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On the Possibility of Obtaining a Stable, Dynamically Equilibrated Plasma Column in the Curved Sections of a Racetrack Theta Pinch

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ON THE POSSIBILITY OF OBTAINING A STABLE, DYNAMICALLY EQUILIBRATED PLASMA COLUMN IN THE CURVED SECTIONS OF A RACETRACK THETA PINCH

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ABSTRACT

Attention is drawn to the fact that the application of an oscillating axial current of reasonable amplitude and frequency could lead to a stable, dynamic, toroidal equilibrium of the plasma column in the curved end sections of the racetrack theta-pinch configuration. The parameters of an experiment aimed at testing the feasibility of this technique are presented. Finally, the r.f. power requirements of racetrack and fully toroidal theta-pinch configurations are compared.

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1. INTRODUCTION

There is currently a resurgence of interest in linear magnetic fusion systems with particular emphasis on linear theta-pinches (see, for example, the minutes of the recent Workshop on End-Stoppering of Linear Magnetic Fusion Systems, Santa Fe, New Mexico, Oct. 12 - 14, 1977). One of the systems to receive attention is the so-called racetrack theta-pinch configuration (Fig. 1) in which one tries to preserve as much as possible the linear geometry while concentrating the problems associated with field curvature to short U-bend sections at the ends. The means by which the equilibrium and stability of the plasma column in these curved sections are to be assured are as yet unspecified.

In a recent Los Alamos Scientific Laboratory Internal Report entitled "On the possibility of obtaining a dynamically equilibrated plasma column in the Scyllac 'derated' sector experiment" we have described how the application of an oscillating axial current of reasonable amplitude and frequency could lead to a stable, dynamic toroidal equilibrium of the curved plasma column in the Scyllac sector experiment. It is tacitly assumed that the technique could eventually be applied to obtain equilibrium in a fully toroidal experiment.

The technique proposed in the Los Alamos report is equally suited to obtaining the required stable equilibrium in the curved sections of the racetrack theta-pinch configuration and the purpose of this report is to draw attention to this fact, to discuss the physics and technological problems to be faced and to present the parameters of a proposed experiment aimed at testing the feasibility of the technique. In addition, a comparison is made between the racetrack and fully toroidal theta-pinch configurations.

2. Physics and technological problems

2.1 Equilibrium

One possible method of obtaining a toroidal equilibrium of the plasma column in the curved sections of the ractetrack θ-pinch is to pass a steady current I_z through the plasma. That is, in these curved sections, one produces a pinched plasma which is confined by a helical magnetic field. After a small outward displacement of the curved plasma column, the plasma feels a restoring force due to the compression of the poloidal component of the magnetic field between the plasma and a conducting shell placed around the discharge vessel and, in this manner, a toroidal equilibrium is established. Theoretical and experimental investigations indicate that there is a critical value of β below which such an equilibrium is stable and above which the amount of I_z needed for toroidal equilibrium exceeds the Kruskal-Shafranov limit, thus making the equilibrium unstable. Typically, the critical β-value is about 5%.

This constraint on the β-value, which is rooted in stability considerations, can, in principle, be circumvented by oscillating the direction of the axial current I_z. The theory of this dynamic stabilization scheme, which was originally investigated by Weibel[1] and which has been re-examined by Lister, Troyon and Weibel[2] indicates that provided the ratio of the amplitude of the oscillating field to that of the basic theta pinch field is less than some critical value, the configuration is stable. This result was obtained for a field free plasma ($\beta = 1$) surrounded by a vacuum. The influence of damping, which provides a stabilizing mechanism against parametric instabilities, was examined using both a viscous fluid - and Vlasov model for the plasma.

