AUGUST 1979 LRP 156/79

SEPARATION OF ISOTOPES

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Abstract :

A new method for separating ions with different charge to mass ratio is described. It is based on the pondero-motive effect near the ion cyclotron resonance.

A method for isotope separation by means of the ion cyclotron resonance has been described by J.M. Dawson et al [1]. Here we propose a method using the pondero-motive effect in a magnetic field. Consider a magnetized plasma composed of electrons and two species of ions. We label these three components with the index j which may take the values e, 1 and 2 We choose the z-axis parallel to the B_0 field and assume that the densities n_j are slowly varying functions of z. All equations will be written in the natural system of units [2]. A circularly polarized plane wave travelling along the z-axis

$$E = E_0 \left\{ \cos(\omega t - kz), \; -\sin(\omega t - kz), \; 0 \right\} , \tag{1}$$

$$B = \frac{k}{\omega} E_0 \left\{ \sin(\omega t - kz), \; \cos(\omega t - kz), \; 0 \right\} , \tag{2}$$

satisfies the dispersion equation [3]

$$\Delta \equiv \omega^2 \left[1 - \sum_j \frac{\omega_{pj}^2}{\Omega_j (\omega - \Omega_j)} \right] - k^2 = 0 \tag{3}$$

* This work was performed at the University of California at Los Angeles in the beginning of 1977.

where

$$\omega_{pj} = \frac{q_j^2 n_j}{m_j} \quad , \tag{4}$$

$$\Omega_j = \frac{q_j B_0}{m_j} \quad . \tag{5}$$

We shall be interested in a left circularly polarized wave which is obtained for $\omega > 0$ and which should be either a standing wave, or a wave beyond cutoff. Such a wave produces a pondero-motive force [4] F_j acting on each species j. F_j is the gradient of the quasi potential ϕ_j :

$$F_j = - \nabla \phi_j \tag{6}$$

where

$$\phi_j = \frac{q_j^2 E_0^2}{2 m_j (\omega - \Omega_j) \omega} \quad . \tag{7}$$

If $\Omega_1 < \omega < \Omega_2$ then the pondero-motive force acts in opposite direction on the two species.

Let us assume that

$$n_1 \gg n_2 \tag{8}$$

and

$$0 < \frac{m_1}{m_2} - 1 \ll 1 \quad . \tag{9}$$

Choosing

$$\omega = \frac{1}{2} (\Omega_1 + \Omega_2) \tag{10}$$

we obtain approximately from (3)

$$k^2 = - \frac{\omega_{p1}^2}{\alpha} \tag{11}$$

where

$$0 < \alpha = \frac{\Omega_2 - \Omega_1}{2\Omega_1} \ll 1 . \tag{12}$$

According to (11) the wave is exponentially decreasing away from the source of excitation. (If we had chosen $\Omega_1 > \Omega_2$ then $\alpha < 0$ and the wave would be propagating. In this case we would assume a standing wave.) The quasi potentials become

$$\phi_e = \frac{m_1 E_0^2}{2 B_0^2} \tag{13}$$

and

$$\phi_1 = - \phi_2 = \frac{m_1 E_0^2}{2 \alpha B_0^2} . \tag{14}$$

The pondero-motive force acting on the electrons is much weaker than the one acting on the ions. The electrons therefore simply follow the ions so as to keep the plasma electrically neutral, $n_e = n_1 + n_2$.

In equilibrium the ion densities are

$$n_1 = n_{10} \exp \left(- \frac{m_1 E_0^2}{2 \alpha T B_0^2} \right) , \tag{15}$$

$$n_2 = n_{20} \exp \left(+ \frac{m_1 E_0^2}{2 \alpha T B_0^2} \right) \tag{16}$$

where T is the temperature of the ions. The ion density n_1 given by Eq. (15) can be inserted into the dispersion equation (11) which becomes the non-linear wave equation [5].

$$\frac{d^2E_0}{dz^2} - \frac{1}{\alpha}\,\omega_{p10}^2\,\exp\left(-\frac{m_1 E_0^2}{2\alpha T B_0^2}\right) E_0 = 0 \,. \tag{17}$$

Assuming that the source of excitation is at $z=0$ we can give the general form of the solution of Eq. (17) in Fig. 1. (If $\Omega_1 > \Omega_2$, then $\alpha < 0$ and the solutions are periodic. The ions 1 are accumulated at the maxima of E_0^2 . The ions 2 are collected near the zeros of E_0) It is now obvious that the wave considered here separates the two species in space.

