional to I_1^2 since the induced field B_{FB} is proportional to I_1 and the restoring force F_{FB} to $I_1 \cdot B_{FB}$. This behavior contrasts with the situation where active feedback is used. In the latter case one can always ensure stability at any current I_1 provided enough current is passed through the feedback coil.

The critical current I_c is a measure of how efficient the passive feedback can be. The lower the value, the better it is. Let us now study the dependence of I_c with respect to the self-inductance of the bumper coils. Their self-inductance can be increased by decreasing the radius of the superconducting wire. Figure 3 shows the variation of the self-inductance M_{44} of the horizontal bumper and of the critical current in function of the wire radius. The higher the self-inductance, the higher the critical current I_c.

The dependence of the critical current with respect to the mutual inductance between the bumper coils and the levitated coils was also examined. Since the horizontal bumpers are the main one which provide stability with respect to sliding displacement, we shall mainly study the influence of varying the mutual inductance M_{14} between them and coil 1. M_{14} can be changed by changing either the relative separation or the bumper size. Reducing their relative separation increases M_{14} since this brings the bumpers closer to the levitating coil 1. As a net effect, the critical current I_c also decreases (Fig. 4).

The behavior discussed above and illustrated by figures 3 and 4 can be understood if one computes the current flowing in one horizontal bumper coil when coil 1 is displaced from its equilibrium. Due to flux conservation, we have

$$0 = I_1 \Delta M_{14} + I_4 M_{44} \tag{14}$$

or

$$I_4 = - \frac{I_1 \Delta M_{14}}{M_{44}} \tag{15}$$

In equations (13) and (14), I_1 and I_4 are the currents of coil 1 and of one of the bumper coils, ΔM_{14} the change in the mutual inductance M_{14}. The magnitude of ΔM_{14} depends on the coupling between coil one and one horizontal bumper, and therefore is proportional to M_{14}. According to equation (14) I_4, and therefore the restoring magnetic field B_{FB} is proportional to M_{14} and inversely proportional to M_{44}.

Equation (14) also indicates that increasing the surface enclosed by the horizontal bumper coils does not necessarily leads to an improvement in the stability. It is true that one does increase M_{14}, which has a beneficial effect. However, it also increases M_{44}.

An optimum depending on the configuration can thus be found (see Table II).

IV CONCLUSION

We have shown that one levitated ring in a simple magnetic configuration can be stabilized. The present study can also provide a useful method in design of levitated superconducting Multipole/Surmac. In such a device, the stabilizing coils must be outside the last stable flux surface Ψ_c. One can therefore use the same method as the one presented in this report to check whether the bumper coils located outside Ψ_c can stabilize the levitated coils. We would like to point out, however, that this problem is far more complicated than the case we have discussed. Even in the simplest configuration, a single levitated coil, plasma stability requires a toroidal field. Such an additional field destabilizes other degrees of freedom and was not included in our discussion.

This theoretical study has been supported by experimental evidence. Using high frequency to simulate the superconducting property of flux trapping, Haflinger and Wuerker {16} of TRW did also show that in the coil configuration of figure 1, there is a minimum energy position for the levitated coil and therefore a stable position. More recently experimental evidence using superconducting loops and bumpers {17} has been obtained by the UCLA Plasma Physics Group {18}.

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FIGURE CAPTIONS

Figure 1: Top view and cut view of the coil configuration. Coil 1 is the levitated coil and coils 2 and 3 are the levitating coils. Two sets of horizontal bumper coils are placed above and below the levitated coil. Their shape is better described in the top view. The vertical bumper coil consists of two arcs of the same radius connected by two vertical conductors.

Figure 2: Variation of the square of the frequency with respect to the current in the levitated coil. The current in the levitating coils is 5000 A. The important fact to be noticed is that stability ($\omega^2 > 0$) is only insured for $I > I_c$. The value of I_c depends on various parameters of the coil configuration.

Figure 3: Variation of I_c as a function of the self-inductance M_{44} of the horizontal bumper coils. M_{44} is varied by changing the radius r of the conductor as shown in the figure. As expected, ω^2 is increased with increasing self-inductance.

Figure 4: Variation of I_c as a function of the mutual inductance M_{14} between the horizontal bumper coil and the levitated coil. M_{14} is varied by changing the separation Δz between the upper and lower horizontal bumper coils. Their size is kept constant. The closer to the levitated coil, the better the stability of the system.