For the case of an oscillating axial current, the toroidal equilibrium is dynamic[3]; the plasma column oscillates about some equilibrium position which is displaced towards the outside wall of the curved section of the discharge tube. In the absence of exact calculations, it is reasonable to assume that the outward shift of the equilibrium position (the Shafranov shift) can be estimated by substituting the r.m.s. value of $\tilde{I}_z$ into the following approximate expression for the equilibrium displacement, Δ, of a plasma column situated within a perfectly conducting toroidal shell of minor radius b and major radius R:

$$\frac{\Delta}{b} = \left(\frac{b}{2R}\right) \beta \left(\frac{B_o}{\bar{B}_\theta}\right)^2 \tag{1}$$

B_o is the main axial (toroidal) field and $\bar{B}_\theta$ is the r.m.s. value of the oscillating poloidal field evaluated at the plasma surface, i.e.

$$\bar{B}_\theta = \frac{\mu_o \bar{I}_z}{2\pi a}$$

where $\bar{I}_z$ is the r.m.s. value of $\tilde{I}_z$ and a is the radius of the pinched plasma column. It is also reasonable to assume that the amplitude of the oscillation about the mean equilibrium position will be small if $\tau/4$ (where τ is the period of the oscillating current) is some small fraction of the time taken by the plasma column to drift to the wall in the absence of $\tilde{I}_z$.

2.2 Stability

As mentioned in the last section, one possible method of obtaining a toroidal equilibrium is to pass a quasi-steady current, I_z, along the pinched column. However, if β exceeds a certain critical value, the amount of I_z needed for toroidal equilibrium exceeds the Kruskal-Shafranov limit and a current-driven kink mode is excited. There is a tendency to believe that this kink mode can then be suppressed by alternating the

the current in a time less than the kink mode growth time. There is no justification for this belief since the application of the alternating current will itself induce parametrically excited instability modes which have to be examined in turn.

Lister, Troyon and Weibel[2] have examined the effects of a low frequency oscillating axial current on the stability of a theta pinch. They find the most dangerous instability modes are:-

(i) Long wavelength m = 0 modes which can be wall-stabilized: if

$$\frac{\bar{B}_\theta}{B_o} < \frac{\sqrt{2}\,(a/b)}{[1 - (a/b)^2]^{\frac{1}{2}}} \qquad (2)$$

these modes are absent.

(ii) Parametrically excited m = 1, long wavelength modes. There is a threshold on $\bar{B}_\theta$ for the onset of parametric excitation which defines the limit of stability. Provided

$$\frac{\bar{B}_\theta}{B_o} < \left(\frac{\lambda}{4a}\right) \left(\frac{\gamma}{\eta}\right)^{3/2} \left(\frac{\omega}{\nu}\right)^2 \qquad (3)$$

the threshold is not attained and this instability is inhibited. $\bar{B}_\theta$ is the r.m.s. value of the oscillating field evaluated at the plasma surface, B_o is the main axial field, a is the plasma radius, b is the radius of the conducting shell, λ is the smaller of the ion mean free path and the plasma radius, ω is the angular frequency of the oscillating field, ν is the sound transit frequency across the plasma radius, λ is the ratio of specific heats and η is a geometrical factor given by

$$\eta = \frac{1 + \left(a/b\right)^2}{1 - \left(a/b\right)^2}$$

This stability theory was developed for a field free, uniform plasma column. However, θ-pinches produced in the laboratory are neither uniform nor field free (i.e. $\beta \neq 1$). Consequently, they possess stable continuous hydromagnetic spectra and it is believed that the inclusion of the effects of these spectra in the theory would lead to a less stringent stability criterion than (3)[4].

