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KINETIC MODULATIONAL INSTABILITY OF LANGMUIR TURBULENCE

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ABSTRACT

The envelope modulational instability of a spectrum of random-phase Langmuir waves is examined in the regime where the group velocities are comparable to the ion thermal speed such that the ion kinetic effects are important.

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Recently, there has been interest in the (envelope) modulational instabilities of either coherent or random-phase Langmuir waves[1]. The nonlinear development of such instabilities may lead to the collapse of Langmuir waves and the formation of density cavities[2]. Previous studies on the instabilities have been limited to either the quasi-static ($k_0^2 \lambda_{De}^2 < m_e/m_i$) or the hydrodynamic ($k_0^2 \lambda_{De}^2 > m_e/m_i$) regimes so that ion kinetic effects can be ignored and fluid descriptions are sufficient[1,2]. Here, k_0 is a typical wave number of the Langmuir waves. In this work, we examine the transitional regime ($k_0^2 \lambda_{De}^2 \sim m_e/m_i$), where the Langmuir wave group velocity $u_0 = 3\,(k_0 \lambda_{De}) v_e$ becomes comparable to the ion thermal velocity v_i for plasmas with $T_e \simeq T_i$ and, therefore, one may expect the kinetic effects due to ions to play an important role. We limit our discussion to the one-dimensional case with the Langmuir waves being random-phase plasmons.

The dynamics of the plasmons is described by the following wave kinetic equation[3]

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k}{\partial k}\frac{\partial N_k}{\partial x} - \frac{\partial \omega_k}{\partial x}\frac{\partial N_k}{\partial k} = 0 , \tag{1}$$

where $N_k = |E_k|^2/4\pi\omega_k$ is the plasmon number density, $\omega_k = \omega_p + 3\,k^2 v_e^2/2\omega_p$, $\partial\omega_k/\partial k = 3\,(k\lambda_{De}) v_e$ is the group velocity and the notations are standard. In Eq.(1), we have adopted the "collisionless" description (i.e., the "adiabatic" approximation[1]); which is only valid for low-frequency long-wavelength modulational envelope waves. The modulation occurs due to the effective potential produced by the low-frequency density perturbations; i.e.,

$$\frac{\partial \omega_k}{\partial x} = \frac{1}{2}\frac{\omega_{po}}{n_o}\frac{\partial \tilde{n}_e}{\partial x} . \tag{2}$$

Here, we have let $n_e = n_0 + \tilde{n}_e$ and assumed $\tilde{n}_e/n_0 \ll 1$. $\tilde{n}_e$ in turn is created by the ponderomotive force exerted on the particles by the plasmons; that is,

$$\frac{\tilde{n}_e}{n_0} = \frac{e}{T_e}(\phi + \psi) , \tag{3}$$

where ϕ is the self-consistent low-frequency potential and ψ is the effective ponderomotive potential given by

$$\psi = -\frac{2\pi e}{\omega_{p0} m_e} \sum_k N_k \tag{4}$$

For the ion dynamics, we use the Vlasov kinetic description because we are interested in the regime where the phase velocities of the envelope waves (i.e., the group velocities of the plasmons) are comparable to the ion thermal velocity v_i. Thus, we have

$$\frac{\partial f_i}{\partial t} + v \frac{\partial f_i}{\partial x} - \frac{e}{m_i} \frac{\partial \phi}{\partial x} \frac{\partial f_i}{\partial v} = 0 , \tag{5}$$

where we have neglected the ion ponderomotive potential which is $O(m_e^2/m_i^2)$ smaller than ϕ. Equations (3) and (5) are then related through Poisson's equation

$$-\frac{\partial^2 \phi}{\partial x^2} = 4\pi e \left(\int f_i \, dv - n_e \right) . \tag{6}$$

The above set of equations subject to the various approximations, thus, provides a complete description of the system.

Assuming $N_k = N_k^{(0)} + \tilde{N}_k(x,t)$, $n_e = \tilde{n}_e(x,t)$, $\phi = \tilde{\phi}(x,t)$, $f_i = f_i^{(0)}(v) + \tilde{f}_i(v,x,t)$, and the perturbations be of the form $\exp\left[i(q x - \Omega t)\right.$, it is then straight forward by linearizing Eqs.(1) to (6) to derive the following dispersion relation

$$1 = \frac{q^3}{4 m_0 m_e} \left(\frac{\chi_e \chi_i}{\varepsilon} \right)_{\Omega, q} \int_{-\infty}^{+\infty} dk \, \frac{dN_k^{(0)}/dk}{\Omega - q u_k} . \tag{7}$$

Here, $u_k = \partial\omega_k/\partial k$, $\varepsilon = 1 + \chi_e + \chi_i$, $\chi_e = 1/q^2\lambda_{De}^2$,

$$\chi_i = -\frac{\omega_{pi}^2}{q^2}\int dv \, \frac{df_i^{(0)}/dv}{v - \Omega/q} , \tag{8}$$

and we have replaced the summation in k by an integration with an appropriate normalization constant. Equation (7) along with Eq.(8) is valid over the quasi-static ($|\Omega/q v_i| < 1$), the hydrodynamic ($|\Omega/q v_i| > 1$) and the transitional ($|\Omega/q v_i| \sim 1$) regimes.

Let us concentrate on the transitional regime. Assuming that the plasmon distribution is sufficiently peaked around the wave number k_0 such that Landau damping due to plasmons can be neglected (i.e., $|\Omega - q u_0|^2 > q^2 u_t^2$, $u_0 = 3(k_0\lambda_{De})v_e$, u_t measures the "thermal" spread in the plasmon group velocities), Eq.(7) in this "cold beam" plasmon limit then reduces to

$$\Omega - q u_0 = \pm i\,\frac{\sqrt{3}}{2}\left(\frac{\chi_i}{\varepsilon}\right)^{1/2}_{\Omega, q} q\lambda_{De}\left(\frac{W}{n_0 T_e}\right)^{1/2}\omega_{p0} . \tag{9}$$

Here, $W = \omega_{p0}\int dk\, N_k^{(0)}$ is the turbulence wave energy. Assuming $k_0^2\lambda_{De}^2 > W/n_0T_e$, we then obtain

$$\Omega \cong q u_0 \pm i\,\frac{\sqrt{3}}{2}\left(\frac{\chi_i}{\varepsilon}\right)^{1/2}_{qu_0, q} q\lambda_{De}\left(\frac{W}{n_0 T_e}\right)^{1/2}\omega_{p0} . \tag{10}$$

Note that in both the quasi-static and the hydrodynamic regimes, χ_i is purely real. In the transitional regime, $u_0 \sim v_i$ or $k_0\lambda_{De} \sim (T_i m_e/T_e m_i)^{\frac{1}{2}}$, however, χ_i is generally complex. Thus, the effects due to thermal ions interacting with the modulational envelope wave produce a real frequency shift and the instability growth rate remains essentially unchanged. For example, with $T_e \simeq T_i$ and $u_0 \simeq v_i$, we have

$$\Omega \simeq q u_0 + \frac{\sqrt{3}}{2}\left(-\sin\frac{\pi}{8} + i\cos\frac{\pi}{8}\right) q \lambda_{De} \left(\frac{W}{n_0 T_e}\right)^{1/2} \omega_{po} \,. \quad (11)$$

Finally, we note that the validity of the "cold beam" plasmon approximation requires $W/n_0 T_e > (\lambda_{De} \Delta k)^2$, where Δk is the typical width of the turbulence spectrum.

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