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INSTABILITY OF COUPLED LANGMUIR AND ION-ACOUSTIC SOLITONS

K. Appert and J. Vaclavik

Centre de Recherches en Physique des Plasmas

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

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ABSTRACT

It is shown that one-dimensional coupled Langmuir and ion-acoustic solitons are unstable against perturbations transverse to their motion. The growth rate of the corresponding instability is obtained.

The long wavelength Langmuir turbulence associated with the heating of plasma by powerful lasers or relativistic electron beams has been receiving much attention in recent years[1,2]. Zakharov[3] and Rudakov[4] have pointed out the importance of the Langmuir solitons in a turbulent plasma and formulated the coupling between Langmuir solitons and the linear ion-acoustic waves. Recent investigations[5-7] of the equations derived by Zakharov have shown that for the solitons moving with a group velocity very close to the ion-acoustic speed the nonlinearity and the dispersion of the ion-acoustic waves are important. Taking these terms into account, Nishikawa et al.[5] have obtained coupled Langmuir and ion-acoustic soliton solution. It is well-known[8-10] that the Langmuir solitons are unstable with respect to perturbations perpendicular to the direction of their motion. On the other hand, Kadomtsev and Petviashvili[11] have shown that the ion-acoustic solitons are stable with respect to such perturbations. A question arises whether the coupled Langmuir and ion-acoustic solitons preserve the stability property of the ion-acoustic solitons or the Langmuir solitons. In this paper we show that the coupled Langmuir and ion-acoustic solitons are unstable against perturbations transverse to their motion. The growth rate of the corresponding instability is obtained.

The basic equations describing the nonlinear dynamics of one-dimensional Langmuir and ion-acoustic waves in a system of coordinates moving with the ion-acoustic speed can be taken in the form[5,7]

$$i\varepsilon\left(\frac{\partial E}{\partial t} - \frac{\partial E}{\partial X}\right) + \frac{3}{2}\frac{\partial^2 E}{\partial X^2} - \frac{n}{2}E = 0\,, \qquad (1)$$

$$\frac{\partial n}{\partial t} + \frac{1}{2}\frac{\partial}{\partial X}\left(\frac{\partial^2 n}{\partial X^2} + n^2 + |E|^2\right) = 0 , \tag{2}$$

where n is the low-frequency ion-density perturbation and E is the electric field of Langmuir oscillations with $\exp(-i\omega_{pe}t)$ factored out. The equations (1) and (2) are in dimensionless units; the units of time, space, density perturbation, and electric field are, respectively, $(\varepsilon\omega_{pe})^{-1}$, λ_D, n_0, and $4(\pi n_0 T_e)^{\frac{1}{2}}$. Here ω_{pe}, λ_D, n_0, T_e, and ε^2 are, respectively, the plasma frequency, Debye length, average density, electron temperature, and electron-to-ion mass ratio.

The equations (1) and (2) possess a stationary solution of the soliton type given by[5]

$$E = -a f(\xi) e^{iS} , \quad S = (X + t/2)\varepsilon/3 + 3X_0/(20\varepsilon), \tag{3}$$

$$f(\xi) = 3^{1/2} 8 \,\mathrm{sech}\left[(2/3)^{1/2}\xi\right] \tanh\left[(2/3)^{1/2}\xi\right],$$

$$n = -a\, g(\xi) , \quad g(\xi) = 12\, \mathrm{sech}^2\left[(2/3)^{1/2}\xi\right], \tag{4}$$

where $\xi = a^{\frac{1}{2}}(X + X_0)$, a is the soliton amplitude, and $X_0 = 20\ at/3$ is the soliton phase.

In the following, we investigate the stability of the soliton solution (3) and (4) against perturbations in the y direction. In this case Eqs.(1) and (2) no longer apply; however, if the y-coordinate dependence is weak

($\partial/\partial y \ll \partial/\partial x$) they can be corrected by adding a quantity $\partial\varphi/\partial y$ to the right-hand side of Eq.(2). The relation defining the function φ was established by Kadomtsev and Petviashvili[11] as $\partial\varphi/\partial x = -\tfrac{1}{2}\,\partial n/\partial y$. Hence one can see that Eq.(2) must be replaced by

$$\frac{\partial}{\partial x}\left[\frac{\partial n}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 n}{\partial x^2} + n^2 + |E|^2\right)\right] = -\frac{1}{2}\frac{\partial^2 n}{\partial y^2} \quad . \tag{5}$$

In general, the system of Eqs.(1) and (5) is rather complicated so we confine ourselves to the limiting case of very large wavelengths along y when $\partial X_0/\partial y \ll a$. In this case Eqs.(1) and (5) may be solved by the Krylov-Bogolyubov perturbation method[11]. Accordingly, we introduce a new independent variable $\xi = a^{\frac{1}{2}}(X + X_0)$, where a and X_0 are slowly varying functions of y and t. Moreover, we put $E = \psi\exp(is)$. On neglecting a small term of the order ε Eqs.(1) and (5) can then be rewritten as

$$\frac{3}{2}a\,\frac{\partial^2\psi}{\partial\xi^2} - \left(a + \frac{n}{2}\right)\psi = \left(\frac{3}{20}\frac{\partial X_0}{\partial t} - a\right)\psi \, , \tag{6}$$

$$\frac{a}{2}\frac{\partial^2}{\partial\xi^2}\left[a\frac{\partial^2 n}{\partial\xi^2} + \frac{40}{3}an + n^2 + \psi^2\right] = -a^{1/2}\frac{\partial}{\partial\xi}\left[\frac{\partial n}{\partial t} + \frac{1}{a}\frac{\partial a}{\partial t}\frac{\xi}{2}\frac{\partial n}{\partial\xi} + a^{1/2}\frac{\partial n}{\partial\xi}\left(\frac{\partial X_0}{\partial t} - \frac{20}{3}a\right)\right] - \frac{1}{2}\left(\frac{\partial^2 n}{\partial y^2}\right)_{x,t} \, , \tag{7}$$

where the right-hand sides can be considered small.