2.3 Secondary wall-breakdown: The use of limiters

It has been observed[3,5] in a number of $\tilde{I}_z$ stabilization experiments that after an initial brief period (the first half cycle or so) during which the distribution of B_θ outside the dense plasma column has a $^1/_r$ dependence, the axial current switches to the vicinity of the discharge tube wall and remains there. In general, a pinched plasma column is surrounded by a low-density residual plasma which arises from a number of causes, amongst them being the ionization of neutral gas which is not collected during the initial implosion of the pinch, the ionization of adsorbed gas liberated from the discharge tube wall and the ionization of wall material. It is in this tenuous plasma that the applied alternating axial current passes after the initial half cycle. In itself, the fact that the high frequency B_θ field is confined to a skin layer extending inwards from the wall does not invalidate dynamic stabilization schemes. The drawback comes from the observation[5,6] of very large 'anomalous' resistivities in the dilute plasma close to the wall. As a consequence, high frequency power dissipation is excessive and it appears questionable whether such schemes can be applied to the confinement of a thermonuclear plasma.

In a number of experiments[7-9], limiters (diaphragms) have been regularly spaced along the discharge tube with the expressly stated purpose of restricting the diameter of the plasma column. However, it so happens that these limiters also play a role in determining the spatial distribution of the high-frequency axial current. In at least two reported experiments[7,10,11], the use of limiters has clearly allowed the alternating current to flow on the pinched plasma column. The influence of limiters is particularly well illustrated in reference [11] where experimentally measured B_θ magnetic field profiles obtained with and without limiters are compared.

A theoretical analysis has been published[12] which shows that insulating limiters suppress the screening effect of the residual plasma by exciting stationary torsional hydromagnetic waves in the cavities formed by adjacent limiters. The essential conclusion of this analysis is that if the square of a non-dimensional quantity,

$$\varepsilon = \frac{\omega L}{V_A}$$

(where ω is the angular frequency of the axial current, 2L is the spacing between adjacent limiters and V_A is the Alfven speed in the tenuous plasma existing between the dense plasma column and the discharge tube wall) is much less than unity, then most of the oscillating axial current will flow on the central plasma column. One should also expect an accompanying reduction in the high frequency energy losses; however, as yet, this aspect has not been investigated experimentally.

3. A proposed experiment

In this section we consider a specific experimental apparatus in which the dynamic equilibrium of a curved plasma column can be investigated, and we estimate the r.f. power needed in the experiment.

Consider a semi-circular θ-pinch apparatus having the following parameters:-

Major radius, $R = 0.5$ m

Minor radius, $b = 0.03$ m

Length of discharge tube ($= \pi R$) = 1.57 m

B_o (maximum axial magnetic field) = 2.0 Tesla

Suppose that in this apparatus we generate a deuterium θ-pinch plasma having the following properties:

Electron density, $n_e = 10^{22}\ m^{-3}$

Electron temperature, $T_e = 100$ eV

$\beta = 1$

and suppose that we wish to achieve a dynamic toroidal equilibrium of this plasma column with $\Delta/b \leqslant 0.2$. In the absence of experimental results concerning the r.f. energy losses which occur when the r.f. current passes entirely on a pinched plasma column, we will assume that the electrical conductivity is classical and, for the purposes of this present exercise, take a value of $\sigma = 10^6$ mho/m.

The condition for toroidal equilibrium (equation (1)) demands that:

$$\frac{\bar{B}_\theta}{B_o} \geqslant \sqrt{\left(\frac{b}{2R}\right) \beta \frac{1}{\left(\frac{\Delta}{b}\right)}}$$

$$\text{i.e.} \quad \frac{\bar{B}_\theta}{B_o} \geqslant 0.39 \qquad (4)$$

This last inequality, coupled with stability criterion (2), requires that the compression ratio, a/b, be at least 0.27. We note that less restrictive conditions on β and Δ/b than those assumed here would lead to a lower required value of $\bar{B}_\theta/B_o$ and, consequently, a lower allowable value for the compression ratio.

Suppose that the radius of the plasma column is 0.009 m (i.e. $a/b = 0.3$).

Substitution of the experimental parameters into (3) leads to the following inequality which must be satisfied if stability is to be ensured:

$$\frac{\bar{B}_\theta}{B_o} < 0.19 \times 10^{-12} f^2 \qquad (5)$$

If we choose the smallest possible value of $\bar{B}_\theta/B_o$ consistent with (4) (i.e. $\bar{B}_\theta/B_o = 0.39$), then (5) requires that $f > 1.4$ MHz (cf. sound transit frequency, $\nu = 9.2$ MHz).

