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ON THE DIELECTRIC RESPONSE FUNCTION IN TWO-DIMENSIONAL CLASSICAL PLASMAS

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ABSTRACT

A numerical investigation of the static properties of the 2-D electron plasma is carried out on the basis of recent improved theory of the dielectric response function. The static form factor is computed over the thermodynamically stable domain of the plasma parameter γ. The onset of instability is found to be located in a range not far from the theoretical value.

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The longitudinal dielectric response function $\epsilon(\vec{k},\omega)$ play a central part in describing various plasma properties [1]; the fluctuations and correlations are readely calculated with a knowledge of such a dielectric function.

In this note we present preliminary numerical results concerning the static properties of 2 - d one-component classical plasma; a more complete analytical as well as numerical work on the subject will be published elsewhere [19] .

In this dimensionality, the model contains various problems of fundamental interest and we refer the reader to the paper of D. Montgomery [2] . Especially the model arises in connection with practical problems in real plasmas, such as diffusion; it has also been recognized that a remarkable model of strong turbulence exists in the highly magnetized 2 - d. Coulomb plasma [3] . Analogous phenomena to those which have been emerged in recent theoretical studies on this model [2] , are shown to occur similarly in other domains, such as high-beta turbulent regime in magnetohydrodynamics, as reported recently [4] .

Within a theoretical approach, not all basic properties of a classical plasma emerge from the study of the model in the small g limit. For a temperature greater than the Fermi energy, there still exists a region in the density temperature plane where the plasma parameter g is greater than unity and for such plasma, the classical treatment is still applicable.

Lately much attention has, in fact, been given to the study of the static properties of a plasma, in the region, where the dimensionless plasma parameter g is not necessarly small [5,6] and where the conventional random phase approximation is no longer appropriate.

In the 3 - d case, a careful truncation sheme of the BBGKY set of equations, well suited for the purpose (and briefly discussed below in order to situate our 2 - d study), has been advanced by S. Ichimaru [7]; and the approach is basically similar to that employed by the same author in his approach to a theory of strong plasma turbulence [8] . Moreover, the validity of such an approximation (which we call the I approximation) has been checked numerically in the 3 - d case only [6] .

It must be noticed that a part of the remarkable improvement with respect to the conventional calculations of the ramdom phase approximations for short ranged correlations and other approach [9], a 3 - d numerical study of the I approximation reproduces with good occuracy the range for the onset of instability, which was also predicted and located by a statistical mechanics treatment of the model [10,11]. In fact, in [6] pronounced peaks in the static form factor S(k) emerge, as $\varepsilon = (4\pi\rho)^{\frac{1}{2}}.e^3.\beta^{3/2} = g/4\pi = (4\pi\rho\lambda_D^3)^{-1}$ lie in the range $\varepsilon \sim 9.3 - 10.6$, i.e., $\Gamma = \varepsilon^{2/3}.3^{-1/3} \sim 3-3.4$ while in [11], $\Gamma \sim 2.5 - 3.3$, as it was shown using inequality as well as H - stability property of statistical mechanics [12]. Moreover, these values agree also with that recently found in [13] i.e. $\Gamma \sim 3$, where a change in the slope of the dispersion curve has been emerged.

To remember the basic assumption involved in such a truncation sheme of the BBGKY set of equations is essentially that the ternary correlation function can be expressed as a functional of the binary correlation, the physical guide-line in selecting the choice of the functional beeing the correct long range behaviour of the correlation in a weakly non ideal 3 - d plasma [14].

Now, as was noticed in [15,16] interesting effects are also expected to occur in the 2 - d case, where an instability should occur at $\gamma = \beta \cdot e^2 = (2\pi\rho \cdot \lambda_D^2)^{-1} = g/2\pi = 4$ [11]. There the pressure as well as the sound speed vanish. Notice that the value $\gamma = 2$ occurs in the 2 component case and is a temperature below which some thermodynamic quantity cannot be proved to exist [14]. (The partition function diverges there even for a +- pair of particles).

It turns out that even in the 2 - d case, the I approximation can be considered as well and should improve previous calculations (this follows from an analysis of the screaning factors which occur in the O'Neil - Rostoker equations).

