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ABSTRACT

Eigen structures and growth rates are obtained for the ion cyclotron beam-whistler plasma instability near the leading edge of a shock wave propagating along the magnetic field.

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The ion cyclotron beam-whistler plasma instability has been proposed by Golden et al [1,2] as a mechanism for generating turbulence in collisionless shock waves propagating along the magnetic field. In this Letter, we present theoretical results on the eigen structures and growth rates of this instability valid near the leading edge of the shock layer. This has relevance with respect to laboratory experiments [3] that measured stationary magnetic field fluctuations there.

We adopt, as in Refs. [1,2], the Mott-Smith representation for the ion velocity distribution function f_i, which supposes that the shock layer consists of interpenetrating unshocked (upstream (u)) and shocked (downstream (d)) ion flows. Following the assumption of Ref. [1], we treat these ion flows as cold monoenergetic beams so that in the rest frame of the shock front,

$$f_i(z,\underset{\sim}{v}) = n_u(z)\,\delta(\underset{\sim}{v} - v_u\hat{e}_z) + n_d(z)\,\delta(\underset{\sim}{v} - v_d\hat{e}_z), \quad (1)$$

$\hat{e}_z$ being the direction of the static magnetic field and z the distance into the shock layer*. The electrons, on the other hand, are treated as a single warm fluid moving with an average drift velocity $v_e(z)\hat{e}_z$ such that there is no net current, viz.

$$v_e(z) = [1 - \eta(z)]v_u + \eta(z)v_d\ , \quad (2)$$

*Note that the upstream ion density $n_u(z)$ decreases from its upstream value $n_{u\infty}$ at the leading edge (taken to be at z=0) to zero at the trailing edge, while $n_d(z)$ has the opposite dependence on z. The Rankine-Hugoniot shock relations enable one to express the mean velocity v_d of the downstream ions in terms of v_u.

where $\eta(z) = n_d(z)/n_e(z)$ and $n_e(z) = n_u(z) + n_d(z)$.

We assume the following WKB form for the perturbed quantities b(z,t):

$$b(z,t) = \tilde{b}(z)\exp\left[i(k_0 z - \omega_s t)\right]. \tag{3}$$

Here $\omega_s = \omega_0 + i\gamma$, $|\gamma| << |\omega_0|$ and $|k_0| >> |\tilde{b}'(z)|/|\tilde{b}(z)| >> L^{-1}$, L being the typical scale length of the shock. The local analysis of Ref. [1] points to the existence of stationary whistler unstable modes near the leading edge of the shock; they are driven unstable by their interaction with the downstream ion cyclotron drift mode

$$\omega_0 - k_0 V_d + \Omega_i = 0. \tag{4}$$

These zero group velocity whistlers are of particular interest to us since they are the ones which have ample time to grow to sufficiently large amplitude to irreversibly scatter the incoming upstream ions. Consistent with Ref. [1], we therefore assume that (ω_0, k_0) satisfy both the dispersion relation at z=0

$$\epsilon_0(\omega_0, k_0) = \epsilon_r(\omega_0, k_0, z=0) =$$

$$= \left[1 - \frac{\omega_0}{\Omega_i} + \frac{k_0 V_e(z)}{\Omega_i} + \frac{k_0^2 C_A^2(z)}{\Omega_i^2} - \frac{\Omega_i}{\omega_0 - k_0 V_u + \Omega_i}\right]_{z=0} = 0 \tag{5}$$

(where C_A is the Alfvèn speed and Ω_i the ion cyclotron frequency), and the zero group velocity condition equivalent to $(\partial\epsilon_0/\partial k_0)_{\omega_0} = 0$. Upon combining Maxwell's equations with the linearized equations of continuity and momentum, one obtains the following equation for the envelope $\tilde{b}(z)$:

$$\left(K^{-2}\frac{d^2}{dz^2} - \epsilon_i\right)\tilde{b}(z) = 0, \tag{6}$$

where

$$K^{-2} \equiv \frac{1}{2}\left(\frac{\partial^2 \epsilon_o}{\partial k_o^2}\right)_{z=0} = \frac{C_A^2(z=0)}{\Omega_i^2} - \frac{\Omega_i V_u^2}{(\omega_o - k_o V_u + \Omega_i)^3} > 0, \tag{7}$$

$$\epsilon_i \equiv i\frac{\Omega_i}{\gamma}\left[\eta(z) - \frac{\gamma^2}{\tilde{\Omega}^2}\right], \tag{8}$$

and

$$\tilde{\Omega}^{-2} \equiv \Omega_i^{-2} - (\omega_o - k_o V_u + \Omega_i)^{-2} > 0. \tag{9}$$

We note that the ion cyclotron drift mode condition (4) is incorporated into the first r.h.s. term of Eq.(8). In deriving Eq.(6), we have adopted the ordering scheme $|\eta(z)| \ll 1$, $|\gamma/\omega_o| \sim O(\eta^{1/2}) \ll 1$ and have retained all terms to order $\eta^{1/2}$ [i].