It is now possible to estimate the r.f. power requirement of this experiment. For f = 1.4 MHz, $\delta/a \approx 5\%$, where δ is the classical skin depth. We may therefore estimate the total plasma resistance, r_p, by using the expression for surface resistivity developed for a semi-infinite plane conductor:

$$r_p = \frac{\pi R}{2\pi a}\sqrt{\frac{\pi f \mu_o}{\sigma}}$$

$$\text{i.e.} \quad r_p = 66 \text{ m}\Omega$$

The choice of $\bar{B}_\theta/B_o = 0.39$ and a = 0.009 m results in:

$$\bar{I}_z = 34.8 \text{ kA}$$

and it follows that the r.f. power requirement of this experiment is:

$$P = 80 \text{ MW}$$

Whilst this requirement is high, it must be remembered that one only needs a small number of cycles (say, 10 - 15) of r.f. current of this frequency and amplitude in order to investigate the principles of dynamic equilibrium. Such bursts of r.f. energy at this power level are readily available using r.f. line generator techniques[13-16].

As mentioned earlier, the amplitude of oscillation of the plasma column about its mean equilibrium position could reasonably be expected to be small if $^{\tau}/_{4}$ (where τ is the period of the oscillating current) is some small fraction of the time taken by the plasma to drift to the wall in the absence of $\tilde{I}_z$. For the specific experimental conditions considered here, the time to drift to the wall in the absence of $\tilde{I}_z$ is 1.13 μsec[17]. Since $^{\tau}/_{4}$ = 0.18μ sec, the plasma oscillation should be negligible.

The discharge tube will have to be equipped with thin, insulating limiters in order to ensure that the $\tilde{I}_z$ current passes entirely on the pinched plasma column. If we assume that the residual plasma existing between the discharge tube wall and the central plasma column has $n_e = 10^{20}\ m^{-3}$, then

$$\varepsilon = 2.89\ L$$

Now a value of $\varepsilon = 0.3$ ensures that very nearly all of $\tilde{I}_z$ flows on the central plasma column[12]. Hence the spacing between adjacent limiters (= 2L) should be about 20 cms. Electrodes placed in the end limiters could be used to feed the oscillating current into the plasma column.

The aims of the experiment would be fourfold:

(i) To investigate whether or not the presence of thin insulating limiters inhibits the formation of a hot, dense θ-pinch plasma in the first place.

(ii) To examine the influence of the limiters on the radial distribution of $\tilde{I}_z$ and to determine <u>experimentally</u> the maximum acceptable spacing between adjacent limiters.

(iii) To make an accurate measurement of the r.f. energy losses since it is this quantity which eventually determines the economics of the scheme.

(iv) To investigate the dynamic, toroidal equilibrium of a curved θ-pinch.

We note that studies (i) - (iii) could be carried out in a linear discharge tube; this would ease considerably the problems of construction and plasma diagnostics.

The parameters of this proposed experiment are summarized in Fig. 2a and 2b.

4. A comparison of racetrack and fully toroidal configurations

If all the experimental parameters except the value of R and the length of the discharge tube (which we put equal to πR) are kept constant, then the following scaling laws apply:-

$$f \propto R^{-\frac{1}{4}}$$

$$\bar{I}_z \propto R^{-\frac{1}{2}}$$

$$P \propto R^{-1/8}$$

The method described in this report to obtain a dynamic, toroidal equilibrium of a curved plasma column can equally well be applied to a fully toroidal configuration. It is of interest to compare the power requirements of a racetrack and fully toroidal θ-pinch for the case where one has the same total length of plasma. Let:-

R_T be the major radius of the fully toroidal θ-pinch

R_R be the radius of the curved end sections of the racetrack θ-pinch

ℓ be the length of one of the linear sections of the racetrack θ-pinch

P_T be the r.f. power required for the fully toroidal θ-pinch

P_R be the r.f. power required for the racetrack θ-pinch.

