May 1974 LRP 80/74

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Abstract

Coupled equations for ordinary and hybrid waves, propagating perpendicular to a static magnetic field, are derived within the fluid approximation. It is shown that a large amplitude ordinary wave is subject to a decay instability as well as a modulational instability. The latter corresponds to a modulation of the pump amplitude, and the excitation of a long wave length hybrid wave. Characteristic quantities, such as the threshold intensity and the maximum growth rate, are calculated for various cases. In the limit of a vanishing magnetic field, the results for stimulated Raman and Brillouin scattering are recovered.

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Coupled equations for ordinary and hybrid waves, propagating perpendicular to a static magnetic field, are derived within the fluid approximation. It is shown that a large amplitude ordinary wave is subject to a decay instability as well as a modulational instability. The latter corresponds to a modulation of the pump amplitude, and the excitation of a long wave length hybrid wave. Characteristic quantities, such as the threshold intensity and the maximum growth rate, are calculated for various cases. In the limit of a vanishing magnetic field, the results for stimulated Raman and Brillouin scattering are recovered.

Lausanne

1. Introduction

The development of high power lasers has spurred a great deal of interest in the parametric instabilities of a plasma subject to intense electromagnetic radiation. Many authors have explored the possibility of plasma heating by means of powerful laser beams and have tried to explained various observed phenomena, such as anomalous reflection and enhanced absorption of laser light in a plasma (see, for example, DuBOIS 1972). Most of these investigations, however, were limited to the case of an unmagnetized plasma. It is only recently that some attention has been devoted to the more interesting case of a plasma immersed in an ambient magnetic field.

Among the great variety of resonant modes in such a plasma, those propagating perpendicular to the external magnetic field have received a special interest. For instance, STENFLO (1971) has considered the coupling between ordinary waves in a drifting plasma while CANO et al. (1969) have examined the interaction between extraordinary and ordinary waves. In addition, several authors have investigated the interaction between extraordinary waves (DAS 1971, PORKOLAB 1971, TZOAR 1969), and recently KINDEL et al. (1972) have shown that effective heating of both ions and electrons can be achieved by pumping the plasma with a lower hybrid wave.

In this paper, we study the coupling between ordinary and hybrid waves, considering the latter as quasi-electrostatic. Within the well-known parametric approximation, it will be shown that a large amplitude ordinary wave can simultaneously excite another ordinary wave and either an upper or a lower hybrid wave. Besides this decay instability, we also observe a new type of instability, namely a "modulational" instability, which results in a modulation of the pump amplitude and the excitation of a long wave length hybrid wave. Characteristic quantities, such as the threshold pump intensity and the maximum growth rate of the excited waves, are calculated for various cases.

The plan of the paper is as follows. In Section 2, we derive the general coupled equations for ordinary and hybrid waves. In Section 3, the so-called parametric approximation is used to obtain a linearized system for the waves under consideration. Sections 4 and 5 describe the excitation of upper and lower hybrid waves, respectively. Effects of collisions are considered in Section 6, and finally, the results are discussed in Section 7.

2. The Coupled Equations

Let us consider a homogeneous, unbounded plasma immersed in a static magnetic field $\vec{B}_o = B_o \vec{e}_z$ ($\vec{e}_z$ is the unit vector along the z-axis). The steady state of the plasma is characterized by a particle number density n_o, an electron thermal velocity $\overline{v}_e$, an ion thermal velocity $\overline{v}_i$, and a zero drift velocity. Within the fluid description, the waves under consideration, i.e. the ordinary and hybrid waves, are governed by the following equations

$$\text{(1a)} \qquad \frac{\partial n_j}{\partial t} + \nabla \cdot (n_o + n_j)\vec{V}_j = 0$$

$$\text{(1b)} \qquad \frac{\partial \vec{V}_j}{\partial t} + (\vec{V}_j \cdot \nabla)\vec{V}_j = \frac{q_j}{m_j}\left[\vec{E} + \vec{\mathcal{E}} + \vec{V}_j \times (\vec{B}_o + \vec{B})\right] - \frac{\gamma_j \overline{v}_j^2 \nabla n_j}{n_o + n_j}$$

$$\text{(1c)} \qquad \nabla \cdot \vec{\mathcal{E}} = \sum_j q_j n_j / \varepsilon_o$$

$$\text{(1d)} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{(1e)} \qquad \nabla \times \vec{B} = \frac{\partial \vec{E}}{c^2 \partial t} + \mu_o \sum_j (n_o + n_j) q_j \vec{V}_j \quad ,$$

where q_j, m_j, n_j and $\vec{V}_j$ stand for the charge, mass, perturbed number density and velocity of the j^{th} species, respectively. $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields of the ordinary waves and $\vec{\mathcal{E}}$ is the electric field of the hybrid waves. γ_j is a numerical factor.

