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RESONANT GROWTH OF ALFVEN WAVES EXCITED IN THE CONTINUOUS SPECTRUM

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ABSTRACT

The transient behaviour of the build up of Alfvèn waves in the continuous spectrum caused by the density and current profile of a plasma column is calculated. It is shown that the amplitude pattern of the azimuthal eigenfunction is proportional to the momentary spectrum of the exciting force, until saturation occurs. The thickness of the resonant layer and the maximum amplitude, limited by ohmic and viscous dissipation, are deduced for a typical tokamak case. Heating time and power input are also considered. Ideal MHD theory and rough approximation are used in order to give good insight with a simple analytic scheme.

1. INTRODUCTION

In a nonuniform plasma column the axial speed of shear Alfvèn waves depends on particle and current density. When the plasma is excited with a perturbation at given wave number and frequency, the cylindrical surface having the same eigenfrequency resonates and forms a singular surface. Near this surface the plasma executes a beating motion. The radial eigenfunction tends to a logarithmic singularity and the azimuthal component tends to a hyperbolic singularity. Kinetic energy accumulates and resonant absorption sets in when the amplitude reaches a certain limit. Heating by this phenomenon is very efficient, and one advantage lays in the fact that the energy is released in the interior.

The continuous Alfvèn spectrum has been computed numerically by normal mode analysis /1/ and represented in a particular case for the m = 1 mode /2/. Normal mode analysis /3-11/ and Laplace transform technique /12-14/ has been applied on ideal MHD plasma. Kinetic theory leads essentially to the same absorption /15-17/.

Our aim is to calculate the energy accumulation process of torsional Alfvèn waves in a diffuse density profile. The equation of motion is deduced from ideal MHD and applied on the m = 1 case in the approximation for great wavelength. By perturbation method we are separating the kink motion which turns out to be a source term for the local Alfvèn waves. The phenomenon is considered as an initial value problem and is solved by Laplace transform. The amplitude saturation occurs when ohmic and viscous dissipation reaches the injected energy flux. Simple analytical results are obtained and applied to a typical tokamak case.

2. SOLUTION OF THE EQUATION OF MOTION

The basic equations of the ideal MHD theory are reducible to a second order differential equation for the radial motion /18-20/. We consider only the m = 1 mode in the long wavelength approximation $k^2r^2 \ll 1$. The equation may be put in the following form:

$$r\left(rA\xi_r'\right)' + 2rA\xi_r' - \left(\rho\omega^2 - k^2B_z^2/\mu_0\right)' r\xi_r = 0 \tag{1}$$

ξ_r is the radial component of the eigenmode

$$\xi = \left(\xi_r, \xi_\theta, \xi_z\right)\exp i\left(\theta + kz - \omega t\right) \tag{2}$$

The prime denotes partial differentiation with respect to r. A means the Alfvèn determinant

$$A = -\rho\omega^2 + k^2B_z^2(1-\nu)^2/\mu_0 \tag{3}$$

The density ρ and the axial magnetic field B_z are functions of r, and ν which contains the axial equilibrium current j is also a function of r. The definition of ν is

$$\nu = -B_\theta/rkB_z \tag{4}$$

We specify parabolic density and current profiles

$$\rho = \rho_0\left(1 - \varkappa r^2/a^2\right) \tag{5}$$

$$j = j_0\left(1 - \lambda r^2/a^2\right) \tag{6}$$

$\varkappa$ and λ are constant coefficients. The azimuthal field and ν become

$$B_\theta = \frac{\mu_0}{r}\int_0^r j\,r\,dr = \frac{1}{2}\mu_0 j_0 r\left(1 - \lambda r^2/2a^2\right) \tag{7}$$

$$\nu = \nu_0\left(1 - \lambda r^2/2a^2\right) \quad \text{with} \quad \nu_0 = \frac{-j_0}{2kB_z} \tag{8}$$