If we consider equal lengths of plasma, then

$$R_T = \frac{\ell}{\pi} + R_R$$

Now,

$$\frac{P_T}{P_R} = \left(\frac{R_R}{R_T}\right)^{1/8}$$

$$= \left[\frac{1}{1 + \left(\frac{\ell}{\pi R_R}\right)}\right]^{1/8}$$

The power requirements of a racetrack apparatus are therefore always greater than those of a fully toroidal apparatus. However, this disadvantage is probably compensated for by the fact that the racetrack apparatus takes up less room, requires far fewer limiters and can have $\tilde{I}_z$ fed directly into the plasma column rather than being induced as would be the case in a completely toroidal apparatus.

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CAPTIONS

Fig. 1. The racetrack θ-pinch configuration.

Fig. 2a, b The linear and semi-circular versions of the proposed experiment.

$a = 0.009$m	$b = 0.03$ m
n_e (dense plasma) $= 10^{22}$ m^{-3}	$R = 0.05$ m
n_e (tenuous plasma) $= 10^{20}$ m^{-3}	$2L \geqslant 0.2$ m
$T_e = 100$ eV	$B_o = 2.0$ Tesla
$\beta = 1$	$\bar{I}_z \approx 35$ kA
$\Delta = 0.006$ m	$f \geqslant 1.4$ MHz

