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ABSTRACT

The interplay between the Cerenkov and anomalous Doppler interactions in the relaxation of a warm electron beam is investigated by numerical means. The most important feature in the interplay is found to be a non-elastic isotropization. A simple semi-analytical model which allows one to estimate various quantities relevant to the relaxation process is also presented.

I INTRODUCTION

Electron beams are one of the classical features of plasma physics. Interest in the beam-plasma interaction was stimulated by its use for plasma production and heating[1], and is sustained today by REB experiments[2]. However, the fundamental reason for this interest is that an electron beam interacting with a plasma represents, in fact, the simplest system in which velocity-space instabilities occur in a pure form. As such, this interaction is a useful tool for investigating collective processes in plasmas.

The "classical" theory of the bump-on-tail instability nowadays is fundamental to any basic course on theoretical plasma physics, and related experiments are performed in magnetised plasmas ($\omega_{ce} >> \omega_{pe}$) in order to simulate the one-dimensionality of the theory. In contrast, it is less known that, in the same operating regime, the multiplicity of the wave-particle resonances may lead to qualitatively different results, for example, to a partial disappearance of the plateau even in a collisionless plasma. In this paper, we study the interplay between two of these resonances, viz. the so-called Cerenkov and anomalous Doppler effects, in the scattering of fast electrons in velocity space. This interplay could explain the inversion of the plateau observed by Kharchenko et al[3]; it might also be related to the anomalous loss of the parallel energy of a beam reported in[4].

Also it cannot be excluded that the interplay may occur in the auroral plasma of those planets with a strong magnetic field, such as Jupiter[5].

In the present work we do not intend to discuss any specific problems associated with laboratory or space plasmas. Rather, as a part of basic plasma physics, we study the initial value problem of the relaxation of a warm high-velocity electron beam in a magnetized plasma. The free energy provided in this way gives rise to a Langmuir turbulence with a broad spectrum, so that the Cerenkov and the anomalous Doppler interactions cooperate closely. The non-separability of the interactions makes analytical treatments suspect[6] and so it is strongly advisable to make use of the much greater potential of numerical calculations. For this purpose, we have introduced a new finite element approximation in order to cast the quasilinear equations in a computer code that is able to treat the wave-particle interactions consistently. Besides the computational results we present simple semi-empirical formulae inferred from parameter studies.

The plan of the paper is now outlined. In Sec. II we formulate the model; successively we discuss the assumptions involved, we present the basic equations, and explain the numerical approach used. The scenario of the beam relaxation is analyzed in Sec. III. In Sec. IV we study the role played by the different parameters. Sec. V, which differs somewhat from the others, contains a simple semi-analytical model able to reproduce the essential features of our numerical experiments. The lengthy calculations, concerning the spectrum are given in Appendix.

II MODEL

A. Assumptions

Let us consider a uniform, collisionless plasma immersed in a strong magnetic field, $\omega_{ce} > \omega_{pe}$ (where ω_{ce} and ω_{pe} are the electron cyclotron and plasma frequencies, respectively). Initially we launch a warm electron beam along the magnetic field [cf. Fig. 1(a)]. It is well known that, in such a situation, plasma waves with the frequencies $\omega_K = \omega_{pe} K_\parallel/K$ grow exponentially and exert a feedback action on the electron distribution. Since we are interested in studying the interplay between the Cerenkov resonance $\omega_K = K_\parallel v_\parallel$ and the anomalous Doppler resonance $\omega_K = K_\parallel v_\parallel - \omega_{ce}$, the beam velocity v_b has to be sufficiently high $v_b > (\omega_K + \omega_{ce})/K_\parallel$. On the other hand we want to avoid the higher harmonics of the cyclotron frequency. Thus, we have to choose the beam velocity in the range

$$\frac{\omega_K + \omega_{ce}}{K_\parallel} < v_b < \frac{\omega_K + 2\,\omega_{ce}}{K_\parallel} . \tag{1}$$

The emission of a plasmon via the anomalous Doppler effect is characterized by an increase in the gyration energy of an electron $\Delta E = \hbar\,\omega_{ce}$, and by a decrease in its parallel energy $\Delta E = -\hbar(\omega_K + \omega_{ce})$. If the magnetic field is sufficiently strong $\omega_{ce} \gg \omega_{pe} > \omega_K$, the plasmon energy may be neglected in comparison with the kinetic energy transferred. For simplicity, we shall assume that the former inequality holds, and so the pitch angle scattering of the electrons may be regarded as elastic. Nevertheless, we should still make two remarks about this simplification. Firstly, the commonly-adopted picture of elastic scattering may fail if a force due to the Cerenkov interaction acts simultaneously on the same particles (cf. Sec. III). Secondly, this approximation makes the system of equations non-conservative (cf. Sec. IIB).

B. Basic equations

We normalize the quasilinear equations[7] according to $k \to k/\lambda_d$, $v \to v \cdot v_{te}$, $t \to t/\omega_{pe}$, $f \to f \cdot n/v_{te}^3$, $\varepsilon_K \to \varepsilon_K \cdot 4\pi n T \lambda_d^3$. Here K is the wavenumber, λ_d is the Debye length, v_{te} is the electron thermal velocity, ω_{pe} is the plasma frequency, and n and T are the electron density and temperature, respectively. The kinetic equation for the electron distribution is then

$$\frac{\partial}{\partial t} f(v_\parallel, v_\perp, t) = \frac{\partial}{\partial v_\parallel} D_0 \frac{\partial f}{\partial v_\parallel} + \left(\frac{\partial}{\partial v_\parallel} - \frac{v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}\right) D_1 \left(\frac{\partial f}{\partial v_\parallel} - \frac{v_\parallel}{v_\perp}\frac{\partial f}{\partial v_\perp}\right), \tag{2}$$

where the diffusion tensor component D_0 takes the Cerenkov interaction into account

$$D_0(v_\parallel, t) = 2\pi \int_{K_\parallel > 0} \frac{d^3K}{(2\pi)^3}\, \mathcal{E}(\underline{k}, t)\, \delta\left(\frac{K_\parallel}{K} - K_\parallel v_\parallel\right), \tag{3}$$

and D_1 takes the anomalous Doppler interaction into account.

$$D_1(v_\parallel, v_\perp, t) = \frac{\pi}{2} \int_{K_\parallel > 0} \frac{d^3K}{(2\pi)^3} \left(\frac{K_\parallel}{K}\right)^2 \mathcal{E}(\underline{k}, t) \left(\frac{K_\perp v_\perp}{\omega_{ce}}\right)^2 \delta(\omega_{ce} - K_\parallel v_\parallel). \tag{4}$$

The nonresonant interaction is neglected since for the range of parameters used the amount of the beam energy transferred to the fluctuations is much smaller than the thermal energy of plasma.

Expressions (3) and (4) show an important common point : neither Dirac operators comprise the variable $v_\perp$. Thus, while the local shape of the distribution function with respect to the parallel velocity is important and so excludes a moment approach, the perpendicular dependance may be treated globally. On the other hand, Eq. (3) and (4) differ in the arguments of the wave-particle operators, which imply different resonant velocities, namely, $v_\parallel = \frac{1}{K}$ for the Cerenkov interaction and $v_\parallel = \omega_{ce}/K_\parallel$ for the anomalous Doppler interaction. Nevertheless, if the spectrum of the fluctuations is broad enough, not only an asynchronous interplay but also a synchronous interplay is possible.