The differential equation (1) possesses a singularity at the point $A(r = r_1) = 0$. This means that a wave can propagate with the local Alfvèn speed ω/k given by

$$\frac{\omega^2}{k^2} = \frac{B_z^2}{\mu_0 \rho_1}(1-\nu_1)^2 \tag{9}$$

If ρ and ν are monotonic functions of r the Alfvèn spectrum spreads out between the following limits

$$\omega_0^2 = \frac{k^2 B_{z0}^2}{\mu_0 \rho_0}(1-\nu_0)^2 \quad \text{and} \quad \omega_a^2 = \frac{k^2 B_{za}^2}{\mu_0 \rho_a}(1-\nu_a)^2 \tag{10}$$

where the indices 0 and a indicate the values at r = 0 and r = a. It is further legitimate to neglect the last term of equation (1) when β is small. From the pressure balance and the relation (9) the ratio of the last two terms is

$$\frac{k^2 (B_z^2)'}{\mu_0 \omega^2 \rho'} = - \frac{\beta}{(1-\beta)(1-\nu)^2} \tag{11}$$

This value is effectively small, expect in a region near $\nu = 1$ which is always avoided in practice.

We are considering the phenomenon as an initial value problem and will use the Laplace transform method for solving equation (1). By writing $\omega = i\,\partial/\partial t = is$, where s is the Laplace variable, the transformed equation becomes

$$r\left(r\hat{A}\,\hat{\xi}_r'\right)' + 2r\hat{A}\,\hat{\xi}_r' + \rho' s^2 r\,\hat{\xi}_r = 0 \tag{12}$$

The circumflex indicates the Laplace transform.

The initial value of ξ_r is the eigenfunction of the kink, as we will see. We replace then ξ_r by $\xi_k + \xi_r$ where ξ_r holds now for the perturbation of the eigenfunction. Following perturbation theory, we neglect the perturbation itself in the last term of the equation, and we are able to write

$$(r\hat{A}\,\hat{\xi}_r')' + \frac{2}{r}(r\hat{A}\,\hat{\xi}_r') \cong -\rho' s^2 \hat{\xi}_k \tag{13}$$

The integration of this first order differential equation is immediately fulfilled if ξ_k is known. In the second member $s^2\hat{\xi}_k$ is the initial acceleration, and multiplied by ρ' it forms the driving force. Therefore a density gradient must exist for exciting the Alfvèn waves.

The kink eigenfunction may be obtained by the following argument: experimentally the plasma is set in motion by a helical winding having an m = 1 configuration and a wavelength $2\pi/k$. The sinusoidal driving current J $\sin\omega t$ is switched on at time t = 0. This corresponds to an infinite value of the Laplace variable. Setting $s \to \infty$ equation (12) yields

$$\frac{\partial^2}{\partial t^2}\left[(r\rho\,\xi_r')' + 2\rho\,\xi_r' + \rho'\xi_r\right] \cong 0 \tag{14}$$

It is easy to verify that the same equation is a consequence of the incompressibility condition $\mathrm{div}\,\xi = 0$, and of the equation of Newton $\rho\ddot{\xi} = -$ grad p (by eliminating p and ξ_θ, and knowing that ξ_z is negligeable for long wavelengths). Thus the non singular solution of (14) gives the initial eigenfunction of the kink. By introducing the profile (5) we find the expansion

$$\ddot{\xi}_r \sim 1 + \varkappa r^2/4a^2 + \cdots \tag{15}$$

We see that the initial acceleration is weekly dependent on r. That allows us to assume a rectangular ξ_k profil without loosing much in precision:

$$\xi_k = \xi_o \sin\omega t \tag{16}$$

or

$$\hat{\xi}_k = \xi_0 \frac{\omega}{s^2 + \omega^2} \tag{17}$$

The first integration of (13) can now be fulfilled

$$\hat{\xi}_r' = \frac{\xi_0 H \omega s^2}{r \hat{A} (s^2 + \omega^2)} \tag{18}$$