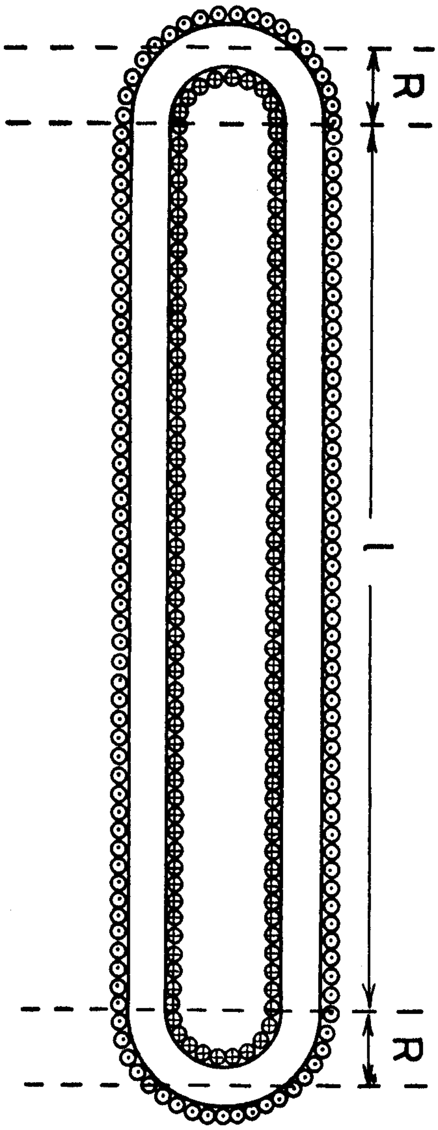

FIG. 1

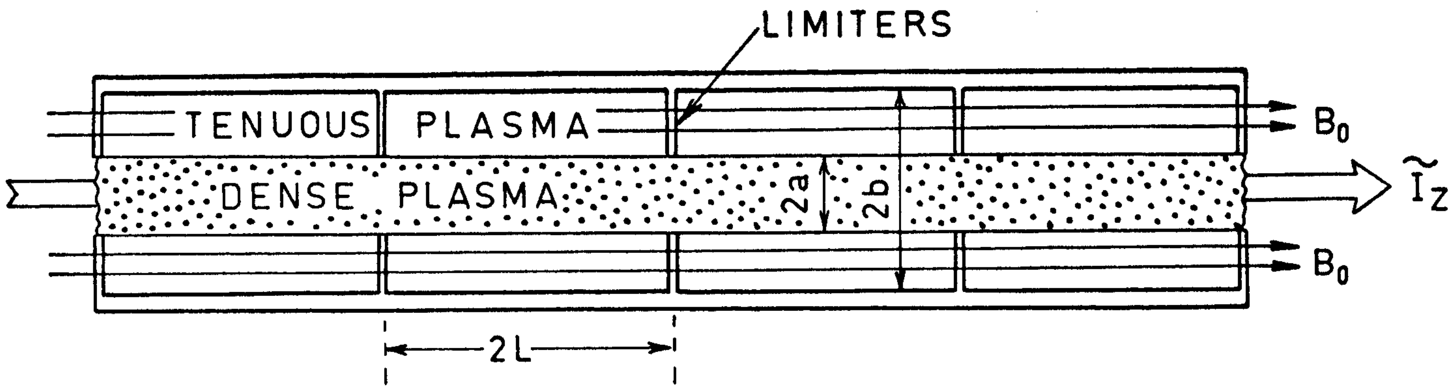
LIMITERS
TENUOUS PLASMA
DENSE PLASMA
2a
2b
B_0
B_0
$\tilde{I}_Z$
2L

Fig. 2a

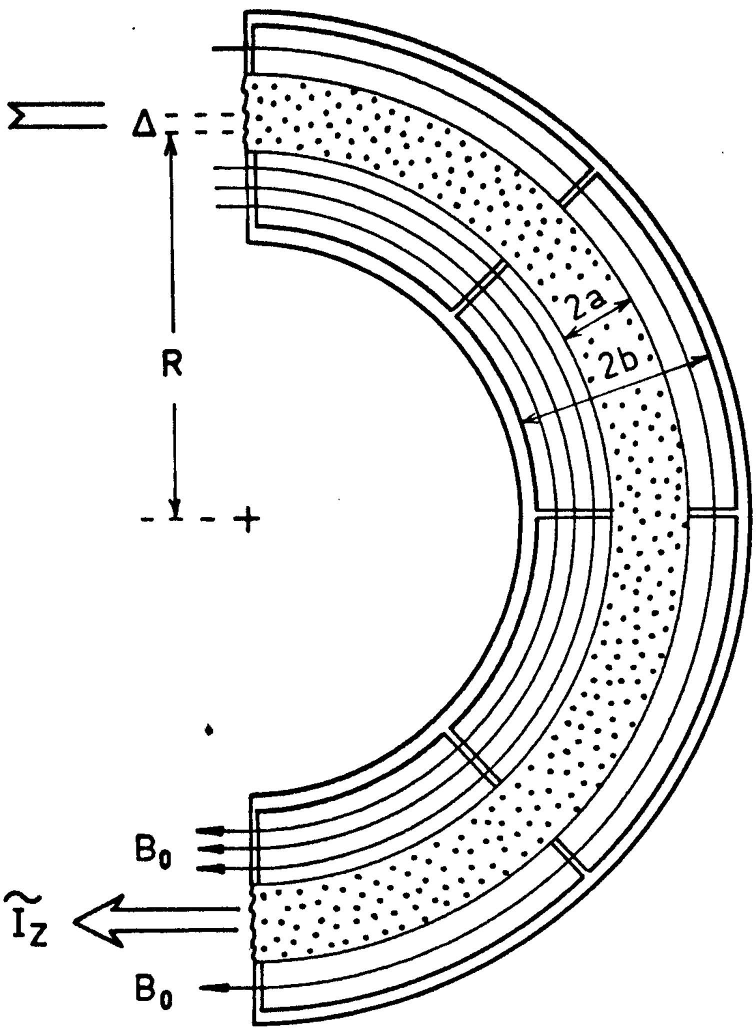
Δ
R
2a
2b
B_0
$\tilde{I}_Z$
B_0

Fig. 2b