Examination of the non-linear terms of Eq.(1), reveals that there are two types of non-linearity: the first (e.g. $\vec{V}_j \times \vec{B}$, $n_j \vec{V}_j$) corresponds to the coupling between hybrid and ordinary waves while the second (e.g. $\nabla\cdot(n_j V_j)$, $n_j \nabla n_j$) is due to the self-coupling of the hybrid waves. Since our purpose is to study the interaction between hybrid and ordinary waves, non-linear terms of the second type will be neglected.

For waves propagating along the x-axis, Eq.(1) then reduces to

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial V_x}{\partial x} = 0 \tag{2a}$$

$$\frac{\partial V_x}{\partial t} - \frac{q}{m}\left(\mathcal{E} + V_y B_0\right) + \frac{\gamma \bar{V}^2}{n_0}\frac{\partial n}{\partial x} = -\frac{q}{m} V_z B \tag{2b}$$

$$\frac{\partial V_y}{\partial t} + \frac{q}{m} V_x B_0 = 0 \tag{2c}$$

$$\frac{\partial V_z}{\partial t} - \frac{q}{m} E = V_x\left(\frac{q}{m} B - \frac{\partial V_z}{\partial x}\right), \tag{2d}$$

where we have omitted the subscript j, referring to either the ions (i) or the electrons (e).

After some algebra, the above system can be expressed as

$$\frac{\partial^2 n_i}{\partial t^2} - \gamma_i \bar{V}_i^2 \frac{\partial^2 n_i}{\partial x^2} + \omega_{ci}^2 n_i + \omega_{pi}^2 (n_i - n_e) = \left(\frac{m_e}{m_i}\right)^2 n_0 \frac{\partial}{\partial x}\left(V_z \frac{\partial V_z}{\partial x}\right) \tag{3a}$$

$$\frac{\partial^2 n_e}{\partial t^2} - \gamma_e \bar{V}_e^2 \frac{\partial^2 n_e}{\partial x^2} + \omega_{ce}^2 n_e + \omega_{pe}^2 (n_e - n_i) = n_0 \frac{\partial}{\partial x}\left(V_z \frac{\partial V_z}{\partial x}\right) \tag{3b}$$

$$\frac{\partial^2 V_z}{\partial t^2} - c^2 \frac{\partial^2 V_z}{\partial x^2} + (\omega_{pe}^2 + \omega_{pi}^2) V_z = -\frac{\omega_{pe}^2}{n_0} V_z \left(n_e + \frac{m_e}{m_i} n_e\right) \tag{3c}$$

where ω_{pj} and ω_{cj} are the plasma and cyclotron frequencies.

By taking the Fourier transform in space and time of Eq.(3), and neglecting terms of order m_e/m_i compared to 1, we finally obtain

(4a) $$\mathcal{D}(\omega,k)\, n(\omega,k) = -\frac{n_0}{2\omega_{pe}^2} k^2 \int V(\omega',k')\, V(\omega-\omega',k-k')\, d\omega'\, dk'$$

(4b) $$D(\omega,k)\, V(\omega,k) = \frac{\omega_{pe}^2}{n_0} \int V(\omega',k')\, n(\omega-\omega',k-k')\, d\omega'\, dk',$$

where we have introduced the notation

(4c) $$\mathcal{D}(\omega,k) \equiv \frac{1+\chi_i+\chi_e}{\chi_e(1+\chi_i)}$$

(4d) $$\chi_j \equiv \frac{\omega_{pj}^2}{\omega_{cj}^2+\gamma_j \bar{v}_j^2 k^2-\omega^2}$$

(4e) $$D(\omega,k) \equiv \omega^2-c^2k^2-\omega_{pe}^2 ,$$

and where the quantities $n(\omega,k)$ and $V(\omega,k)$ are the Fourier transforms of the electron density (induced by the hybrid waves) and the electron transverse velocity (induced by the ordinary waves), respectively.