The evolution of the waves obeys the quasilinear equation

$$\frac{\partial}{\partial t} \mathcal{E}(\underline{K},t) = 2(\gamma_0 + \gamma_1)\,\mathcal{E}(\underline{K},t) \quad , \tag{5}$$

where γ_0 corresponds to the Cerenkov interaction

$$\gamma_0 = \frac{\pi}{2}\frac{K_\parallel}{K^3}\int d^3v\; K_\parallel \frac{\partial f}{\partial v_\parallel}\;\delta\left(\frac{K_\parallel}{K} - K_\parallel v_\parallel\right) \quad , \tag{6}$$

and γ_1 to the anomalous Doppler interaction

$$\gamma_1 = \frac{\pi}{8}\frac{K_\parallel}{K^3}\int d^3v\; \left(\frac{K_\perp v_\perp}{\omega_{ce}}\right)^2 K_\parallel \left(\frac{\partial}{\partial v_\parallel} f - \frac{v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp} f\right)\delta(\omega_{ce} - K_\parallel v_\parallel). \tag{7}$$

We note that, consistent with the symmetry of the wave-particle interaction, the reducing factor $(k_\perp v_\perp/2\omega_c)^2$ between the anomalous Doppler interaction and the Cerenkov one is the same, both for diffusing particles and for growing waves. This reduced "efficiency" of the anomalous Doppler effect plays a role in its competition with the Cerenkov effect, as we shall see later.

It can easily be checked that Eqs. (2) - (7) constitute a set of equations which conserves the number of particles and momentum, but not the energy due to the assumption of elastic scattering.

Finally, the beam-like initial distribution is modeled by

$$f(v_\parallel, v_\perp, t=0) = \frac{1}{1+\xi}\left\{\left[\frac{1}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2}\left(v_\parallel^2 + v_\perp^2\right)\right]\right.\right. \tag{8}$$
$$\left.\left. + \frac{\xi}{(2\pi)^{3/2} v_{t_\parallel} v_{t_\perp}^2} \exp\left[-\frac{1}{2}\left(\frac{v_\parallel - v_b}{v_{t_\parallel}}\right)^2 - \frac{1}{2}\frac{v_\perp^2}{v_{t_\perp}^2}\right]\right]\right\} ,$$

where the free parameters, controlling the relaxation phenomena, are ξ the ratio of beam to bulk density, v_b the velocity of the beam, $v_{t_\parallel}$ the longitudinal spread of the beam, and $v_{t_\perp}$ the perpendicular spread of the beam.

C. Numerical Procedure

A finite element approach has been used to discretize Eqs. (2) - (7). It has been shown elsewhere[8] that this method leads, in a natural way, to consistent numerical schemes for quasilinear equations. In this paper we would like to describe a cylindrically-symmetric magnetized plasma. After integration over the ignorable variable in Eqs. (2) - (7) we are left with a two-dimensional problem given essentially by the same equations. At this point the method described in[8] could be used. For the purpose of a magnetized plasma however, further simplifications turn out to be possible. They consist of a substantial modification of the finite element method, which is outlined in the following paragraphs.

Remembering the remarks in Sec. IIB about the global character of the distribution function with respect to the perpendicular direction, we may introduce special semi-finite elements, assuming a shape a priori in that direction. This amounts to using a moment approach in the perpendicular velocity space where the local shape of the distribution is not important while keeping a detailed kinetic approach in the parallel velocity space where the local shape is crucial.

In this context we seek a solution of Eq. (2) of the form

$$f_N = \sum_{j=1}^{N} f_j(t)\ \Psi_j(v_\parallel)\ \frac{1}{2\pi T_j(t)}\ \exp\left[-\frac{1}{2}\ \frac{v_\perp^2}{T_j(t)}\right] , \tag{9}$$

where $\Psi_j(v_\parallel)$ is the usual roof function. For the moment we cannot offer to a skeptical reader a more convincing argument for the choice of a Maxwellian shape than its practical handling, however, an a posteriori justification will be given later (cf. Sec. III).

Let us project the diffusion equation (2) on the test functions $\Psi_i(v_\parallel)$ by means of the scalar product

$$(\Psi_i, f) = \int_{-\infty}^{+\infty} dv_\parallel \int_0^{\infty} 2\pi\, v_\perp\, dv_\perp\ \Psi_i\, f \ .$$

As a result we obtain a set of N ordinary differential equations of first order for the expansion coefficients $f_j(t)$.

$$\sum_{j=1}^{N} A_{ij} \dot{f}_j = \sum_{j=1}^{N} [B^0_{ij} + B^1_{ij}] f_j \quad , \quad i = 1, \ldots, N \quad , \tag{10}$$

where

$$A_{ij} = \int_{-\infty}^{+\infty} dv_\parallel \, \psi_i \, \psi_j \quad ,$$

$$B^0_{ij} = - \int_{-\infty}^{+\infty} dv_\parallel \, \dot{\psi}_i \, \dot{\psi}_j \, D_0(v_\parallel) \quad ,$$

$$B^1_{ij} = - \int_{-\infty}^{+\infty} dv_\parallel \, \dot{\psi}_i \left(\dot{\psi}_j T_j + \psi_j v_\parallel \right) \bar{D}_1 (v_\parallel) \quad ,$$

with $D_0(v_\parallel)$ defined in Eq. (3), and

$$\bar{D}_1 (v_\parallel) = \frac{\pi}{2} \int_{k_\parallel > 0} \frac{d^3k}{(2\pi)^3} \left(\frac{k_\parallel}{k} \right)^2 \mathcal{E}_k \, \frac{2k_\perp^2}{\omega_{ce}^2} \, \delta (\omega_{ce} - k_\parallel v_\parallel) \quad .$$

The expansion coefficients $T_j(t)$ are obtained by the projection of Eq. (2) on the test functions $\Psi_i \cdot \frac{1}{2} v_\perp$. Thus, we have a second system of ordinary differential equations for the quantities $g_j(t) = f_j(t) \cdot T_j(t)$:

$$\sum_{j=1}^{N} A_{ij} \dot{g}_j = \sum_{j=1}^{N} [B^0_{ij} + C^1_{ij}] g_j \quad , \quad i = 1, \ldots, N \quad , \tag{11}$$

where

$$C^{1}_{ij} = -\int_{-\infty}^{+\infty} dv_{\parallel} \left(2\dot{\psi}_i - \psi_i \frac{v_{\parallel}}{T_j} \right) \left(\dot{\psi}_j T_j + \psi_j v_{\parallel} \right) \overline{D}_1 (v_{\parallel}) \quad .$$

Finally, we also use finite elements for the discretization of the wave spectrum, in contrast to what has been done in[8]. The finite k space is subdivided into small rectangles of irregular size. The basis functions take the value one on a small rectangle and are zero elsewhere. Integrals over small rectangles are performed numerically using nine integration points. This method has the advantage over that of equally-spaced discrete waves that it produces a high density of Cerenkov and anomalous Doppler interaction lines in velocity space, $1 - Kv_{\parallel} = 0$ and $\omega_{ce} - K_{\parallel} v_{\parallel} = 0$, respectively, even with a rather coarse wavenumber grid.

III SCENARIO OF THE BEAM RELAXATION

For convenience, we first introduce two abbreviations. For a wave with wavenumber K, the resonant velocity for the Cerenkov interaction $v_{\parallel} = \frac{1}{K}$ will be noted V_r^c, while V_r^d will stand for the resonant velocity of the anomalous Doppler interaction $v_{\parallel} = \omega_c / K_{\parallel}$.