The characteristic length of the separation region is

$$L = \frac{\sqrt{\alpha}}{\omega_{p10}} = \sqrt{\frac{\alpha\, m_1}{q_1^2 n_{10}}} \,. \tag{18}$$

According to (15) and (16) the efficiency of separation is given by the ratio

$$\frac{n_{10}}{n_{1min}} = e^{\rho} \tag{19}$$

where the separation parameter ρ is

$$\rho = \frac{m_1 E_{0\,max}^2}{2\alpha T B_0^2} \tag{20}$$

In practice one wishes to achieve a prescribed reduction in the densities of the more abundant species, n_1 which is equivalent to prescribing the separation parameter ρ In terms of this parameter the required field strength is

$$E_{0\,max} = (2\alpha\rho)^{1/2}\left(\frac{T}{m_1}\right)^{1/2} B_0 \quad . \tag{21}$$

The transverse dimension of the separation device must be considerably larger than the Larmor radius of the ions which can be calculated from the transverse velocity

$$V_\perp = \frac{E_0}{\alpha B_0} = \left(\frac{2\rho}{\alpha}\right)^{1/2}\left(\frac{T}{m_1}\right)^{1/2} . \tag{22}$$

The Larmor radius of both species of ions is

$$R = \left(\frac{2\rho}{\alpha}\right)^{1/2}\left(\frac{T}{m_1}\right)^{1/2}\frac{m_1}{q_1 B_0} \quad . \tag{23}$$

Let us assume that the ions of species 2 are evacuated at $z=0$ as fast as they arrive. The mass flow of these ions along the tube in the negative direction then is simply

$$\Gamma = m_2 n_2 V_\parallel = (m_2 T)^{1/2} n_{20} \tag{24}$$

where n_{20} and T are the density and temperature at large values of z where the wave has negligible amplitude.

The energy density carried by the wave is

$$W = \frac{1}{2\omega} \frac{\partial \Delta}{\partial \omega} \langle E^2 \rangle \tag{25}$$

where Δ is given by (3) and the time average $\langle E^2 \rangle$ is simply E_0^2.
Thus we find approximately

$$W = \frac{\omega_{p1}^2 E_0^2}{2\alpha^2 \Omega_1^2} = \frac{1}{2} m_1 n_1 V_\perp^2 \tag{26}$$

This last equation shows that nearly all of the wave energy is kinetic energy of the more abundant ions.

As an example, let us consider the following mixture of ions :

$$m_1 = 3.97 \; 10^{-25} \; kg \; ,$$

$$\alpha = 6.3 \; 10^{-3} \; ,$$

$$m_2 = (1 - 2\alpha) \, m_1 \; ,$$

$$q_1 = q_2 = e = 1.6 \; 10^{-19} \; As \; .$$

We choose

$$B_0 = 2.5 \; Vs/m^2 \; ,$$

$$n_1 = 10^{17} \; m^{-3} \; ,$$

$$T = 10^4 \; {}^\circ K \; ,$$

$$\varsigma = 10 \; .$$

We find

$$E_{o\,max} = 524\ V/m\ ,$$

$$L = 0.863\ m\ ,$$

$$R = 3.32\ 10^{-2}\ m\ ,$$

$$V_\perp = 3.32\ 10^{4}\ m/sec\ ,$$

$$\Gamma = 2.3\ 10^{-5}\ kg\ sec^{-1}\ m^{-2}\ .$$

The separation method described here is based on cyclotron resonance. Collisions tend to wash out the resonances Ω_1 and Ω_2 and affect unfavorably the phase relation between the field and the transverse velocity. Such effects must be held to sufficiently low level which requires

$$\alpha \tau \Omega_1 \gg 1 \tag{27}$$

where τ is the collision time of the ions for impulse exchange. Landau damping is inoperative since the wave is cut off so that its phase velocity is infinite. Ion-electron collisions depend on the electron temperature. Assuming $T_e = 4\ 10^4\ {}^\circ K$ we find

$$\alpha \Omega_1 \tau_{ie} = 12.3\ .$$

If the degree of ionisation is ε then the ion-neutral collision time is

$$\tau_{in} = \frac{\varepsilon}{1-\varepsilon}\ \frac{1}{\sigma n V_\perp} \tag{28}$$

where σ is the ion-neutral cross-section. Assuming $\sigma = 5\ 10^{-19}\ m^2$

and $\varepsilon = 0.9$ we obtain

$$\alpha \Omega_1 \tau_{in} = 34$$

These values seem already sufficiently large. If not, it should be possible to raise T_e to 10^5 °K and ε close to unity so that $\alpha \Omega_1 \tau$ for all collisions becomes quite large.

Spatial non-uniformity of the wave field might conceivably have an unfavorable effect on the ion orbits, by introducing drifts. However, we expect that such drifts only cause the ions to precess about the z -axis. Therefore, a separation device which is large compared to R in the transverse direction can accomodate most orbits. Nevertheless, it will be desirable to produce wave fields which are as uniform as possible in the transverse direction.