where

$$H = -\frac{1}{r^2} \int_0^r \rho' r^2 dr \tag{19}$$

By identifying the transformed Alfvèn determinant

$$\hat{A} = \rho s^2 + k^2 B_z^2 (1-\nu)^2 / \mu_0 \tag{20}$$

with

$$\hat{A} = \rho (s^2 + \omega_r^2) \tag{21}$$

and using formula (9) we define a frequency

$$\omega_r = \omega \frac{B_z}{B_{z1}} \sqrt{\frac{\rho_1}{\rho}} \cdot \frac{1-\nu}{1-\nu_1} \tag{22}$$

which is the Alfvèn frequency at radius r. The expression (18) becomes

$$\hat{\xi}_r' = \frac{\xi_0 H \omega s^2}{r \rho (s^2 + \omega^2)(s^2 + \omega_r^2)} \tag{23}$$

The obtained solution behaves like an oscillator with eigenfrequency ω_r excited by a force proportional to $(\xi_0 H/r\rho)\sin\omega t$. When $\omega_r = \omega$ the amplitude grows linearly in time, and a singular surface sets up at radius r_1 after a very long time. The plasma column is comparable to a juxtaposition of independent oscillators laying between r_i and $r_i + dr$ and resonating at $\omega_r(r_i)$. Their displacements correspond to ξ_r'. A well known mechanical analog behaves the same way: the frequency indicator for motor generators consisting of vibrating lamellas.

We have still to execute the inverse transform of (23):

$$\xi_r' = \frac{\xi_0 H}{r\rho(1-\omega_r^2/\omega^2)}\left[\sin\omega t - \frac{\omega_r}{\omega}\sin\omega_r t\right] \tag{24}$$

It is useful to expand this result for a small frequency shift $\Delta\omega = \omega_r - \omega \ll \omega$:

$$\xi_r' \cong \frac{-\xi_0 H\omega}{2r\rho\Delta\omega}\left[\sin\omega t(1-\cos\Delta\omega t) - \cos\omega t\sin\Delta\omega t\right] \tag{25}$$

where

$$\Delta\omega = \omega_r'(r-r_1) \quad and \quad \omega_r' = \omega\left[\frac{B_z'}{B_z} - \frac{\rho'}{2\rho} - \frac{\nu'}{1-\nu}\right] \tag{26}$$

The rms value of (25) is given by

$$\langle \xi_r' \rangle = \frac{\xi_0 H\omega}{\sqrt{2}\, r\rho\Delta\omega}\sin(\Delta\omega t/2) \tag{27}$$

The function (25) represents a beating motion with the nodes travelling towards the singular surface. The depth of the singular layer may be defined as follows: it is the spacing of the two **nodes** closest to the surface

$$\Delta r = \frac{4\pi}{|\omega_r'|\, t} \tag{28}$$

We remark, in addition to this, that the Fourier spectrum $g(\omega_r)$ of a wave train $\sin\omega t$ with duration t is

$$g(\omega_r) = \frac{\sin(\omega_r-\omega)t/2}{\omega_r-\omega} \tag{29}$$

The above function has the same form as (27), except for a slowly varying geometrical factor. This shows in another manner that the plasma responds like a Fourier analyser.

In the figure 1 we represent the normalized function $a\xi'_r/\xi_0$ calculated with formula (24). The chosen parameters are $\varkappa = \lambda = \frac{1}{2}$ and $\nu_a = -0.75$, and the singular surface is placed at radius $r_1 = a/\sqrt{2}$. We show an eighth period sequence going from $\omega t = 20\pi$ to 20.75π, and on the same scale, a sequence going from 100π to 100.75π.

It is not necessary to know the eigenfunction ξ_r, but we calculated it for interest. The figure 2 shows $\langle\xi_r\rangle/\xi_0$ at the 50th period.