In order to solve Eq.(6), $\eta(z)$ must be prescribed. Near the leading edge $\eta(z) \simeq n_d(z)/n_{u\infty}$, and we assume its profile to be

$$\eta(z) = \begin{cases} 0, & z \leq 0 \\ (z/L)^2, & z \geq 0 \text{ and } |z| \ll L. \end{cases} \tag{10}$$

Equation (6) can then be solved for the positive -and negative- z regions and by matching the solutions at z=0. If one makes the ansatz,

$$\left|\arg\left(\gamma^{1/2} e^{-i\pi/4}\right)\right| < \frac{\pi}{2}, \tag{11}$$

one then has for $z < 0$,

$$\tilde{b}_- = A_- \exp(K_- z), \tag{12}$$

where

$$K_- = (K^2 \Omega_i \gamma e^{-i\pi/2} / \tilde{\Omega}^2)^{1/2}. \tag{13}$$

The other solution is discarded due to its divergence at $z = -\infty$. For $z > 0$, Eq.(6) can be reduced to the standard equation for parabolic cylinder functions [4],

$$\left[\frac{d^2}{d\xi^2} - \left(\frac{1}{4}\xi^2 + a \right) \right] \tilde{b}_+ = 0. \tag{14}$$

Here

$$\xi = z/\alpha,$$

$$\alpha = \left(\frac{\gamma L^2}{4 K^2 \Omega_i} \right)^{1/4} e^{-i\pi/8}, \tag{15}$$

and

$$a = -(\gamma^3 \Omega_i)^{1/2} (KL/2\tilde{\Omega}^2) e^{i\pi/4} \tag{16}$$

The solution, well behaved at $z = \infty$, is then

$$\tilde{b}_+ = A_+ U(a, \xi). \tag{17}$$

From the matching conditions that $\tilde{b}_+ = \tilde{b}_-$ and $d\tilde{b}_+/dz = d\tilde{b}_-/dz$ at $z = 0$, one obtains the following conditions for the eigenvalues:

$$\gamma_n^3 = 4 a_n^2 \tilde{\Omega}^4 \Omega_i^{-1} (KL)^{-2} e^{-i\pi/2}; \quad n = 0, 1, \ldots \tag{18}$$

and a_n is the n_{th} root of the following equation:

$$-a = \left[\frac{\Gamma(3/4 + a/2)}{\Gamma(1/4 + a/2)} \right]^2 . \tag{19}$$

In general, Eq.(19) must be solved numerically. However, it can be shown that a_n must be a negative real number and solutions can be obtained in some limiting cases. Let $a_n = -|a_n|$. For $|a_n| \ll 1$, we have $|a_0| \simeq 0.1$. For $|a_n| \gg 1$, solutions only occur near the poles of the r.h.s. of Eq. (19), i.e., $a_n \simeq -(2n + 3/2)$, where n is a large positive integer. The eigenvalue γ_n of the unstable mode (Re $\gamma_n > 0$), which we label $\tilde{\gamma}_n$, is then given by

$$\tilde{\gamma}_n = \left[4|a_n|^2 \tilde{\Omega}^4 \Omega_i^{-1} (KL)^{-2} \right]^{1/3} e^{-i\pi/6} \quad ; n = 0, 1, \ldots \tag{20}$$

We note that $\tilde{\gamma}_n$ satisfies the ansatz Eq.(11). Moreover, since $\tilde{\gamma}_n$ is complex, the unstable mode experiences a real frequency shift. Since $|a_n|$, $|(K_-)_n|$, $|\alpha_n|^2$ and the turning point $|z_{tn}|^2 = |4 a_n \alpha_n^2|$ increase with $\tilde{\gamma}_n$, our results indicate that modes with higher growth rates are peaked deeper inside the shock front and extend further into the shock. This is physically expected because the instability is driven by finite η and η increases with z. We also note that the validity of the approximations made here requires $|k_0| \gg |(K_-)_n|, |\alpha_n|^{-1} \gg L^{-1}$ and $|z_{tn}| \ll L$, thereby giving upper and lower bounds to the acceptable values of $\tilde{\gamma}_n$.

Finally, we remark that Ref. [2] considers the case where the shocked ion cyclotron drift mode is more realistically treated as a hot Maxwellian beam in the local approximation; our above eigenmode analysis can be readily extended to this more general case [5].

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