One notes, unfortunately, that the above system as such is still untractable. However, in certain cases, it is possible to reduce the convolution integrals of (4) to finite sums, and to obtain a coupled system in the so-called parametric approximation.

3. The Parametric Approximation

Here, we consider a plasma subject to a large amplitude ordinary wave. We assume that this wave is externally-driven and acts as a pump of constant power feeding energy into small disturbances inside the plasma. We then determine the conditions under which these disturbances will grow in the form of an ordinary and a hybrid wave.

Let $\vec{V}_o$ and $\vec{v}$ be the velocities induced by the pump and by the perturbed fields, respectively, we now assume that

(5) $$\vec{V} = \vec{V}_o + \vec{v} \ , \qquad \text{with} \qquad \vec{v} \ll \vec{V}_o \ .$$

For a monochromatic pump wave,

(6a) $$\vec{E}_o = 2\bar{E}_o \, \vec{e}_z \sin(\omega_o t - k_o x) \ ,$$

the Fourier transform of the total velocity is

(6b) $$V(\omega,k) = V_o \left[\delta(\omega-\omega_o)\,\delta(k-k_o) + \delta(\omega+\omega_o)\,\delta(k+k_o) \right] + v(\omega,k) \ ,$$

with

(6c) $$V_o = \frac{e\bar{E}_o}{m_e \omega_o} \ .$$

On substituting (6a) into (4a-b) and keeping only terms of first order in V_o, we readily obtain the following linear system:

(7a) $$\mathcal{D}(\omega,k)\, n(\omega,k) = -\frac{n_o V_o k^2}{\omega_{pe}^2} \left[v(\omega-\omega_o, k-k_o) + v(\omega+\omega_o, k+k_o) \right]$$

(7b) $$D(\omega,k)\, v(\omega,k) = \frac{\omega_{pe}^2 V_o}{n_o} \left[n(\omega-\omega_o, k-k_o) + n(\omega+\omega_o, k+k_o) \right] .$$

This equation shows the typical properties of the parametric coupling process: each component $n(\omega,k)$ is coupled to $v(\omega \pm \omega_o, k \pm k_o)$, which in turn couple with $n(\omega,k)$ and $n(\omega \pm 2\omega_o, k \pm 2k_o)$. One thus has an infinite set of linear equations coupling various Fourier components at beat frequencies $\omega \pm N\omega_o$ and wave numbers $k \pm Nk_o$ $(N = 1, 2, 3, \ldots)$. However, if $n(\omega,k)$ is a resonant response of the plasma, $n(\omega \pm N\omega_0, k \pm Nk_0)$ will usually be off resonance and the corresponding amplitudes will remain negligibly small compared to those of $n(\omega,k)$. Within this approximation, Eq.(7) can be reduced to a closed system for $n(\omega,k)$, $v(\omega-\omega_0, k-k_0)$ and $v(\omega+\omega_0, k+k_0)$:

(8a) $$\mathcal{D}(\omega,k)\; n(\omega,k) = -\frac{n_0 V_0 k^2}{\omega_{pe}^2}\left[v(\omega-\omega_0, k-k_0) + v(\omega+\omega_0, k+k_0)\right]$$

(8b) $$D(\omega\pm\omega_0, k\pm k_0)\, v(\omega\pm\omega_0, k\pm k_0) = \frac{\omega_{pe}^2 V_0}{n_0}\; n(\omega,k) \;.$$

From Eq.(8) we deduce the following dispersion relation

(9a) $$\mathcal{D}(\omega,k) = -k^2 V_0^2 \left(\frac{1}{D^+} + \frac{1}{D^-}\right) ,$$

where we have introduced the notation

(9b) $$D^{\pm} \equiv D(\omega \pm \omega_0 , k \pm k_0) \;.$$

In the following sections, Eq.(9a) will be solved separately for the excitation of upper and lower hybrid waves.

4. Parametric Excitation of the Upper Hybrid Wave

For the simultaneous excitation of an ordinary wave and an upper hybrid wave, the dispersion relation (9a) becomes

(10a) $$\omega^2 - \omega_H^2 = \omega_{pe}^2 k^2 V_0^2 \left(\frac{1}{D^+} + \frac{1}{D^-}\right) ,$$

where ω_H is the well-known upper hybrid frequency, i.e.