3. ENERGY ABSORPTION AND SATURATION

When the depth of the singular surface gets smaller than the plasma radius, the azimuthal speed exceeds the radial speed, and therefore the kinetic energy is mainly localized in the azimuthal motion. Thus we only need to know ξ_θ for calculating the accumulated energy. By solving the system of equations (57) in Gruber's thesis /19/ we find

$$\xi_\theta = i\left(\xi_r + r\xi'_r\right) \cong ir\xi'_r \tag{30}$$

The energy density $\frac{1}{2}\rho\omega^2 r^2 \langle\xi'_r\rangle^2$ can easily be integrated over the volume, because the expression (27) contributes mainly in the region near r_1. The accumulated kinetic energy per unit length becomes

$$E \cong \frac{\pi^2 r_1 \xi_0^2 H^2 \omega^4 t}{4\rho\,|\omega'_r|} \tag{31}$$

The energy increases proportionally in time, so the absorbed power is constant. From the outside the plasma appears like a constant ohmic resistor.

As H contains ρ' at the first power, and as the term containing ρ' in (26) is dominant, the absorbed power is nearly proportional to the density gradient.

(31) may be written in a more explicit form by introducing (5), (9) and (26), (B_z' is negligeable):

$$W = \frac{k_o^2 \omega B_z^2 V \xi_o^2}{2\mu_o q_o^2} C_2 \tag{32}$$

Here W is the absorbed power, V is the plasma volume and the new quantities k_0 and q_0 are defined below. C_2 is a geometrical factor:

$$C_2 = \frac{\pi \varkappa (r_1/a)^4 q^2 (1-\nu_1)^2}{8(1-\varkappa r_1^2/a^2)\, C_1}$$

$$C_1 = 1 - \frac{\nu' a^2 (1-\varkappa r_1^2/a^2)}{(1-\nu_1)\varkappa r_1} \tag{33}$$

The absorbed power is strongly dependent on the position r_1 of the singular layer. The formula (32) is only valid when $a - r_1$ is greater than the depth (28).

We have the following useful relation

$$q k_o = q_o k \tag{34}$$

q_0 = safety factor for a tokamak

q = $1/\nu_a$ per definition (see (8))

k_0 = $-1/R$ where R is the major radius of the tokamak

k = wave number of the exciting coil

From (8) we obtain:

$$k\nu_1 = -\frac{1-\lambda r_1^2/2a^2}{(1-\lambda/2)\,q_0 R} \tag{35}$$

The saturation of the accumulated kinetic energy occurs when the dissipation level caused by resistivity and viscosity reaches the power input. (Other energy conversion processes are not considered). The heating thus begins after a certain accumulation time which we shall calculate.

Concerning the ohmic dissipation, we take the Spitzer resistivity $\eta = m_e/0.743\ ne^2\tau_e$. The axial component j_z of the current is dominant near the singular layer. From Gruber's thesis we find

$$\mu_0 j_z = -(1-\nu)k\left(3\,\xi_r' + r\,\xi_r''\right)B_z \cong -(1-\nu)kr\,\xi_r''\,B_z \tag{36}$$

Let ε_1 be the thermalized energy density

$$\frac{\partial\varepsilon_1}{\partial t} = \eta\,\dot{j}_z^2 \tag{37}$$

By introducing (9) the ohmic dissipation per unit volume is

$$\frac{\partial\varepsilon_1}{\partial t} \cong \frac{\eta\rho}{\mu_0}\left(\omega r\,\xi_r''\right)^2 \tag{38}$$

Further, the viscous dissipation /21/ is determined by

$$\frac{\partial\varepsilon_2}{\partial t} \cong \mu_3\left(\frac{\partial v_\theta}{\partial r}\right)^2 \tag{39}$$

where $\mu_3 = 0.513\ nKT\,\tau_i(1+\Omega_i^2\tau_i^2)^{-1}$ is the transversal coefficient of viscosity, and $\Omega_i\,\tau_i$ is the product of the ion gyrofrequency and the collision time. From (30) the azimuthal speed is $|v_\theta| = \omega r\,\xi_r'$, and we can write

$$\frac{\partial\varepsilon_2}{\partial t} \cong \mu_3\left(\omega r\,\xi_r''\right)^2 \tag{40}$$