Levitated coil radius	=	10 cm
Levitated coil radius	=	8 cm
Mass of the levitated coil	=	1 kg
Inner radius of horizontal bumper	=	7 cm - 9 cm
Outer radius of horizontal bumper	=	11 cm - 13 cm
Vertical bumper radius	=	7 cm
Spacing between levitating coils	=	10 cm
Spacing between horizontal bumpers	=	5 cm - 9 cm
Vertical bumper height	=	4 cm
Conductor radius	=	.39 cm
Angular opening of bumper coil	=	1 radian
Current in levitating coils	=	5000 A

Table I: Typical numerical values used in the numerical calculations.

Inner radius {cm}	Outer radius {cm}	M_{14} {H}	M_{44} {H}	I_c {A}
11	9	$4.61\ 10^{-9}$	$6.09\ 10^{-8}$	8361
12	8	$8.74\ 10^{-9}$	$9.4\ \ 10^{-8}$	6472
13	9	$1.17\ 10^{-8}$	$1.22\ 10^{-7}$	8162

Table II: Variation of the mutual inductance M_{14}, self-inductance M_{44} and the criteria of current I_c in function of the size of the horizontal bumper coils. Their spacing is fixed to 5 cm. The calculations show that there exists an optimum in the size of the bumper coils.

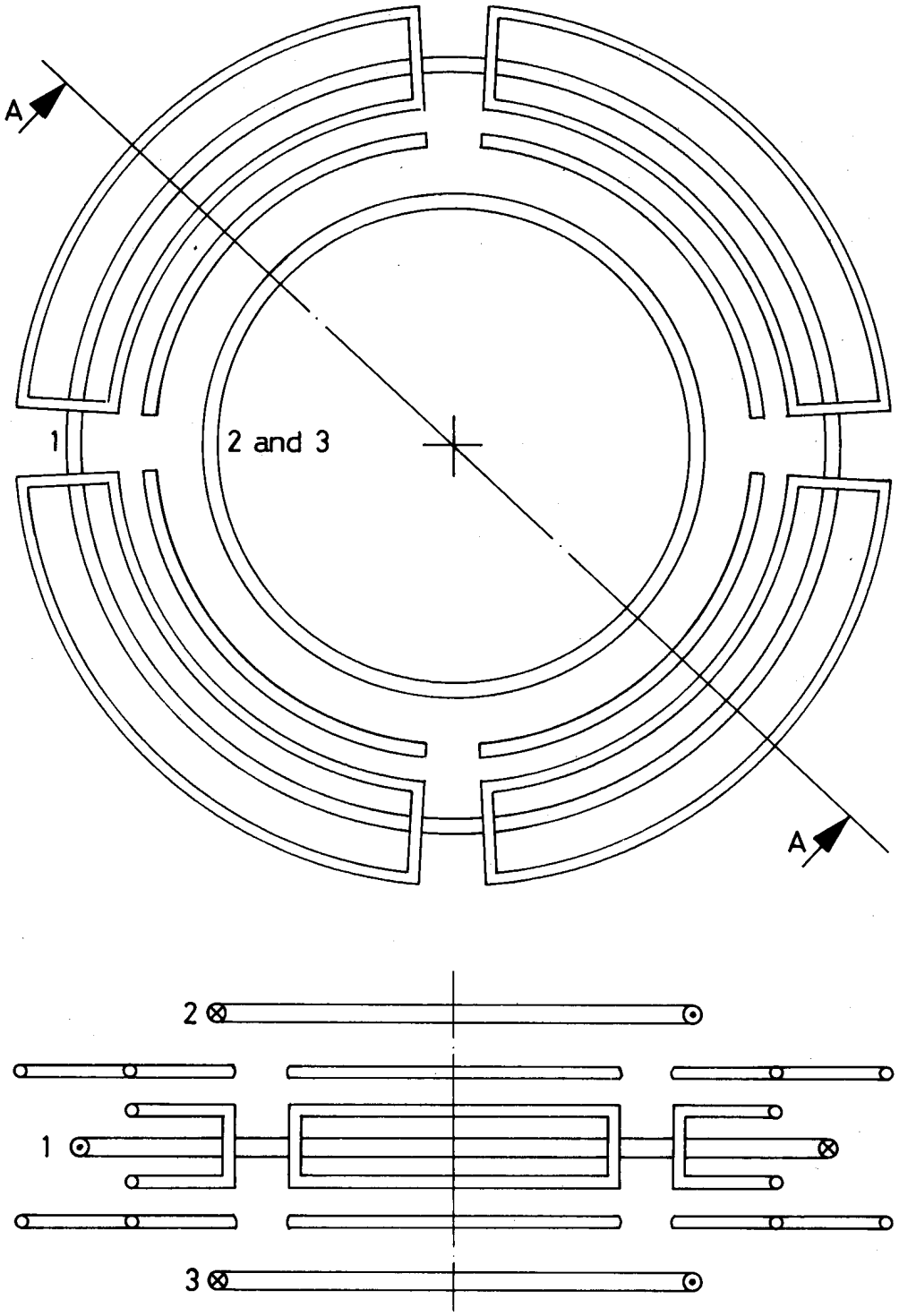

FIG. 1

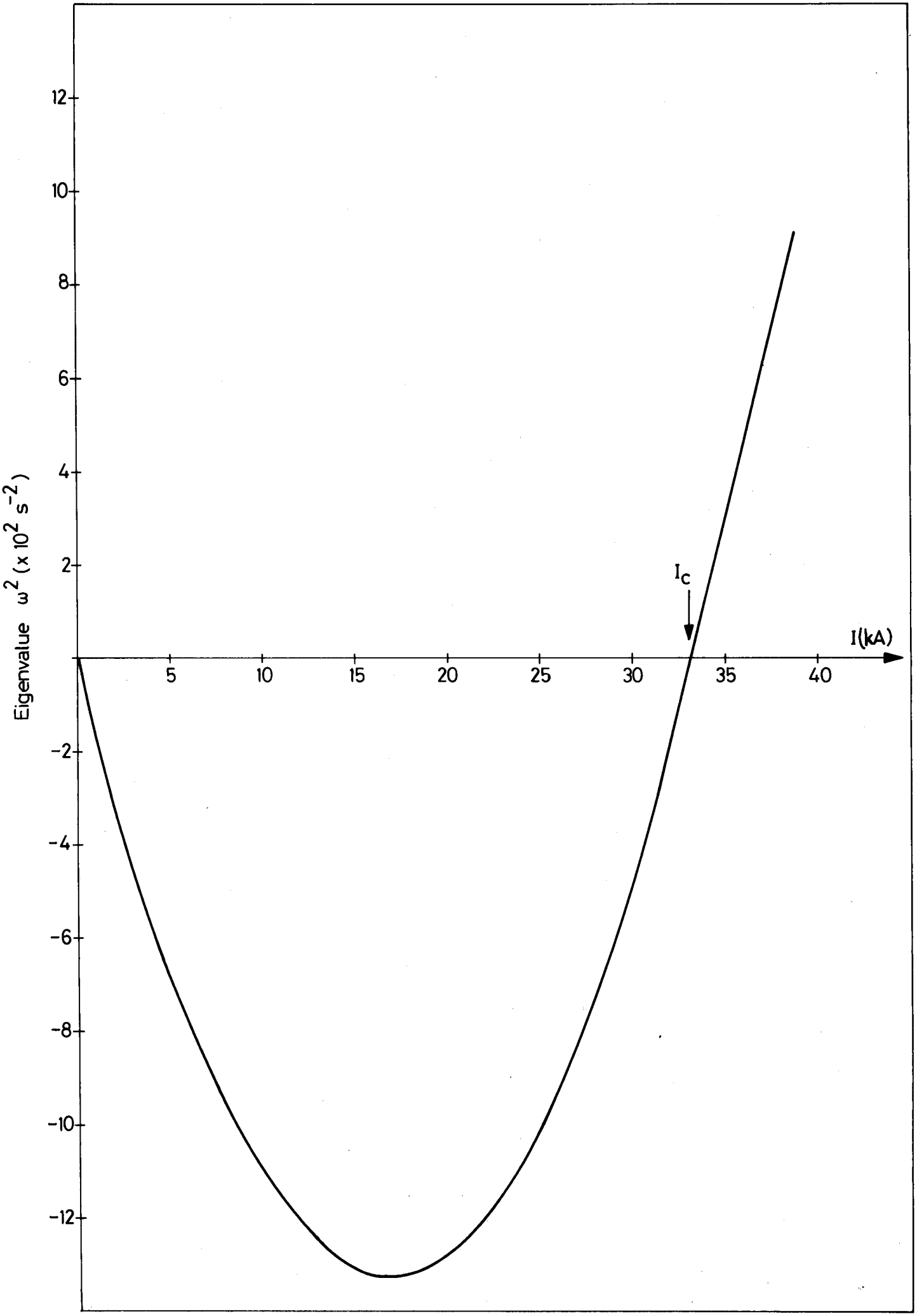
Eigenvalue ω^2 (x 10^2 s^{-2})
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
5
10
15
20
25
30
35
40
I(kA)
I_c