(10b) $$\omega_H = \left(\omega_{pe}^2 + \omega_{ce}^2 + \gamma_e k^2 \bar{V}_e^2\right)^{1/2} \;.$$

Examination of (10a) shows that the coupling process is most effective under the resonant condition

(11a) $$\omega_0 = \omega_H + \Omega ,$$

where Ω is the frequency of the ordinary wave at the shifted wave number $k - k_0$.

Taking account of the linear dispersion of the waves under consideration, this condition is fulfilled by two hybrid waves with

(11b) $$k = k_0 \pm k_0 \left[1 - \frac{(2\omega_0 \omega_H - \omega_H^2)}{c^2 k_0^2} \right]^{1/2} .$$

In the neighborhood of these values, it is easily seen that, in general , $1/D^+$ is negligibly small compared to $1/D^-$, and Eq.(10a) can be transformed to

(12a) $$(\omega - \omega_H)^2 - \delta(\omega - \omega_H) + K = 0 ,$$

where we have introduced the notation

(12b) $$\delta = \omega_0 - \omega_H - \Omega$$

(12c) $$K = \frac{\omega_{pe}^2 k^2 V_0^2}{4\, \omega_H \Omega} .$$

From the solutions of (12a), i.e.

(13) $$\omega = \omega_H + \frac{\delta}{2} \pm \frac{1}{2}(\delta^2 - 4K)^{1/2}$$

we see that if $\delta^2 > 4K$, there exist two undamped modes near the upper hybrid frequency: one of these modes is the well-known upper hybrid wave with a frequency shifted by $\frac{1}{2}(\delta - \sqrt{\delta^2 - 4K})$, due to the pump effect, and the other is a new mode which essentially arises from the beating of the pump wave with an ordinary wave. While the first mode will be damped, the second mode will start to grow when the pump intensity exceeds the threshold value

(14a) $$K_m = \frac{\delta^2}{4}$$

or

(14b) $$V_{om}^2 = \frac{\delta^2 \omega_H \Omega}{\omega_{pe}^2 k^2} .$$

Above this threshold value, the growth rate of the excited waves attains its maximum value

(14c) $$\Gamma_M = \sqrt{K} = \frac{\omega_{pe} k V_o}{2\sqrt{\omega_H \Omega}}$$

under the matching conditions (11).

In obtaining these results, we have assumed that the pump can effectively couple only two waves, due to the fact that $1/D^+$ is negligibly small compared to $1/D^-$. While this is well justified for hybrid wave numbers of the order of k_o, it definitely fails if $k \ll k_o$. The latter condition applies to the forward scattering process where the hybrid wave number becomes, for $\omega_o \gg \omega_H$,

(15a) $$k = k_o - k_o \sqrt{1 - (2\omega_o \omega_H - \omega_H^2)/c^2 k_o^2} \approx \frac{\omega_o \omega_H}{c^2 k_o} \approx \frac{\omega_H}{c}$$

and

(16a) $$D(\omega \pm \omega_o, k \pm k_o) = \pm 2(\omega_o \omega - c^2 k_o k) + \omega^2 - c^2 k^2$$
$$\approx \pm 2(\omega_o \omega - c^2 k_o k) .$$

Thus, both D^+ and D^- must be taken into account in the dispersion relation (9a), which now becomes

$$\omega^2 - \omega_H^2 = 2\omega_{pe}^2 k^2 V_o^2 \left[\frac{\omega^2 - c^2k^2}{(\omega^2 - c^2k^2)^2 - 4(\omega_o \omega - c^2 k_o k)^2} \right] \tag{17a}$$

For $ck = \omega_H + \delta$, with $\delta \ll \omega_H$, Eq.(17a) can be approximated by

$$\omega - \omega_H \approx \frac{-\omega_{pe}^2 k^2 V_o^2}{2\omega_o^2(\omega - ck)} \tag{17b}$$

or

$$(\omega - \omega_H)^2 - \delta(\omega - \omega_H) + K = 0 \tag{17c}$$

with

$$K = \frac{\omega_{pe}^2 k^2 V_o^2}{2\omega_o^2} = \frac{\omega_{pe}^2 V_o^2 (\omega_H + \delta)^2}{2\omega_o^2 c^2} . \tag{17d}$$

The threshold value for the pump intensity then is

$$V_{om}^2 = \frac{\delta^2 \omega_o^2 c^2}{2\omega_{pe}^2 (\omega_H + \delta)^2} , \tag{18a}$$

above which the growth rate attains its maximum value

$$\Gamma_M = \sqrt{K} = \frac{\omega_{pe}\, \omega_H V_o}{\sqrt{2}\, \omega_o c} \tag{18b}$$

when $\delta = 0$.