In a first stage, which is short compared with the complete relaxation, the usual Cerenkov interaction leads to the formation of a one-dimensional plateau on the electron distribution (cf. Figs. 1.a,b,c.). Simultaneously, a broad spectrum of the plasma waves is excited. This spectrum is independent of the initial level of the fluctuations, as long as the level is chosen to be sufficiently low. Thus, we may consider this parameter as being irrelevant to the relaxation phenomena. Wherever the distribution function has changed in $v_{\parallel}$-space, forming a plateau, there exist waves of substantial energy with the corresponding V_r^c. Those with the smallest V_r^c (near the bulk of the distribution function) have their V_r^d corresponding to the end of the tail and so can diffuse the fast electrons via the anomalous Doppler effect. This diffusion would tend to make the tail isotropic by converting the longitudinal energy into perpendicular energy (cf. Fig. 2). However, the Cerenkov effect makes this situation unreal. Immediately after the start of the isotropization process the slope of the electron distribution, integrated over $v_{\perp}$, becomes sufficiently positive to destabilize the plasma waves via the Cerenkov effect again. As a result, a part of the electrons with high gyration energy is pushed from the region of positive slope down to the bulk. For the same reason the level of turbulence is raised and the perpendicular temperature increases all along the tail. This dominating character of the Cerenkov effect over the anomalous Doppler effect is due to the factor $(K_{\perp} v_{\perp} / 2\omega_{ce})^2$, as mentioned in Sec. IIB. It is important to emphasize that their interplay never results in the formation of a bump; the plateau has only a very small positive slope which does not appear in the figure [cf. Fig. 1.(d)]. We may point out another interesting feature about the interplay. The retraction of the plateau leaves a negative slope at the end of the tail so that waves with their V_r^c here, previously excited by the Cerenkov effect, are now damped by the inverse Cerenkov effect. As a result, some electrons are pushed back to the high velocity domain. Thus, they may be diffused by the anomalous Doppler interaction more than once and so acquire a very high perpendicular energy. On the other hand, all electrons at the end of the tail may gain perpendicular energy but not all are able to complete their diffusion due to anomalous Doppler interaction, since the Cerenkov effect interferes and pushes them down to the bulk. In view of this situation, it is clear that the true dependence of the distribution function on $v_\perp$ may not be simply constructed from the initial longitudinal dependence via the diffusion of the electrons along the lines of constant energy in the way shown in Fig. 2. This phenomenon, which we shall call "non-elastic isotropization", justifies the use of the ansatz. From one standpoint it lays by the heels the objection which could arise a priori against the ansatz : the concept of isotropization seems somehow incompatible with the idea of an imposed perpendicular shape. From another point of view, the "non-elastic isotropization" may not be expected to lead to some exotic dependence of the distribution function on $v_\perp$; moreover, since the detailed shape is not important as explained in Sec. IIB, we choose the convenience of a Maxwellian shape.

Let us come back to the scenario of the relaxation. As the plateau shrinks, its level rises so that it goes deeper into the bulk. Thus, the waves with a smaller V_r^c grow, the lower boundary of the V_r^d is shifted down, and a new section of the plateau is destroyed by the anomalous Doppler effect. At the end of this competition between the Cerenkov interaction, which tends to flatten the distribution, and the anomalous Doppler interaction, which tends to destroy the plateau, the perpendicular temperature is uniform along the tail. A shortened plateau stays between the bulk and the point D defined by min $\{V_r^d\}$ [cf. Fig. 1.(e)]. The edge point C near the bulk is defined by $\min\{V_r^c\}$, and is related to D by the obvious relation

$$D = \omega_{ce} \cdot C \quad . \tag{12}$$

Beyond the D point the distribution function is isotropic, and hence stable with respect to the generation of waves via the anomalous Doppler effect. As far as the spectrum is concerned, there remain only large fluctuations with their V_r^c situated on the plateau. It is worth mentioning that the final state of the distribution is found to be affected only slightly by the initial spread of the beam (cf. Fig. 6 for an example). A warmer beam relaxes more slowly during the first stage. However, at the end of the stage the distribution has no memory of the initial spread - nevertheless this information remains in the spectrum. There are less fluctuations with a high V_r^c, but those which have their V_r^c close to the C point, and so start diffusing the fast electrons via the anomalous Doppler effect, remain nearly unchanged.

IV PARAMETER STUDIES

Apart from the C point and the D point, the final state is described by the perpendicular spread of the shortened tail. Since the spread is uniform all along it, the concept of perpendicular temperature $T_\perp$ may be introduced to summarize the code results. Moreover, the final perpendicular temperature is an adequate measure for the efficiency of the interplay in thermalizing a beam. Even if the net result of the relaxation plasma does not depend on the initial spread of the beam, it does depend on the other free parameters ξ, V_b , ω_α. We are going to show their respective role in plots (Figs. 3-6) where the code results are indicated by crosses; the solid line refers to the semi-analytical model to be presented (cf. Sec. V).

A) <u>role of ξ</u> (ratio between beam and bulk density)

The only essential change resulting from an increase in ξ is a rise in the level of the plateau appearing at the end of the first stage. This rise does not change the ratio of electrons which are diffused by the anomalous Doppler interaction to the others. Hence, we expect no change in $T_\perp$. However, we observe a slight increase in $T_\perp$ (Fig. 3). This effect may be easily understood in a more accurate description. The rise in the level of the plateau is accompanied by a shift of the C point toward the bulk; according to Eq. (12) the corresponding shift of the D point is more pronounced, $\Delta D = \omega_{ce} \cdot \Delta C$, and the relative number of the electrons which are diffused by the anomalous Doppler interaction is a little higher. An increase in ξ also results in a rise in the level of turbulence and so accelerates the diffusion of the electrons by the fluctuations. The effect appears in Fig. 4 where we consider the complete relaxation time versus ξ; the former may be defined as the time from the launching of the beam until the growth rate of the perpendicular temperature has dropped by two orders of magnitude.

B) <u>role of v_b</u> (beam velocity)

It is natural to anticipate that v_b has a strong influence on the perpendicular temperature (cf. Fig. 5). A little increase in v_b is so drastic, not only because the kinetic energy of the beam is proportionnal to the square of its velocity, but also, because of the more pronounced retraction of the tail. Indeed, the more the tail shrinks the more waves are damped by the inverse Cerenkov effect. As explained in Sec. III, this damping promotes the "more-than-once" diffusion of the electrons and increases the perpendicular temperature.

As for the relaxation time, an increase in v_b leads to a rise in the level of turbulence and thus stimulates the diffusion of the particles by the fluctuations. On the other hand, the particles have to travel a longer distance, and the net result is a relaxation time slightly increasing with the beam velocity v_b.

C) <u>role of ω_{ce}</u> (electron cyclotron frequency)

It is clear that a higher magnetic field requires a higher velocity of the electron for the resonance condition to be fulfilled. Thus, due to an increase in the ratio ω_{ce}/ω_{pe}, fewer electrons may be diffused by the anomalous Doppler effect and the final perpendicular temperature drops (cf. Fig. 6). The weak dependence of the relaxation time on ω_{ce} may be understood without invoking a subtle interplay of the anomalous Doppler effect with the Cerenkov one. Simply, the diffusion tensor component D_1 [Eq. (4)] becomes smaller with increasing ω_{ce} whereas the distance the electrons have to travel becomes shorter [cf. Eq. (12)]. The two effects tend to balance each other.

V ANALYTICAL MODEL

At the level of understanding of the topic, which our numerical experiments have led us to, it appears that a simple analytical model which includes the salient features observed in the relaxation process can be suggested. Moreover, it has the advantage of yielding very simple formulae which are far more flexible in use than a code. At this point, we should comment the attempt[6] to treat the relaxation in a purely analytical way. In spite of sophisticated analytical means, the authors were obliged to separate the part of the spectrum due to the Cerenkov effect from that due to the anomalous Doppler effect. All the study was biased by this basic assumption which led to artefacts in the dynamics of the relaxation and dubious results.