FIG. 2

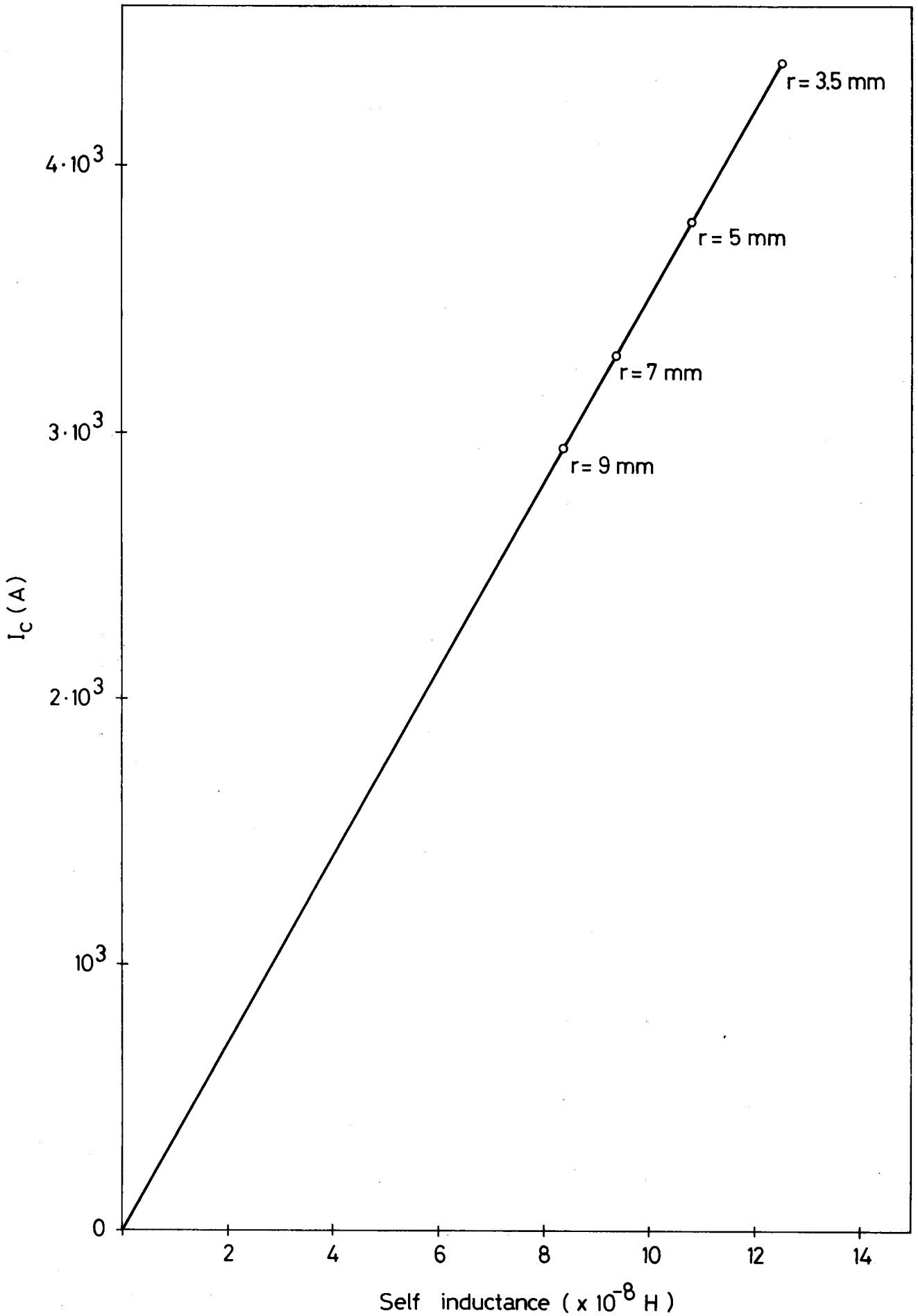
r = 3.5 mm
r = 5 mm
r = 7 mm
r = 9 mm
4·10^3
3·10^3
2·10^3
10^3
0
I_c (A)
2
4
6
8
10
12
14
Self inductance (x 10^-8 H)