The expressions (38) and (40) are similar. Their sum can be put in correlation with two diffusion coefficients

$$\frac{\partial \mathcal{E}}{\partial t} \cong (D_1 + D_2)\rho(\omega r \xi_r'')^2 \tag{41}$$

the classical ohmic diffusion coefficient

$$D_1 = \frac{\eta}{\mu_0} \tag{42}$$

and viscous diffusion coefficient

$$D_2 = \frac{\mu_3}{\rho} \tag{43}$$

Recalling (25) and taking the rms value, we obtain

$$\frac{\partial\langle\mathcal{E}\rangle}{\partial t} = (D_1+D_2)\rho\left[\frac{\xi_0 H\omega^2}{2\rho\omega_r'}\right]^2\left[1-\cos\alpha x+\alpha^2x^2/2-\alpha x\sin\alpha x\right]x^{-4} \tag{44}$$

with $\alpha = \omega_r' t$ and $x = r - r_1$. For a narrow singular layer the integration over the volume becomes elementary, and the dissipation per unit length yields

$$\frac{\partial E}{\partial t} = \frac{\pi^2}{6\rho}(D_1+D_2)\, r_1\, \xi_0^2 H^2\omega^4|\omega_r'|\, t^3 \tag{45}$$

The accumulation time t_a is now defined as the time obtained by equating (45) with the input power E/t:

$$(\omega t_a)^3 = \frac{3\omega}{2(D_1+D_2)}\left[\frac{B_z'}{B_z} - \frac{\rho'}{2\rho} - \frac{\nu'}{1-\nu}\right]^{-2} \tag{46}$$

The depth of the singular layer is given by the expression (28) at $t = t_a$

$$\Delta r = 4\pi(2/3)^{1/3}\left(\frac{D_1+D_2}{\omega}\right)^{1/3}\left[\frac{B_z'}{B_z} - \frac{\rho'}{2\rho} - \frac{\nu'}{1-\nu}\right]^{-1/3} \tag{47}$$

The two expressions above are similar to the results cited by J. Kappraff, J.A. Tataronis and W. Grossmann /16/. The term containing ν' arises from the fact that we treated the m = 1 mode instead of the zero mode. The quotient of the diffusion coefficients is

$$\frac{D_1}{D_2} = \frac{0.087}{\beta}\left(1+\frac{T_i}{T_e}\right)\left(\frac{T_i}{T_e}\right)^{1/2} \tag{48}$$

In the case of a low β tokamak thermalization by ohmic dissipation is dominant. (The nonlinear processes may modify the situation).

4. NUMERICAL RESULTS

We quote the results for a tokamak having the following typical data:

major radius	R	=	1 m			
minor radius	a	=	0.2 m			
magnetic field	B_z	=	2 Vsec/m^2			
temperatures	KT_e	=	1000 eV	KT_i	=	500 eV
axial density	n_0	=	2.10^{19} m^{-3}			
axial β (computed)	β	=	3.10^{-3}			
safety factor	q_0	=	3			

The same parabolic profile is assumed for the density and the current: $\varkappa = \lambda = \frac{1}{2}$. The excitation frequency is chosen in order to resonate at $r_1 = a/\sqrt{2}$. The excitation coil has a periodic structure around the minor and the major circumference, expressed by the coordinates θ and z. Its surface current is given by

$$j_e \cos(\theta + kz) \sin\omega t$$

If the coil is formed by N turns wound at a radial distance r_e and fed be a current J, the transverse field near the axis is:

$$B_e = \frac{\mu_o}{2} \dot{\gamma}_e \sin\omega t = \frac{\mu_o N J}{4 r_e} \sin\omega t$$

Shielding effects, helicity and toricity are neglected. We take a positive wave number k = 1/R, that means the coil and the field have opposite helicity. In this case $q = - q_0$.