The final distribution function, to which the relaxation process described in Sec. III leads, may be characterized by the following quantities : the positions of the C and D points, and the perpendicular spread of the tail. As mentioned in Sec. II, the spread may be characterized by a single quantity $T_\perp$, thus, three quantities have to be determined. However, we have only two relations available: the conservation of particle number, and Eq. (12) relating D and C. In order to eliminate the remaining degree of freedom, one should know the spectrum and use the conservation of momentum. Instead, we take the initial beam velocity v_b into account in the simplest manner and sketch the dynamics of the relaxation as shown in Fig. 7. where V' is related to v_b by

$$\frac{V'}{V_b} = \frac{1}{\omega_{ce}} = \frac{C}{D} \quad . \tag{13}$$

In other words, V' is the V_r^c for waves having their V_r^d at v_b.
The C point is given by conservation of the number of particles

$$\frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{C^2}{2}\right)\left[(D-C) + \frac{1}{2}(v_b - D)\right] = \xi \; ,$$

therefore,

$$C = \left\{ 2 \ln\left[C\left(\frac{\omega_{ce}}{2} - 1\right) + \frac{v_b}{2}\right] - 2 \ln\left(\xi \sqrt{2\pi}\right)\right\}^{1/2} \tag{14}$$

which can be solved in a few iterations with a simple pocket calculator.

A. Estimate of $T_\perp$

Let us assume that the plateau retracts as far as the D point in the manner shown in Fig. 7. The diffusion mechanism stops when the slowest of the electrons, which are able to emit a plasmon via the anomalous Doppler effect, are prevented from doing so by the shape of the distribution. At this moment, the edge point of the distribution reaches the D point so that $\gamma_1 \to 0$ (marginal stability for the anomalous Doppler effect).

On introducing the Maxwellian ansatz, already discussed in detail (cf. Sec. III).

$$f(v_\parallel, v_\perp, t) = F(v_\parallel, t) \frac{1}{2\pi T_\perp(v_\parallel, t)} \exp\left(-\frac{1}{2} \frac{v_\perp^2}{T_\perp(v_\parallel, t)}\right) \tag{15}$$

into Eq. (7), we obtain

$$\gamma_1 \sim \left(\frac{\partial}{\partial v_\parallel} F + \frac{v_\parallel}{v_\perp} F \right)_{v_\parallel = \frac{\omega_{ce}}{k_\parallel}} .$$

Thus, the end of the diffusion is characterized by

$$\lim_{\varepsilon \mapsto 0} \left(\left. \frac{\partial F}{\partial v_\parallel} \right|_{v_\parallel = D+\varepsilon} + \frac{D}{T_\perp} F \Big|_{v_\parallel = D+\varepsilon} \right) = 0 . \tag{16}$$

We are well aware of a small inconsistency when considering both the Maxwellian ansatz (15) and the straight line between the D and v_b points in Fig. 7. However, since it enables us to write down convenient formulae with reasonable accuracy, we will retain it without more ado. Therefore, Eq. (16) yields

$$T_\perp = D \left(v_b - D \right) = \omega_{ce} \cdot C \left(v_b - \omega_{ce} \cdot C \right) . \tag{17}$$

Thus, by means of Eqs. (14) and (17) we are able to estimate the final perpendicular temperatures. The various dependences on the relevant parameters are shown in Figs. 3, 5, and 6 with the "experimental" points obtained from the code.

B. Estimate of the relaxation time τ

The sketch of the relaxation comprises two qualitatively different stages. The first one, dominated by the Cerenkov interaction (time t_1, in Fig. 7) may be considered as instantaneous when compared with the complete relaxation process. Nevertheless, it has the important role of increasing the level of fluctuations so that the initial level can be neglected. The second stage may be seemingly characterized by a change in the wave spectrum resulting from a transfer of the fluctuation energy from high V_r^c to low V_r^c near the bulk. As the plateau shrinks, the waves are damped by the inverse Cerenkov effect in the domain D–v_b, and re-emitted via the anomalous Doppler effect in the domain C–V', the total fluctuation energy remaining constant within a few per cent (cf. Fig. 8). Thus, it is plausible to assume that the relaxation is controlled by this transfer. Let us try to use this assumption to estimate the relaxation time.

By analogy with the formula pertinent to pure Cerenkov effect[9] we write for τ

$$\tau = \alpha \cdot \frac{1}{\langle \gamma_1 \rangle} \ln \left(\frac{W_1 + W_2}{W_1} \right) . \tag{18}$$

Here, α is some empirical factor, W_1 is the initial (with respect to the second stage) fluctuation energy of the waves with their V_r^c situated between C and V', W_2 is the amount of energy transferred, and $<\gamma_1>$ is a mean value of the growth rate of the waves in the domain C-V'. Combining Eqs. (7) and (15) we obtain

$$\langle \gamma_1 \rangle = \frac{\pi}{4} \left\langle \frac{K_\parallel}{K} \left(\frac{K_\perp}{K} \right)^2 \right\rangle \frac{X \cdot \mathcal{H}}{\omega_{ce}^2} \quad . \tag{19}$$

Here, $X = (D+V_b)/2$ is the averaged velocity of the electrons which emit the plasma waves in the region C-V', and

$$\mathcal{H} = \frac{1}{(2\pi)^{1/2}} \exp\left(- \frac{C^2}{2}\right) \tag{20}$$

is the height of the plateau.

Hence, we find

$$\tau = \frac{4}{\pi} \frac{\alpha}{\left\langle \frac{K_\parallel}{K} \left(\frac{K_\perp}{K} \right)^2 \right\rangle} \frac{2\,\omega_{ce}^2}{D + V_b} \frac{1}{\mathcal{H}} \ln\left(1 + \frac{W_2}{W_1}\right) \quad . \tag{21}$$

Let us assume that the factor $\alpha / \left\langle (K_\parallel/K)(K_\perp/K)^2 \right\rangle$ is independent of the parameters, which seems to be justified a posteriori by the results, and can be determined empirically from numerical experiments. It remains to estimate the ratio W_2/W_1. Since W_2 and W_1 are to be evaluated at the end of the first stage, one can use the method indicated by Parail and Pogutse [Eq. (20)] to obtain the spectrum. We present these calculations in the appendix. Finally, we find the following formula for the relaxation time

$$\tau = \frac{10^3}{V_b + \omega_{ce} \cdot C} \frac{\omega_{ce}^2}{\varkappa} \ln \left(1 + \frac{W_2}{W_1} \right) . \tag{22}$$

Since the only drastic parameter controlling τ is the magnitude of the beam in Fig. 6 we show the behavior of τ with respect to ξ.

VI CONCLUSION

Our aim has been to study the interplay between the Cerenkov and anomalous Doppler interactions in the relaxation of a warm electron beam. We have investigated this problem with the assistance of a code which includes as few ad hoc assumptions as possible.

The most important feature in the process considered is found to be a non-elastic isotropization. We emphasize that this new phenomenon is the consequence of the cooperation, either synchronous or asynchronous, between the two interactions; it is not due to the plasmon emission via the anomalous Doppler effect since we treat the case $\omega_{ce} >> \omega$.

Also we have shown and discussed the role played by the different parameters in the beam dynamics. Moreover, we have presented a simple semi-analytical model, based on our numerical experiments, which allows one to estimate various quantities relevant to the relaxation process without the need of a code.