A potential threat to the method might be instabilities associated with the large amplitude ion cyclotron wave. However, the experiments reported in 1) do not seem to show such effects.

References :

1) J.M. Dawson et al. Phys. Rev. Lett 37, 1547 (1976)

2) E.S. Weibel, American Journal of Physics 36, 1130 (1968)

3) T.H. Stix, The Theory of plasma waves, McGraw-Hill (1962)

4) H. Motz, C.J.H. Watson, Advances in Electronics and Electron Physics 23, 153 (1967)

5) E.S. Weibel: pp 60 - 67 in "The plasma in a magnetic field (edited by R.K.M. Landshoff), Stanford Univ. Press, (1958). Equations of this type have since been frequently discussed in the litterature.

Figure Captions :

Fig. 1: Qualitative dependence of the electric field amplitude E_0 and the densities n_1 and n_2 on z.

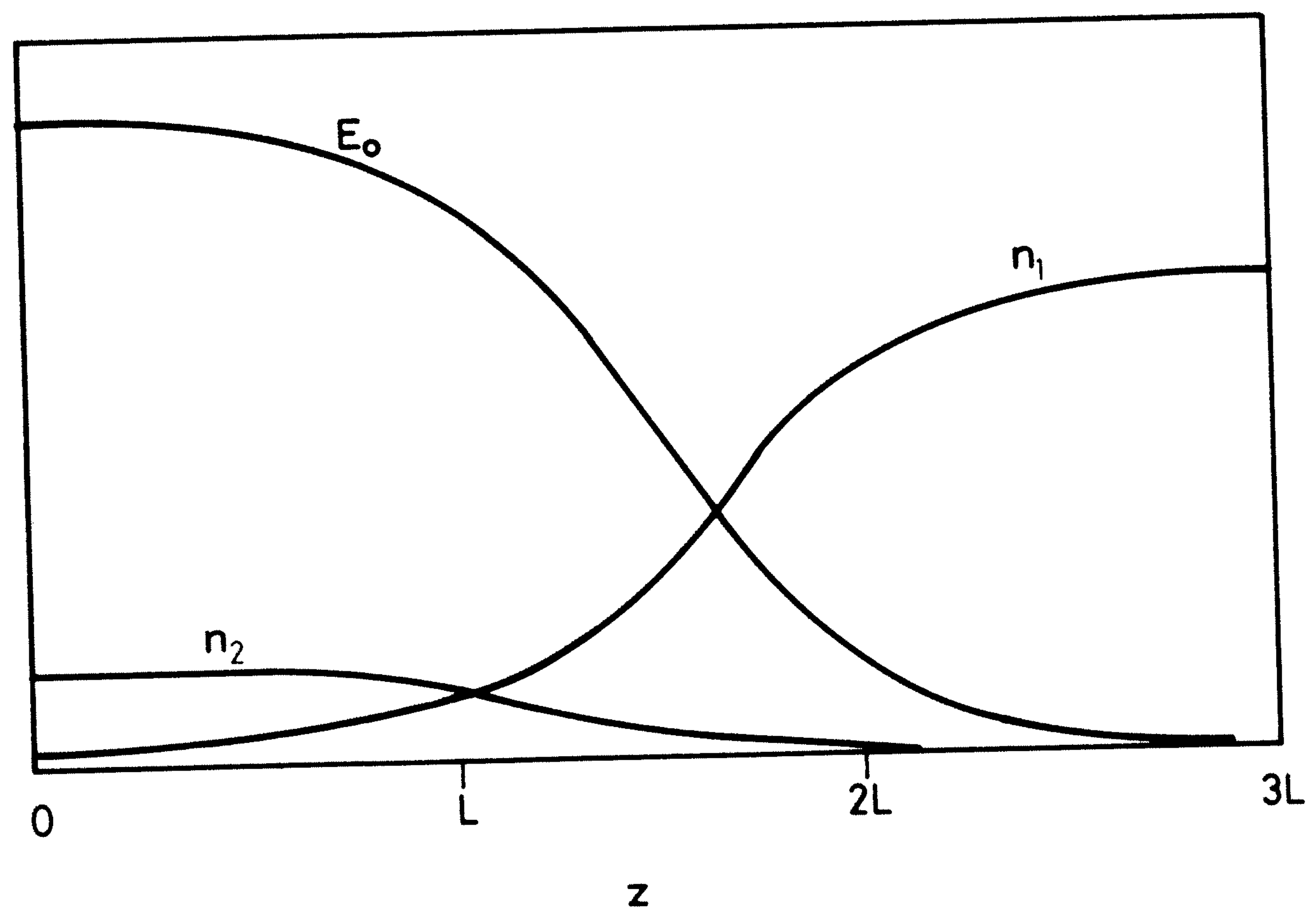

Fig.1