In considering a system of electrons in a 2 - d box with positive charge background, the 2 nonlinear selfconsistent equations to be solved, and relating the structure factor $S(x)$ to the screaning function $1 + u$ are then given by

$$S(x) = \frac{x^2}{x^2 + 1 + u(x)}$$
$$u(x) = \frac{\gamma}{2\pi} \cdot x \cdot \int_0^\infty dy \int_0^{2\pi} \cos\theta \, S(y) \, S(|\vec{x} - \vec{y}|) \qquad (1)$$

with the dimensionless variable $k/k_D = x$ and $k_D^2 = (2\pi \rho \beta e^2)$,
$\gamma = g/2\pi = \beta \cdot e^2$, $\beta = (kT)^{-1}$, $|\vec{k}| = k$.
S is related to the radial correlation function by $\rho g(x) = \sum_{\vec{k}} (S(k) - 1) \cdot e^{i\vec{k}\cdot\vec{x}}$ and to the static dielectric function by the fluctuation dissipation theorem.

Notice that for $\gamma = 0$, S takes on his Debye value : and gives rise to an unphysical short range behaviour of the correlations. For each γ, the correct long wavelenght limit can be found with (1) and is given by

$$u(x) \sim \alpha x^2 + \beta x^4$$

with

$$\beta = \frac{\gamma}{16} \cdot \int_0^\infty \left(\frac{dS}{dy}\right)^2 \cdot \frac{dy}{y}$$

and where α is connected with the sound speed. It can be shown that the compressibility sum rules is satisfied for each γ , contrary to the 3 - d case.

A numerical solution of the nontrivial set of equation (1) in the range up to $\gamma \sim 3.$ and $x \sim 10$ has been found using digital computer and a short description of the method used is now breafly presented together with the new results.

We first transform the set of equations (1) for $S(x)-1 = (\rho g)(x)$ and $u(x)$ with the variables change $X = e^{-x}$, $Y = e^{-y}$ into the form

$$U(X) = -\frac{\gamma}{\pi} \cdot \ln X \cdot \int_0^1 dY \, \frac{1+G(Y)}{Y} \int_0^{\pi} d\theta \cos\theta \, G(Z)$$
$$G(X) = -\frac{1+U(X)}{(\ln X)^2 + 1 + U(X)} \qquad (2)$$

where $Z = \exp -\left[(\ln X)^2 + (\ln Y)^2 - 2 \ln X \cdot \ln Y \cdot \cos\theta \right]^{1/2}$

$$u'(-\ln X) \equiv U(X)$$
$$(\rho g)'(-\ln X) \equiv G(X)$$

The integral occuring in (3) is then discretized by means of Gauss method, i.e. :

$$U(X_j) = -\frac{\gamma}{4} \cdot \ln X_j \sum_{k=1}^{n} W_k^n \, \frac{1+G(X_k)}{X_k} \sum_{\ell=1}^{m} W_k^m \cos\theta_k \, G(Z_{j,k}^{\ell})$$
$$G(X_j) = -\frac{1+U(X_j)}{(\ln X_j)^2 + 1 + U(X_j)}$$

and where $j \in [1;n]$; $\{X_j\}, \{\theta_j\}, \{W_j^n\}, \{W_j^m\}$,
are given by the method.
Having first computed the set $\{Z_{j,k}^{\ell}\}$, we have then deduced $G(Z_{j,k}^{\ell})$ interpolating $\{G(X_j)\}$ by means of cubic spline method [18] . The relations $G(0) = 0$ and $G(1) = -1.0$ have also been employed. Having thus calculated the set $\{U(X_j)\}$ we have deduced the new set $\{G(X_j)\}$. The iteration has been stopped if

$$\max_{j \in [1,n]} \left(\left| \overset{i+1}{G}(X_j) - \overset{i}{G}(X_j) \right| \right) < 10^{-8}$$

The method can be improved [19] in considering the decomposition of the interval [0,1] in many pieces [0,A], [A,B], [B,1]. It will then be possible to work with different density of points, i.e. especially in the interval [0,A] to precise the structure of the peaks or oscillations which have been emerged in the static form factor.

Moreover, the initial value problem has been solved, using for any value of γ the corresponding result of the step $\gamma - \delta$. In the first step ($\gamma = 0.05$) the initial value for g(k) has been chosen equal to his Debye limit (u = 0).

These results obtained for the different set of γ will then be used in [19] as initial values to further approach the correlation function g(k) solution of (1) and this in a much bigger domain, containing smaller wavelength.

Results on (ρ g)(k) for different value of γ are given in Fig. 1,2. Above $\gamma = 1.6$, g(k) begins to overshot the zero level and at $\gamma \sim 3.4$ a peak emerges, and will be pronounced for γ approaching the value 4. A careful examination of the screaning function 1 + u in this region is in progress. Moreover it is expected that in the short range domain, g(r) violates the physical condition $g(r) \gtrsim -1$ above some $\gamma_0 < 4$, but this only on a reduced scale, as in the 3-dimensional case. It is expected that this is related to the change of $\beta(\gamma)$ at γ_0 and thus to a change in the structure of the long wavelength part of the fluctuations of Ornstein Zernike type, occuring in the system [6,19].