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Appendix : calculation of W_1 and W_2

In the units used, Eq. (20) given by Parail and Pogutse reads

$$\bar{\mathcal{E}}\left(K=\frac{1}{v}\right)=\pi v^5\int_c^v dx\,\left[F_2(x)-F_1(x)\right]\ ,\quad v>c\ , \tag{A1}$$

where

$$\bar{\mathcal{E}}\left(K=\frac{1}{v}\right)=\int_0^{\pi/2}\frac{d\varphi}{2\pi}\ \sin\varphi\ \mathcal{E}_K\ ,$$

$$F_{2,1}=\int 2\pi\, v_\perp\, dv_\perp\, f_{2,1}\ ,$$

and the indices 1,2 refer to the initial and final distributions (with respect to the Cerenkov stage), respectively. From Eq. (8) and the fact that $F_2 = \mathcal{H}$ it follows that

$$F_2(x)-F_1(x)=\mathcal{H}-\frac{\xi}{(2\pi\, v_{t_\parallel})^{1/2}}\exp\left[-\frac{(x-v_b)^2}{2\,v_{t_\parallel}^2}\right]$$
$$-\frac{1}{(2\pi)^{1/2}}\exp\left(-\frac{x^2}{2}\right)\ ,\quad x>c\ . \tag{A2}$$

We now combine Eqs. (A1) and (A2) to obtain the quantity $\bar{\varepsilon}$ which appears in the expression for the wave energy

$$\frac{1}{2\pi}\int dv \, \frac{1}{v^4} \, \bar{\mathcal{E}}\left(k = \frac{1}{v}\right) \quad .$$

1. In the estimate of W_1, we neglect the term corresponding to the beam and find

$$W_1 = \frac{1}{2}\int_c^{v'} dv \, v \int_c^{v} dx \left[\varkappa - \frac{1}{(2\pi)^{1/2}} \exp\left(\frac{-x^2}{2}\right)\right] .$$

Performing the integration we obtain

$$W_1 = \frac{\varkappa}{2}\left(\frac{v'^3}{3} + \frac{c^3}{6} - \frac{v'^2 c}{2}\right) + \frac{1}{4}\left[\varkappa \cdot c - \frac{v'}{(2\pi)^{1/2}} \exp\left(-\frac{v'^2}{2}\right)\right]$$
$$- \left(\frac{v'^2}{2} - \frac{1}{4}\right)\left[P_1(c) - P_1(v')\right] \quad , \tag{A3}$$

where $$P_1(x) = \int_x^{\infty} dy \, \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{y^2}{2}\right) \quad .$$

2. In the estimate of W_2 we disregard the bulk term so that

$$W_2 = \frac{1}{2}\int_D^{v_b} dv \, v \int_c^{v} dx \left\{ \varkappa - \left\{ \frac{1}{(2\pi \, v_{t_\parallel}^2)^{1/2}} \exp\left[-\frac{(x - v_b)^2}{2\pi \, v_{t_\parallel}^2}\right]\right]\right\} ,$$

i.e.,

$$W_2 = \frac{\mathcal{H}}{2}\left[V_b^2\left(\frac{V_b}{3} - \frac{C}{2}\right) - D^2\left(\frac{D}{3} - \frac{C}{2}\right)\right]$$

$$+ \frac{\xi}{(2\pi V_{t_\parallel}^2)^{1/2}} \exp\left[-\frac{(V_b - D)^2}{2 V_{t_\parallel}^2}\right] \frac{V_{t_\parallel}^2}{4}\left(V_b + D\right)$$

$$- \frac{\xi}{4}\left(V_b^2 + V_{t_\parallel}^2 - D^2\right) P_2(V_b - D) \tag{A4}$$

$$- \frac{\xi V_{t_\parallel}^2}{2} \frac{V_b}{(2\pi V_{t_\parallel}^2)^{1/2}} \quad ,$$

where

$$P_2(x) = \int_x^\infty dy \, \frac{1}{(2\pi V_{t_\parallel}^2)^{1/2}} \exp\left(-\frac{y^2}{2 V_{t_\parallel}^2}\right) \quad .$$

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FIGURE CAPTIONS

Fig. 1. Typical evolution of the electron distribution function. The parameters used are : $\xi = 10^{-3}$, $v_b = 10$, $\omega_{ce} = 2$.

a) Launching time.

b) Pure Cerenkov stage, $t \sim 10^3$.

c) End of the first stage; leading electrons start being pitch-angle scattered by the anomalous Doppler effect on the fluctuations previously excited, $t \sim 10^4$.

d) Cerenkov and anomalous Doppler effect cooperate closely to destroy the tail; note that there is no appreciable beam formation, $t \sim 8 \times 10^4$.

e) Final state, $t \sim 4 \times 10^6$.

Fig. 2. Velocity distribution (represented by equi-lines) we would find if we omitted the Cerenkov effect during the second stage.

Fig. 3. Influence of the beam density on the final perpendicular temperature. The parameters used are: $v_b = 10$, $\omega_{ce} = 2$. The crosses show the code results whereas the solid line is calculated from the formula (17).

Fig. 4. Dependence of the relaxation time on the beam density. τ is defined as the time after which T increases so slowly that $\left[T_{\perp}(2\tau) - T_{\perp}(\tau)\right]/T_{\perp}(\tau) < 3\%$. The bars account for the uncertainty due to the "discret" code outputs whilst the solid line represents the formula (22). The parameters used are: $v_b = 10$, $\omega_{ce} = 2$.

Fig. 5. Influence of the beam velocity on the final perpendicular temperature. The parameters used are: $\xi = 10^{-3}$, $\omega_{ce} = 2$. The crosses show the code results whereas the solid line is calculated from the formula (17).

Fig. 6. Influence of the magnetic field on the final perpendicular temperature. The parameters used are: $v_b = 14$, $\xi = 10^{-3}$. The crosses show the code results whereas the solid line is calculated from the formula (17). The plot shows also the influence of the initial spread of the beam: (x) $V_{t_{//}} = 2$, $V_{t_{\perp}} = 1$; (0) $V_{t_{//}} = 4$, $V_{t_{\perp}} = 1$; ($\square$) $V_{t_{//}} = 2$, $V_{t_{\perp}} = 2$.

Fig. 7. Model for the relaxation of the distribution function (cf. Sec. V).

Fig. 8. Sketch of the transfer of the fluctuation energy during the second stage (cf. VB). C.R.V. stands for Cerenkov resonant velocity.