For the calculation of the heating power (32) we assume a kink amplitude $\xi_0 = 10^{-3}$ m. We deduce the related current producing this amplitude by means of the dispersion relation with second member

$$\left[\left(\frac{q_o \omega}{k_o v_A}\right)^2 - 2q(q-1)\right]\xi_o \cong \frac{\mu_o \dot{\gamma}_e q_o (1-q)}{B_z k_o}$$

This equation is valid for great aspect ratio, and we assume steady state. (The damping should be included, but can be neglected in our example). v_A is a velocity defined by $v_A^2 = B_z^2/\mu_0 \bar{\rho}$. From (34) we have q = - 3 and $q_0/k_0 = - 3$ R. The current-amplitude relation becomes

$$NJ = 5 \cdot 10^5 \; \xi_0 = 500 \text{ Amp.turns}$$

for a coil with radius $r_e = \sqrt{2}\, a$. The resonance term, in brackets in the dispersion relation, provides a favorable enhancement effect of the kink amplitude.

The angular frequency and the heating power

$$\omega = 11 \cdot 10^{-6} \text{ sec}^{-1} \qquad W = 2.3 \cdot 10^6 \text{ Watts}$$

are within the existing RF technology.

It is useful to know the reactive energy E_i of the circuit. Estimating the coil inductance to be $L \cong 2\mu_0 RN^2$ we obtain

$$E_i = 0.31 \text{ Joule}$$

The low quality factor Q of the circuit

$$Q = \frac{\omega E_i}{W} = 1.5$$

reveals very strong coupling to the plasma.

For the accumulation time and the depth of the resonant layer we find

$$t_a = 55 \cdot 10^{-6} \text{ sec} \qquad \Delta r = 12 \cdot 10^{-3} \text{ m}$$

Finally, the heating period t_2 necessary for the injected energy to reach the thermal energy is

$$t_2 = 1.8 \cdot 10^{-3} \text{ sec}$$

Here are some other important quantities:

- field at the plasma surface = 11 Gauss
- maximum field in the resonant layer: $b_\theta \cong -(1-\nu)\, kr\, \xi'_r\, B_z = 1400$ Gauss
- enhancement of the field $= 0.21\, \omega t_a = 127$
- maximum azimuthal speed in the resonant layer: $v_\theta = 0.083\, \omega t_a\, \omega \xi_0 = 0.55 \cdot 10^6$ m/sec
- sound speed $c_s = 0.38 \cdot 10^6$ m/sec.

5. CONCLUSION

We present a simple analytical calculation of the energy accumulation mechanism of shear Alfvèn waves in a plasma column having a diffuse density and current profile. The equation of motion is obtained from the ideal MHD theory and formulated in the long wavelength approximation $k^2r^2 \ll 1$. By a perturbation method it is possible to separate the movement of the kink and to consider it as a source term for the Alfvèn waves. The equation of motion is handled as an initial value problem and solved by means of the Laplace transformation /14/.

The kinetic energy of the Alfvèn wave is carried essentially by the azimuthal speed, which increases linearly in time. The depth of the resonant layer varies inversely with time, and consequently the total energy increases linearly.

Assuming thermalization by ohmic and viscous dissipation, we obtain saturation of the energy after an accumulation time of the order of 50 μsec for typical tokamak parameters. With a helical coil having one wavelength in the toroidal direction, a current of 500 Ampere-turns gives rise to a plasma amplitude of 1 mm at a frequency below 2 MHz. The power input is 2 MW. It is favorable to wind the coil in the opposite direction to the field lines, because the driving force is greater, and the frequency can be set closer to the eigenfrequency of the kink. So the coupling to the plasma is improved, but exact resonance should be avoided for reason of stability.

In the calculated example the azimuthal speed exceeds the sound speed. Furthermore the calculated electron drift speed in the resonant layer is much greater than the ion thermal speed. It is to expect that nonlinear processes and turbulence reduce the accumulation time. In order to avoid a dangerous energy concentration, the frequency must be swept over the Alfvèn continuum /10/.

The periodicity of the excitation coil fixes the working point in the diagram of the dispersion relations /2/. It is important that at this point the Alfvèn frequency increases with radius. The reason is the following: from Vlasov theory and taking into account finite Larmor radius effects, A. Hasegawa and L. Chen /7/ demonstrate the creation of a new modified Alfvèn wave by mode conversion in the resonant layer. This wave propagates in the direction of decreasing Alfvèn frequency, thus, in our case, towards the interior. Otherwise the wave would be partly reflected and would heat the outer part of the plasma column.