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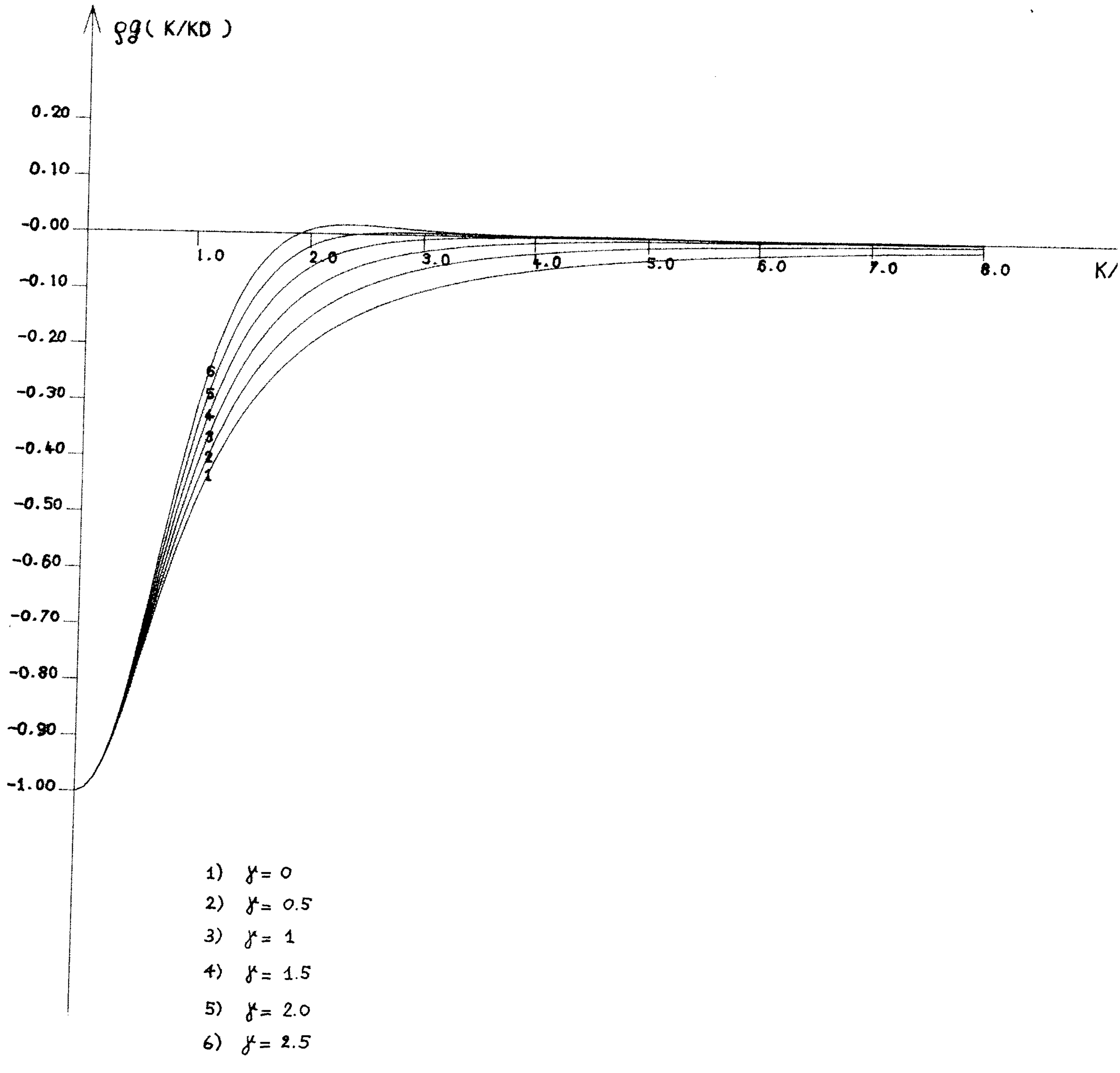
ρg(K/KD)
0.20
0.10
-0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
K/
1
2
3
4
5
6
1) γ = 0
2) γ = 0.5
3) γ = 1
4) γ = 1.5
5) γ = 2.0
6) γ = 2.5

fig 1

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