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ABSTRACT

The effects of plasma waves on the runaway production rate is studied. For a wave packet with phase velocities along the electric field containing the critical velocity for runaway, the runaway production rate is found to be enhanced by many orders of magnitude in the presence of plasma waves.

FIGURE CAPTION

Figure 1. Runaway rates $A/n\nu$ versus D_0 , the ratio of the quasilinear to the collisional diffusion for three different electric fields E in unit of the critical field E_0 . v_1 and v_2 are chosen to be 3 and 20 times v_e respectively. For $D_0 = 0$, $A/n\nu$ = $4.0 \cdot 10^{-7}$, $4.4 \cdot 10^{-15}$, $3.4 \cdot 10^{-41}$, for E = 3%, 1%, 0.3% respectively. For $D_0 \to \infty$, $A/n\nu$ = $2.5 \cdot 10^{-4}$, $5.3 \cdot 10^{-5}$, $4.5 \cdot 10^{-6}$, for E = 3%, 1%, 0.3%.

Recently the possibility of using r.f. to drive a d.c. current in plasma for the operation of a steady-state Tokamak has been of considerable interest. Both theoretical analysis[1-9] and experimental demonstration[8-11] have shown the feasibility of significant current generation by lower hybrid waves excited in plasmas. Because of the finite plasma resistivity, this current could induce an electric field in plasma in addition to an externally driven inductive field. This electric field can accelerate electrons to become runaways, which can carry substantial fractions of the current and lead to plasma instabilities[12,13,14]. In fact the presence of waves can greatly enhance the runaway production rate because the quasilinear diffusion of electrons by waves can enhance the flow in velocity space across the critical velocity beyond which the electric field acceleration dominates over the collisional drag and the electron becomes runaways.

In this Letter, we will show that the runaway production rate can be enhanced many orders of magnitudes by the plasma waves for realistic parameters, if the critical velocity is within the range of the phase velocities of the wave packet. This important effect has been hithertofore neglected in the theoretical analysis but has been shown in at least two experiments. In the lower-hybrid wave heating experiments on ATC Tokamak, Boyd et al.[16], observed a rapid, order of magnitude increase in the intensity of the synchrotron radiation, reaching to maximum after the r.f. power has been tuned off. They attributed this enhanced radiation to increased number of runaway electrons and estimated about 10% of the r.f.

power went into their production. A recent Letter by Yamamoto et al.[11] reports the generation of r.f.-driven current carried primarily by the runaway electrons. They observed that, concurrent with the r.f. pulse, there is a marked decrease of the loop voltage because the current carried by runaways is less resistive and a sudden enhancement of the electron cyclotron radiation due to the pitch angle scattering of runaways by unstable modes[14].

Consider a wave packet of plasma wave in a magnetic field with frequency $\omega_K = \omega_p k_{\parallel}/k$ few times the lower hybrid frequency. For simplicity we assume its spectrum, $W_K = |E_K|^2/8\pi$, is uniform between $v_1 \leq \omega_K/K_{\parallel} \leq v_2$ and zero outside, maintained at steady state by an external source S_K such that

$$(2\gamma_L + \nu_{ei}) W_K = S_K \tag{1}$$

where γ_L is the Landau damping rate and ν_{ei} is the collisional damping rate of the wave. (We have also studied the runaway production rate in the presence of waves with power-law spectra $W_k \sim k^{-\alpha}$, $\alpha = 3,4$, and found it to be insensitive to the shape of spectrum). The quasilinear diffusion coefficient in the velocity component v along the magnetic field $\vec{B}$ is

$$D(v) = \left(\frac{e}{m}\right)^2 \int \frac{\sin\theta \, d\theta \, k^2 dk}{(2\pi)^2} |E_k|^2 \cos^2\theta \; \pi \delta(\omega_K - kv\cos\theta) \tag{2}$$

$$= \begin{cases} D_w (v_e/v)^3 & v_1 \leq \frac{\omega_K}{K_{\parallel}} \leq v_2 \\ 0 & \text{outside} \end{cases}$$

where $D_\omega = \left(\frac{e}{m}\right)^2 |E_K|^2/(8\pi\lambda_D^3\omega_p)$.

Note that the quasilinear coefficient has the same velocity dependence as the collisional diffusion $D_c = \nu v_e^2 (v_e/v)^3$. For simplicity, we employed the Vedenov model[17] for collision term which well approximates the diffusive process towards Maxwellian due to electron - electron collision, but neglects the pitch angle diffusion.

Since we consider here the one-dimension quasilinear diffusion by waves along the electric field parallel to the magnetic field, its effect on pitch angle diffusion is not important. The kinetic equation for the electron in the velocity component along $E \parallel B$ is :

$$\frac{\partial f}{\partial t} + \frac{e}{m} E \frac{\partial f}{\partial v} = \frac{\partial}{\partial v} D \frac{\partial f}{\partial v} + \frac{\partial}{\partial v}\left[\nu \left(\frac{v_e}{v}\right)^3 \left(v f + v_e^2 \frac{\partial f}{\partial v}\right)\right] + A\delta(v) \tag{3}$$

where the first term on the right is the quasilinear diffusion by waves with $D(v)$ given by Eq. (2), the second term is the collision term and A is the source necessary for maintaining the steady state in the presence of the electric field E , for which we shall solve Eq. (3) by setting $\frac{\partial f}{\partial t} = 0$.