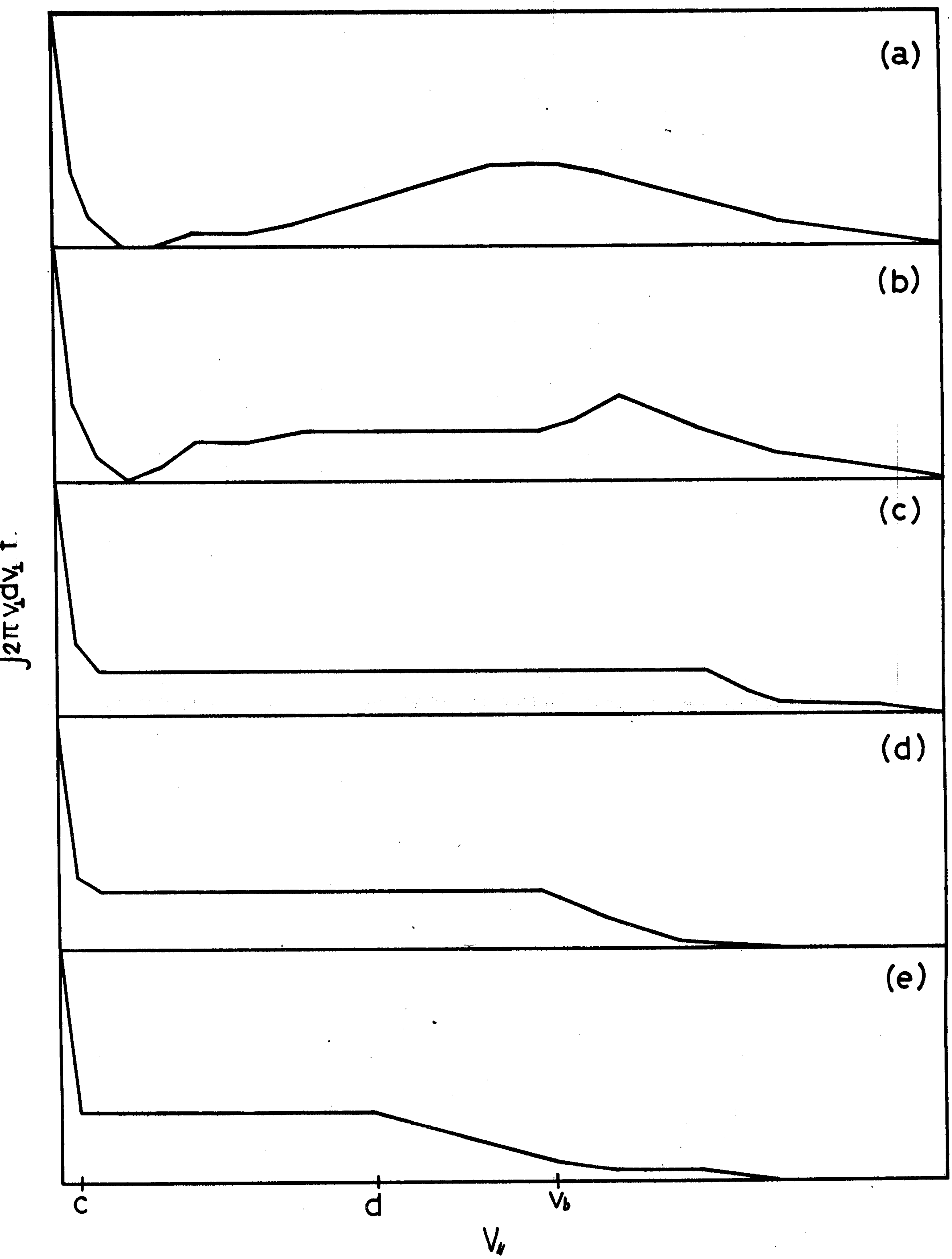


Fig. 1

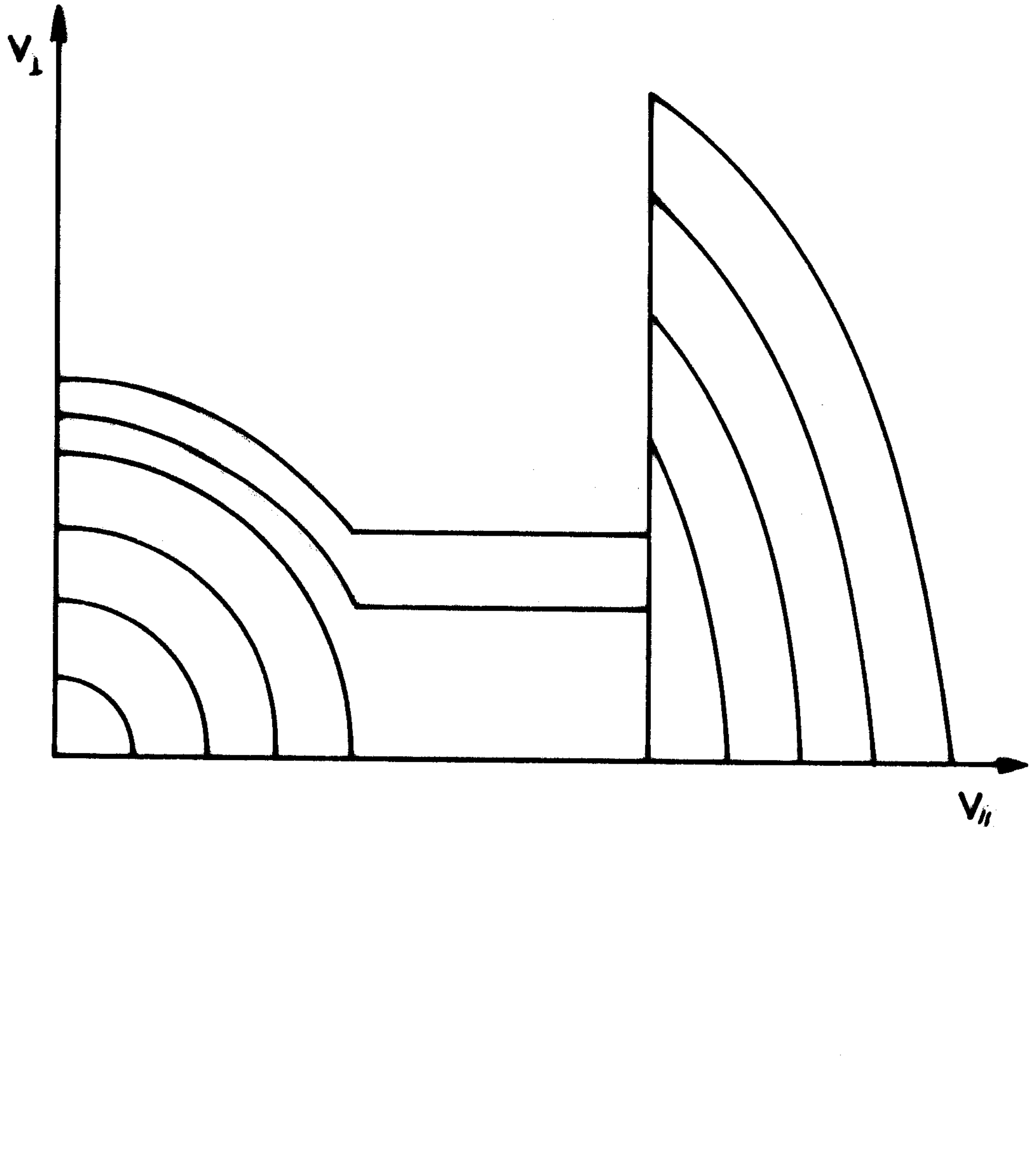


Fig 2

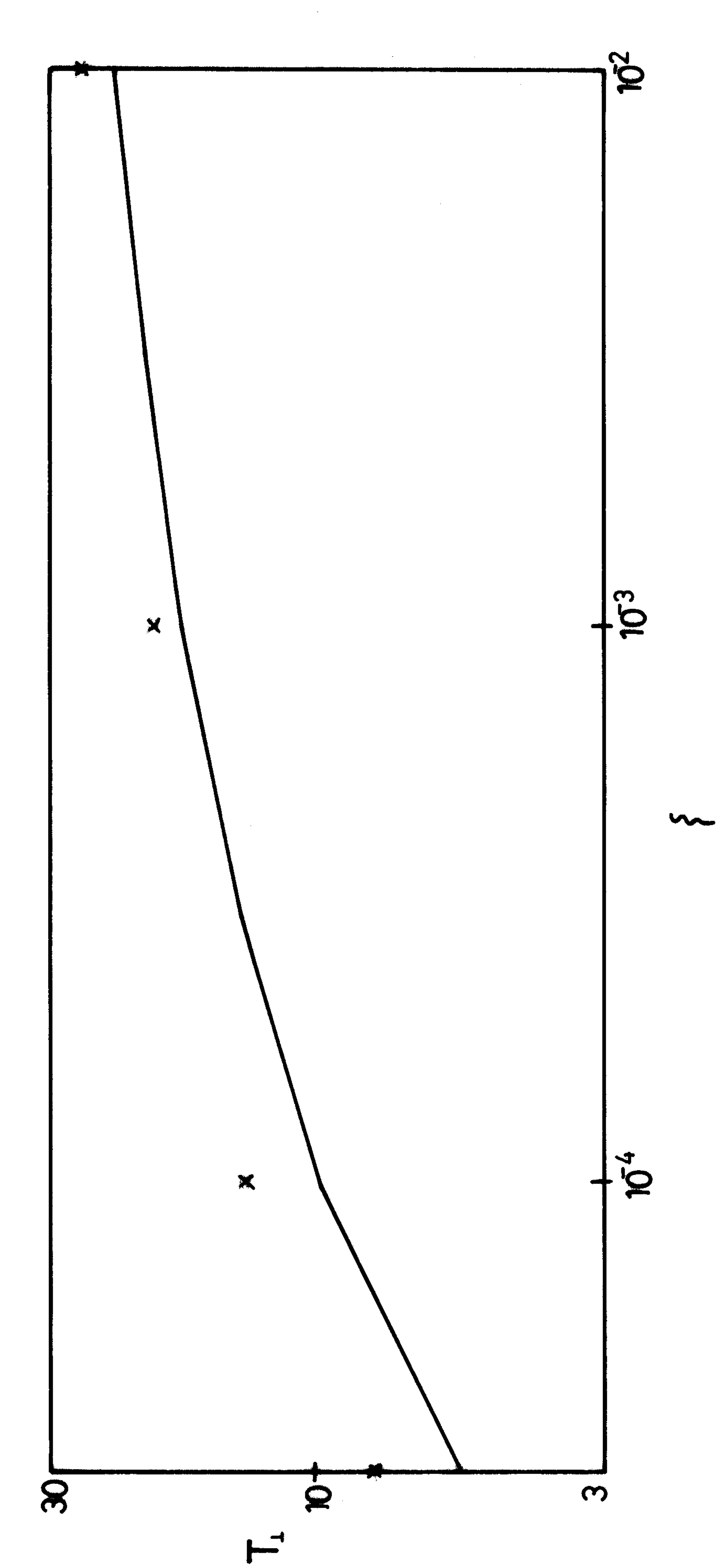