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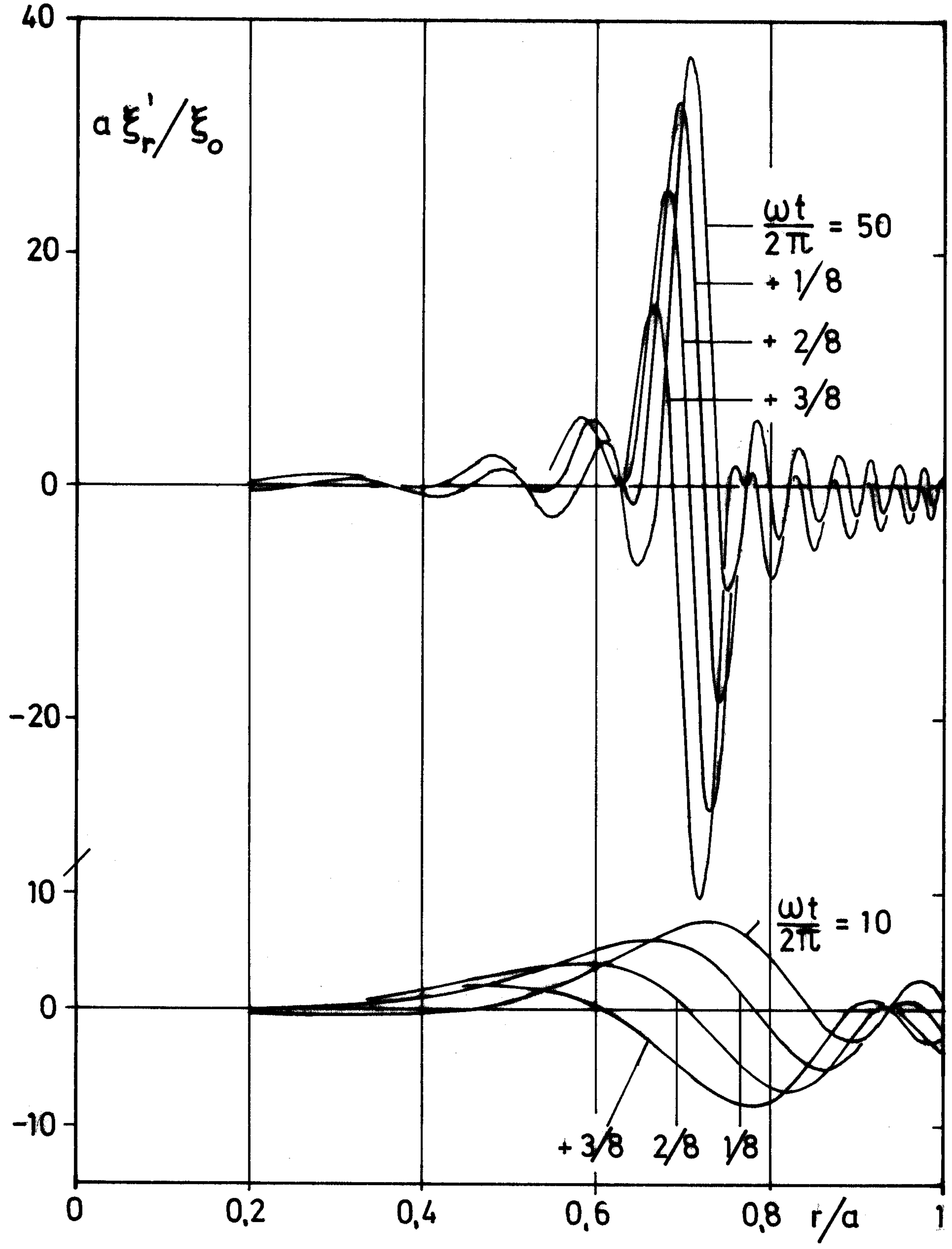

Figure 1: Formation of the singular layer.