Integrating Eq. (3) over v once, we have a first order equation

$$\frac{\partial f}{\partial v} = \frac{\left[\frac{e}{m} E - \nu (v_e^3/v^2)\right] f - A}{D_T (v_e/v)^3} \tag{4}$$

where $D_T = \nu v_e^2 + D_w = \nu v_e^2 (1 + D_0)$,

$D_0 = D_w / \nu v_e^2 \approx (\omega_p/\nu)(W/nT)(k_1 \lambda_D)^{-3}$, is the ratio of the quasilinear to collisional diffusion coefficient, $W = |E_k|^2 / 8\pi \; k_1^3$, k_1 is the maximum wave number. For $v \to \infty$, we find from Eq. (3) $f(\infty) = mA/eE$ which then serves as a boundary condition. Integrating Eq. (4), once again to find

$$f(v) = \exp\left[\left(E\frac{u^4}{4} - \frac{u^2}{2}\right)/(1 + D_0)\right]\left\{ C - \frac{A(\nu v_e)^{-1}}{1 + D_0}\int^{u} du\, u^3 \exp\left[\left(-E\frac{u^4}{4} + \frac{u^2}{2}\right)/(1 + D_0)\right]\right\} \tag{5}$$

where C is to be determined from the boundary conditions, and E is now expressed in the unit of $E_0 = m\nu v_e / e$ the critical field for the thermal electron to runaway and $u = v/v_e$.

Changing variables to $x = (E^{1/2} u^2 - E^{-1/2})(1 + D_0)^{-1}/2$ and $y = (E^{1/2} u^2 - E^{-1/2})/2$, we find solution to Eq. (3) satisfying the boundary conditions at $u \to \infty$: $f(\infty) = A(\nu v_e)^{-1}/E$ and f be continuous at $u_1 = v_1/v_e$ and $u_2 = v_2/v_e$:

$$f(u) = \frac{A(\nu v_e)^{-1}}{E}\left[1 + E^{-1/2}\int_{y}^{\infty} \exp(y^2 - y'^2)\, dy'\right] \quad \text{for} \quad u > u_2 \tag{6a}$$

$$f(u) = \frac{A(\nu v_e)^{-1}}{E}\left[1 + E^{-1/2}\int_{y_2}^{\infty} \exp(y_2^2 - y'^2 + x^2 - x_2^2)\, dy' + E^{-1/2}(1 + D_0)^{-1/2}\int_{x}^{x_2} \exp(x^2 - x'^2)\, dx'\right] \quad \text{for} \quad u_1 \le u \le u_2 \tag{6b}$$

$$f(u) = \frac{A(\nu v_e)^{-1}}{E}\Big[1 + E^{-1/2}\int_{Y_2}^{\infty} dy' \exp\left(X_1^2 - X_2^2 - Y_1^2 + Y_2^2 - y'^2 + y^2\right)$$

$$+ E^{-1/2}(1+D_o)^{-1/2}\int_{X_1}^{X_2} dx \exp\left(y^2 - Y_1^2 + X_1^2 - x^2\right)$$

$$+ E^{-1/2}\int_{y}^{Y_1} dy' \exp\left(y^2 - y'^2\right)\Big] \quad \text{for} \quad u < u_1 \tag{6c}$$

where $Y_i = E^{1/2}(u_i^2 - u_c^2)/2$, $X_i = Y_i(1+D_o)^{-1/2}$, $i = 1, 2$, $u_c = E^{-1/2} = v_c/v_e$ is the critical velocity for runaway. Now we may evaluate $f(u=0)$ from (6c) and set it equal to that of Maxwellian $f_o = n_o/(2\pi)^{1/2} v_e$ at $u = 0$ to determine the source strength A needed to maintain a steady state with a given $f(0)$.

$$A = \frac{n_o \nu}{(2\pi)^{1/2}} E^{3/2} e^{-1/4E}\Big[E^{1/2} e^{-1/4E} + \frac{e^{-D_o X_1^2}}{(1+D_o)^{1/2}}\int_{X_1}^{X_2} dx\, e^{-x^2} +$$

$$\exp\left(D_o(X_2^2 - X_1^2)\right)\int_{Y_2}^{\infty} dy\, e^{-y^2} + \int_{-\frac{1}{2E^{1/2}}}^{Y_1} dy\, e^{-y^2}\Big]^{-1} \tag{7}$$