FIG. 3

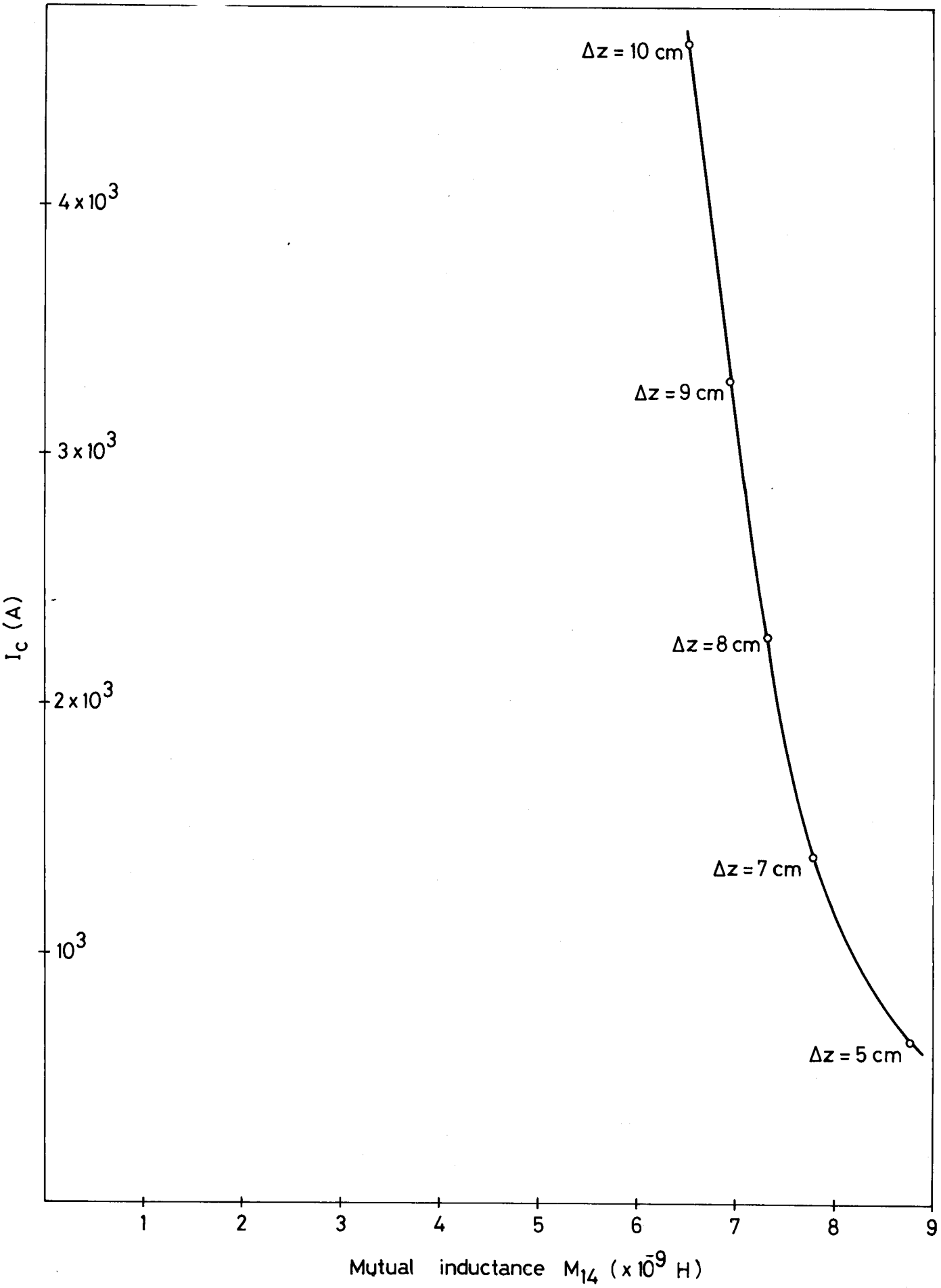
Δz = 10 cm
Δz = 9 cm
Δz = 8 cm
Δz = 7 cm
Δz = 5 cm
4 x 10^3
3 x 10^3
2 x 10^3
10^3
I_c (A)
1
2
3
4
5
6
7
8
9
Mutual inductance M_14 (x 10^-9 H)

FIG. 4