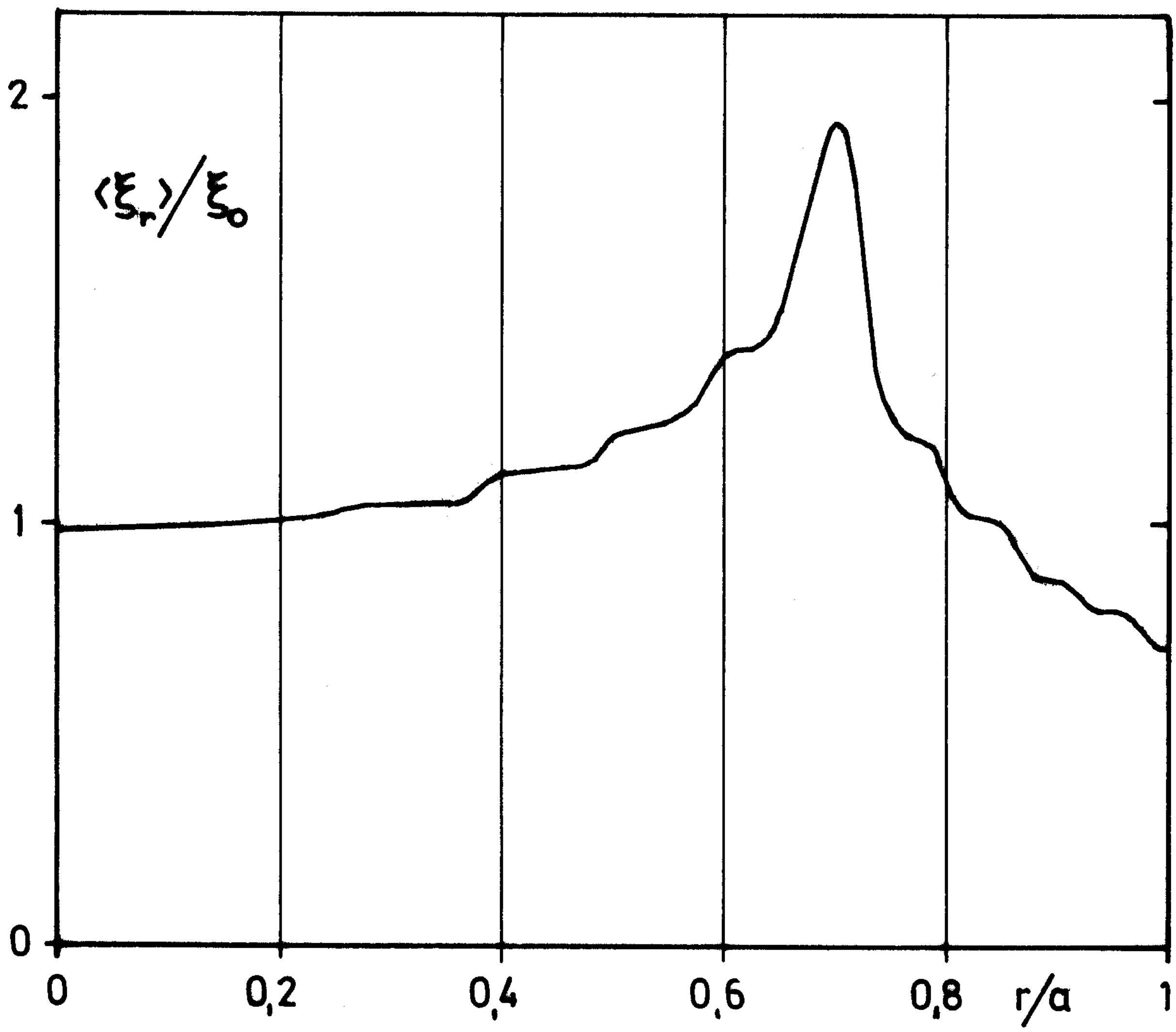

Figure 2: rms. value of the eigenfunction at the 50th period.