This is just the runaway production rate in the steady state since $A = \frac{eE}{m} f(u=\infty)$ which is just the flux in the velocity space along E moving to infinity. Eq. (7) is evaluated numerically and the runaway production rate as a function of $D_o = D_w/D_c$ is shown in Fig. 1. It is seen that even for moderate values of D_o the runaway production rate is enhanced many orders of magnitude over that without waves $(D_o = 0)$

and, with $D_0 \neq 0$, even a small electric field $E = 10^{-3}$ can have significant rate of runaway production. By setting $D_0 = 0$, we obtain the runaway production rate in the absence of the r.f. waves for $E \ll 1$ [14,15]

$$A_0 = \frac{\sqrt{2}\, n_0 \nu}{\pi} E^{3/2} \exp(-1/4E) \tag{8}$$

In the following we evaluate analytically Eq. (7) for the case $v_1 \ll v_c = E^{-1/2} v_e \ll v_2$ where the quasilinear diffusion by plasma waves is important. In this case, the second term in the parenthesis of Eq. (7) dominates and we obtain the runaway production rate

$$A = \frac{\sqrt{2}\, n_0 \nu}{\pi} E^{3/2} \frac{(1 + D_0)^{1/2}}{[\operatorname{erf}(|x_1|) + \operatorname{erf}(x_2)]} \exp\left[- \frac{1 + 2D_0 (v_1/v_c)^2}{4E(1 + D_0)}\right] \tag{9}$$

Because $(1+D_0)^{-1}$ in the exponent, A is significantly enhanced over Eq. (8) for typically $D_0 > 1$; and A can be substantially large even for $E \ll E_0$. In fact we may define the effective critical electric field for electrons to runaway in the presence of quasilinear diffusion by waves as $E_{eff} = E_0/(1+D_0)$ above which practically all electrons runaway in $E^{-3/2} D_0^{-1/2}$ times the collision period. For very large quasilinear diffusion coefficient $D_0 \gg 1$, one of the last two terms in Eq. (7) becomes dominant and we find the runaway production rate saturates at a value

$$A = \begin{cases} \sqrt{\frac{1}{2\pi}}\, n_o \nu E \exp(-v_1^2/2v_e^2) & \text{for } |x_2| > |x_1| \\ \sqrt{\frac{1}{2\pi}}\, n_o \nu E \exp(-v_1^2/2v_e^2) \left(\frac{u_2^2 - u_c^2}{u_c^2}\right) \exp\left[\frac{(v_2^2/v_c^2 - 1)^2}{4 E D_o}\right] & \text{for } |x_1| > |x_2| \end{cases}$$

which for $D_o \to \infty$ is sensitive to the value of the smallest phase velocity v_1. To minimize the power requirement for maintaining the steady state of waves, we must avoid excessive Landau damping and choose $v_1 \gtrsim 3 v_e$. Knowing A and $f(u)$ in Eq. (6), one may evaluate the Landau damping rate for the wave packet γ_L and substituting into Eq. (1) determine the source level needed for maintaining a steady-state wave spectrum.

For $v_1 << v_c \approx v_2$, we find the third term in the paranthesis of Eq. (7) dominates and the runaway rate is

$$A = \frac{\sqrt{2}\, n_o \nu}{\pi} E^{3/2} \exp\left[- \frac{1 + 2D_o (v_1/v_c)^2}{4 E (1 + D_o)}\right]$$

Again many orders of magnitude enhancement over $D_o = 0$ case is possible.

For v_c well outside the range between v_1 and v_2, the waves have little effect on the runaway generation.

The time needed to establish the steady-state distribution is also considerably shorted by the waves. Instead of the usual $\left(E_0/E\right)^{3/2}\nu^{-1}$ which is the time for acceleration to critical velocity, now it takes only a quasilinear diffusion time to cross the critical velocity :

$$U_c^2/D \quad \sim \left(E_0/E\right) v_e^2/D$$

In conclusion, plasma waves with phase velocities comparable to the critical velocity for runaway can enhance the runaway production rate by many orders of magnitude, even at moderate intensities by quasilinearly diffusing electrons through the critical velocity. This theoretical result explains the experimental observations of enhances synchrotron radiations when lower hybrid waves are injected into Tokamak plasmas.

For the purpose of generating a steady-state current by waves in Tokamaks, it is therefore essential for the phase velocities of the wave packet to lie outside the critical velocity to avoid excess runaway production which leads to runaway-driven instabilities.

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15. The runaway production rate obtained from the full Fokker-Planck equation including pitch angle scattering with effective ion charge Z_{eff} :

$$A = 0.35\, \nu n_o \left(\frac{E_o}{E}\right)^{3/8\, Z_{eff}} e^{-E_o/4E} \quad exp\left[-\left\{2 Z_{eff}\, E_o/E\right\}^{1/2}\right]$$

obtained by R.M. Kulsrud, Y.C. Sun, N.K. Winsor and H.A. Zallon, Phys.Rev.Lett. 31, 690 (1973), and R. Cohen, Phys.Fluids 19, 239 (1976). We note that leading exponent, $exp\left(-E_o/4E\right)$, is not changed by the effective Z but in the presence of the waves, it is this leading exponent that changes by quasilinear diffusion. The other factors are expected to be remained unaffected by waves.

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