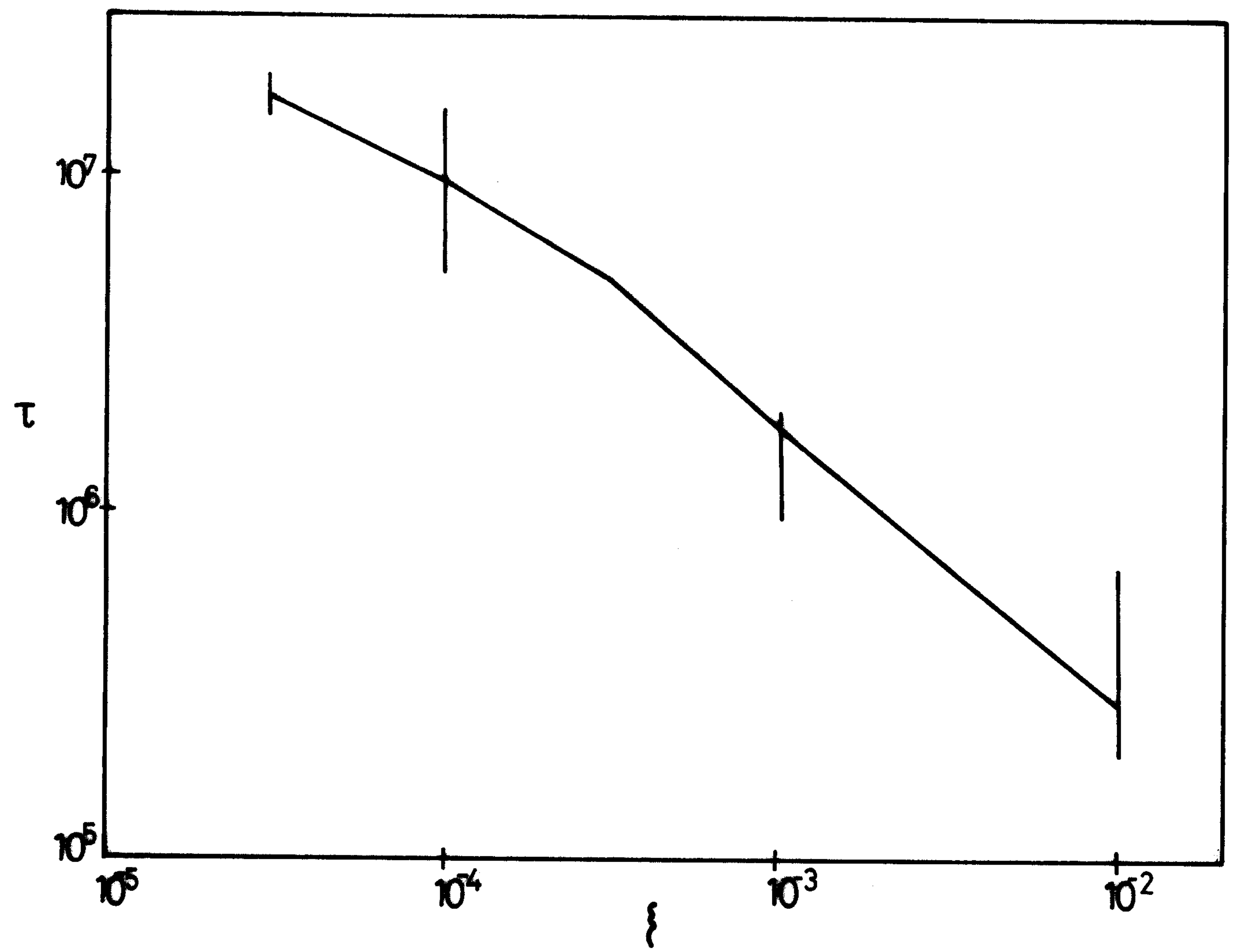


Fig 3



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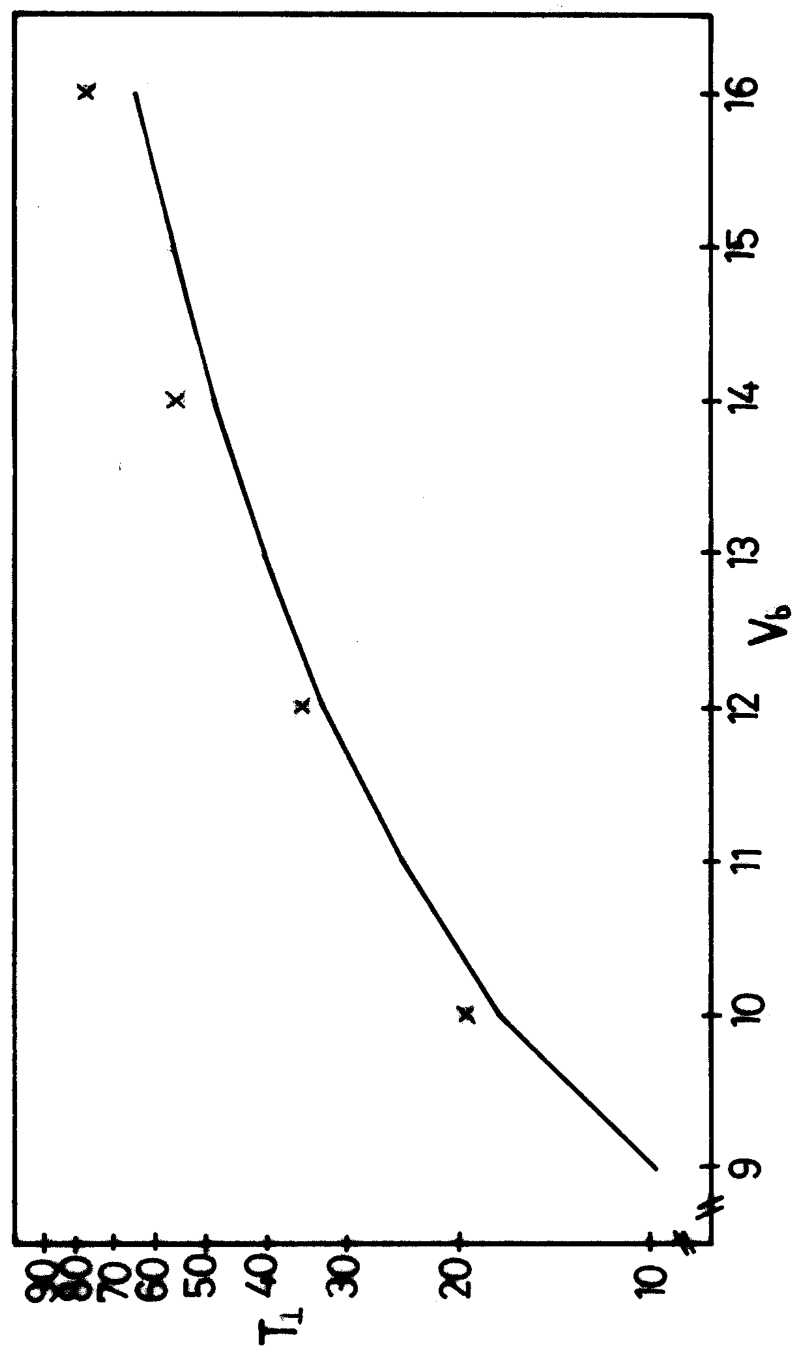