In the zeroth approximation the right-hand sides are neglected and it follows that $\psi_0 = -\,af(\xi)$ and $n_0 = -\,ag(\xi)$. In the first approximation we put $\psi = \psi_0 + \psi_1$ and $n = n_0 + n_1$, linearize the left-hand sides of Eqs.(6) and (7), and substitue ψ_0 and n_0 in the right-hand sides. As we shall see later, the variations of the phase X_0 are much larger than the variations of the amplitude a ($\partial a/\partial t \ll \partial X_0/\partial t, \partial a/\partial y \ll \partial X_0/\partial y$). Consequently, the equations for the first order quantities read

$$\frac{3}{2}\frac{\partial^2 \Psi_1}{\partial \xi^2} + \left(\frac{g}{2} - 1\right)\Psi_1 + \frac{f}{2} n_1 = \left(a - \frac{3}{20}\frac{\partial X_0}{\partial t}\right) f, \tag{8}$$

$$\frac{\partial^2 n_1}{\partial \xi^2} + \left(\frac{40}{3} - 2g\right) n_1 - 2f\Psi_1 = 2\left(\frac{\partial X_0}{\partial t} - \frac{20}{3} a\right) g. \tag{9}$$

It can easily be verified that the well-behaved solution of Eqs.(8) and (9) is

$$\Psi_1 = C\frac{df}{d\xi} + \left(a - \frac{3}{20}\frac{\partial X_0}{\partial t}\right)\left(f + \frac{\xi}{2}\frac{df}{d\xi}\right), \tag{10}$$

$$n_1 = C\frac{dg}{d\xi} + \left(a - \frac{3}{20}\frac{\partial X_0}{\partial t}\right)\left(g + \frac{\xi}{2}\frac{dg}{d\xi}\right), \tag{11}$$

where C is an arbitrary function of y and t. An inspection of the functional form of the relations (10) and (11) shows that the quantities Ψ_1 and n_1 represent small corrections to the amplitude a and to the phase X_0. We can require, without any loss of generality, that the corrections be zero. This means that the amplitude and the phase have been chosen correctly. Thus we have

$$\frac{\partial X_0}{\partial t} = \frac{20}{3} a. \tag{12}$$

For the second order quantities we obtain from Eqs.(6) and (7)

$$\frac{3}{2}\,\frac{\partial^2\psi_2}{\partial\xi^2}+\left(\frac{g}{2}-1\right)\psi_2+\frac{f}{2}n_2=0\,, \tag{13}$$

$$\frac{a^{3/2}}{2}\,\frac{\partial}{\partial\xi}\left[\frac{\partial^2 n_2}{\partial\xi^2}+\left(\frac{40}{3}-2g\right)n_2-2f\psi_2\right]=\frac{\partial a}{\partial t}\left(g+\frac{\xi}{2}\,\frac{dg}{d\xi}\right)+g\,\frac{a}{2}\,\frac{\partial^2 X_0}{\partial y^2}\,. \tag{14}$$

Multiplying Eqs.(13) and (14) by $\frac{df}{d\xi}$ and g, respectively, and integrating over ξ from $-\infty$ to ∞ it can be shown, after some manipulations, that the left-hand side of Eq.(14) becomes zero. Consequently we get

$$\frac{\partial a}{\partial t}=-\frac{2}{3}\,a\,\frac{\partial^2 X_0}{\partial y^2}\,, \tag{15}$$

where we have made use of the relation $\int d\xi\, g(g+\xi/2\; dg/d\xi)=3/4\int d\xi\, g^2$. From Eq.(15) one can see that the variations of the amplitude are indeed much smaller than the variations of the phase.

Combining Eqs.(12) and (15) yields

$$\frac{\partial^2 X_0}{\partial t^2}=-\frac{40}{9}\,a\,\frac{\partial^2 X_0}{\partial y^2}\,. \tag{16}$$

If we set $X_0 \sim \cos(k_\perp y)\exp(\gamma t)$ we find that an instability occurs with the growth rate

$$\gamma=\frac{2}{3}\,(10a)^{1/2}k_\perp\,. \tag{17}$$

In conclusion, we have shown that one-dimensional coupled Langmuir and ion-acoustic solitons are unstable with respect to self-focusing, or bending. As a result of the instability the solitons break up into separate bunches along the y axis. The growth rate is shown to be proportional to the square root of the soliton amplitude, i.e., it is rather large. A question remains as what will be a final state, if any, in the development of the instability discussed here. The investigation of this problem amounts to a general treatment of the nonlinear equations (1) and (5). The work on this topic is in progress.

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