In summary, we have shown that a pump wave with a frequency well above the hybrid frequency can resonantly excite three forward waves, i.e. one upper hybrid and two ordinary waves, with frequencies ω_H, $\omega_o \pm \omega_H$, and wave numbers $\frac{\omega_H}{c}$, $k_o \pm \frac{\omega_H}{c}$, respectively.

Now, assuming that these two ordinary waves have the same initial amplitude E_1, the total transverse electric field inside the plasma can be written as

$$(19)\qquad \begin{aligned} E &= 2\,\bar{E}_0 \sin(\omega_0 t - k_0 x) + \sum_{+-} E_1 e^{\Gamma_M t} \sin\left[(\omega_0 \pm \omega_H)t - (k_0 \pm \frac{\omega_H}{c})\,x\right] \\ &= 2\left[\bar{E}_0 + E_1 e^{\Gamma_M t} \cos \omega_H \left(t - \frac{x}{c}\right)\right] \sin(\omega_0 t - k_0 x)\ , \end{aligned}$$

which just represents a pump wave with a modulated amplitude.

Thus, this three-wave coupling process can be considered as a modulational instability, in contrast to the scattering instability which is due to a two-wave coupling process.

5. Parametric Excitation of the Lower Hybrid Wave

Well below the electron plasma frequency, electrostatic waves can propagate in the lower hybrid mode according to the linear dispersion relation

$$(20a)\qquad \omega^2 = \frac{\omega_{pi}^2}{1+A} + \omega_{ci}^2 + \gamma_i k^2 \bar{V}_i^2 \equiv \omega_L^2$$

with

$$(20b)\qquad A = \frac{\omega_{pe}^2}{\omega_{ce}^2 + \gamma_e k^2 \bar{V}_e^2}\ .$$

When coupled to an ordinary wave, these lower hybrid waves follow the dispersion law

$$(21)\qquad \begin{aligned} \omega^2 - \omega_L^2 &= \frac{A}{1+A}\left(-\omega^2 + \gamma_i k^2 \bar{V}_i^2 + \omega_{pi}^2 + \omega_{ci}^2\right) k^2 V_0^2 \left(\frac{1}{D^+} + \frac{1}{D^-}\right) \\ &\approx \left(\frac{A}{1+A}\right)^2 \omega_{pi}^2\, k^2 V_0^2 \left(\frac{1}{D^+} + \frac{1}{D^-}\right)\ . \end{aligned}$$

Note that resonant interaction between the lower hybrid and ordinary waves occurs at

$$k = k_o \pm k_o \left[1 - \frac{(2\omega_o \omega_L - \omega_L^2)}{c^2 k_o^2} \right]^{1/2} . \tag{22}$$

Performing the same calculations as were done in the previous section for the scattering instability, Eq.(21) is reduced to

$$(\omega - \omega_L)^2 - \delta(\omega - \omega_L) + K = 0 \tag{23a}$$

where δ is the frequency mismatch, and

$$K = \left(\frac{A}{1+A}\right)^2 \frac{\omega_{pi}^2 k^2 V_o^2}{4 \omega_L \Omega} . \tag{23b}$$

The threshold value for the pump intensity is then given by

$$\begin{aligned} V_{om}^2 &= \frac{\delta^2 \omega_L \Omega}{\omega_{pi}^2 k^2} \left(\frac{1+A}{A}\right)^2 \\ &= \frac{\delta^2 \omega_L \Omega \left(\omega_{pe}^2 + \omega_{ce}^2 + \gamma_e k^2 \bar{V}_e^2\right)^2}{\omega_{pe}^4 \omega_{pi}^2 k^2} , \end{aligned} \tag{24a}$$

above which the growth rate assumes its maximum value

$$\Gamma_M = \sqrt{K} = \frac{\omega_{pi} k V_o}{2\sqrt{\omega_L \Omega}} \left(\frac{\omega_{pe}^2}{\omega_{pe}^2 + \omega_{ce}^2 + \gamma_e k^2 \bar{V}_e^2}\right) \tag{24b}$$

with k given by Eq.(22).