Fig 5

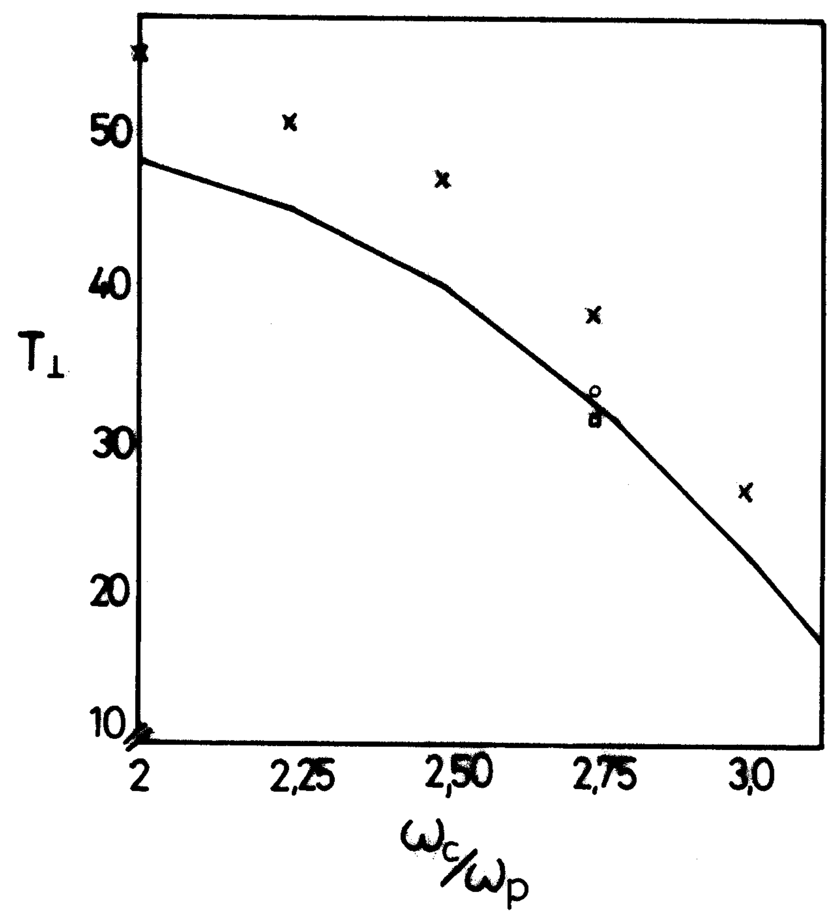

Fig 6

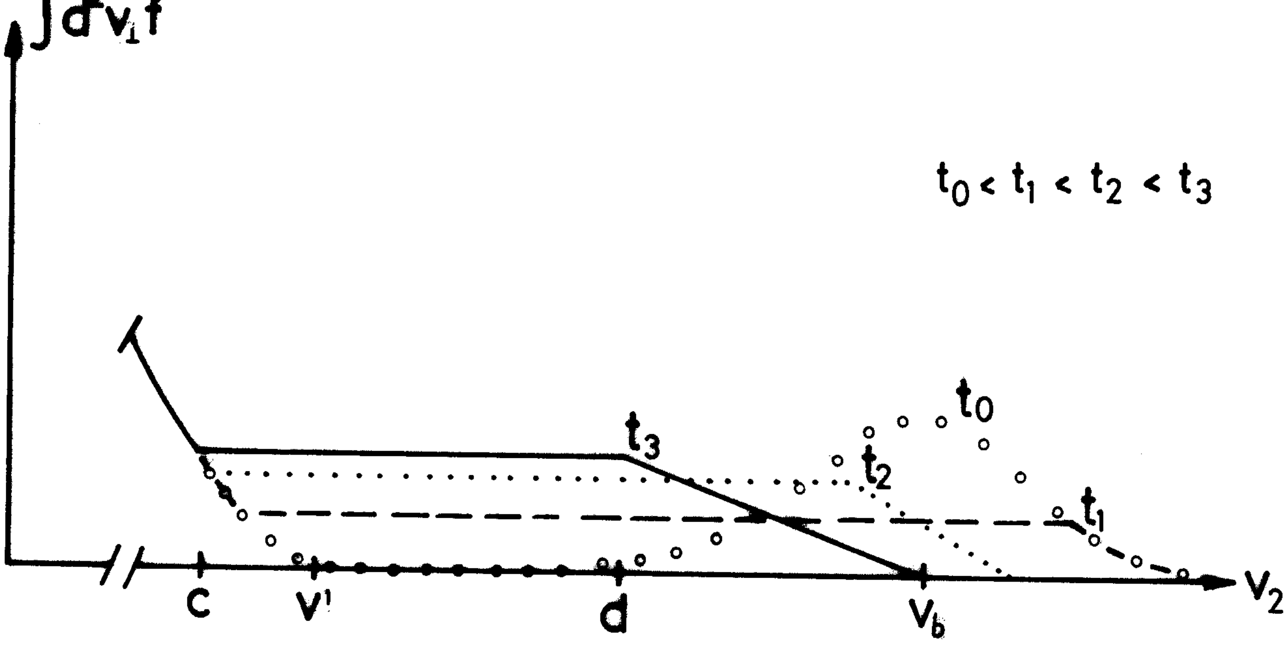

Fig 7

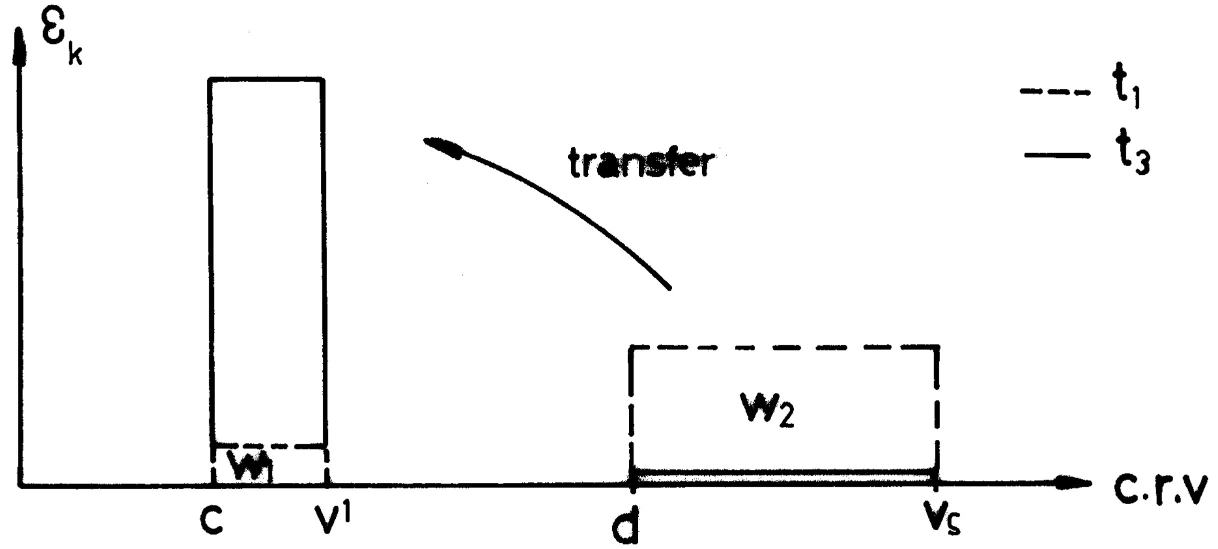

Fig 8