An analogous derivation for the modulational instability, corresponding to the excitation of one lower hybrid and two ordinary waves, with frequencies ω_L and $\omega_o \pm \omega_L$, yields the dispersion equation

(25a) $$(\omega - \omega_L)^2 - \delta(\omega - \omega_L) + K = 0$$

with

(25b) $$K = \left(\frac{A}{1+A}\right)^2 \frac{\omega_{pi}^2 k^2 V_0^2}{2\omega_0^2} .$$

Here, the threshold intensity and the maximum growth rate are

(26a) $$V_{om}^2 = \frac{1}{2}\left(\frac{1+A}{A}\right)^2 \frac{\delta^2 \omega_0^2 c^2}{\omega_{pi}^2 (\omega_L + \delta)^2}$$

and

(26b) $$\Gamma_M = \left(\frac{A}{1+A}\right) \frac{\omega_{pi}\, \omega_L\, V_0}{\sqrt{2}\, \omega_0\, c} .$$

Note that these results are obtained for $k \approx \frac{\omega_L}{c}$ and $\omega_0 \gg \omega_{pe}$.

6. Collisional Effects

For the sake of simplicity, collisional damping of the waves under consideration has been neglected in the foregoing analysis. For weakly damped waves, this can be included by formally replacing ω^2 by $\omega^2 + 2i\Gamma_1\omega$ and $\omega^2 + 2i\Gamma_2\omega$, respectively, in the expressions of $\mathcal{D}(\omega,k)$ and $D(\omega,k)$. Here, Γ_1 and Γ_2 represent the linear damping rates of the hybrid and ordinary waves. If this is done, one finds the following values for the threshold intensity and the maximum growth rate:

(27a) $$K_m = \Gamma_1 \Gamma_2 \left[1 + \frac{\delta^2}{(\Gamma_1 + \Gamma_2)^2} \right]$$

and

(27b) $$\Gamma_M = \frac{1}{2}\left[-(\Gamma_1 + \Gamma_2) + \sqrt{(\Gamma_1 - \Gamma_2)^2 + 4K} \right] ,$$

where K has been defined previously.

As is clear from these expressions, collisions reduce the growth rate of the interacting waves while imposing a minimum value for the threshold intensity of the pump.

7. Conclusion

Within the fluid model and the parametric approximation, it has been shown that an ordinary wave can simultaneously excite another ordinary wave and either an upper or a lower hybrid wave. Moreover, there exists a new type of instability, namely the modulational instability, which corresponds to the excitation of a long wave length (upper or lower) hybrid wave, and a modulation of the pump amplitude. An examination of the characteristic properties of these instabilities shows that

i) in a weak magnetic field, the excitation of lower hybrid waves is easier than the excitation of upper hybrid waves, whereas in a strong magnetic field, the latter are definitely favoured;

ii) the presence of a static magnetic field has a two-fold effect: on the one hand, it prevents the waves from being Landau damped and thus facilitates their excitation; on the other hand, it increases the frequencies of the interacting waves and consequently increases the threshold intensity while decreasing the maximum growth rate;

iii) in the limiting case of a zero magnetic field, the upper and lower hybrid waves become the Langmuir and ion acoustic waves, respectively. Thus, by letting $\omega_{ci} = \omega_{ce} = 0$ in the foregoing expressions, we recover the results for stimulated Raman and Brillouin scattering in an unmagnetized plasma.

Finally, it should be noted that, in the present analysis, the hybrid waves were considered as longitudinal waves. As far as the interaction between ordinary and hybrid waves is concerned, inclusion of a transverse component of the hybrid wave would not change our results in any essential way. A natural extension of this work would be the study of the parametric coupling between ordinary and Bernstein waves, using a kinetic description.

Acknowledgment

The author wishes to thank Dr. F. Hofmann for reading the manuscript.

This work was supported by the Swiss National Science Foundation.

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