May 1974

LRP 81/74

EFFECTS OF THE PUMP SYMMETRY ON THE PARAMETRIC EXCITATION OF TRANSVERSE AND LONGITUDINAL WAVES

Nguyen The Hung

Centre de Recherches en Physique des Plasmas

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

May 1974

LRP 81/74

EFFECTS OF THE PUMP SYMMETRY ON THE PARAMETRIC EXCITATION OF TRANSVERSE AND LONGITUDINAL WAVES

Nguyen The Hung

Abstract

The parametric coupling of transverse and longitudinal waves in a plasma is shown to depend strongly on the structure of the pump wave. Within the fluid model, the threshold pump intensity and the maximum growth rate of the excited waves are obtained for the cases of a linearly polarized, a travelling, and a standing circularly polarized pump wave.

Lausanne

1. Introduction

It is well-known that a strong radiation, acting as a pump, can simultaneously excite transverse and longitudinal waves in a plasma (DuBOIS, 1972). The purpose of this paper is to study the effects of the pump polarization on the wave-wave coupling process. Within the fluid model, we shall derive the coupled equations for the cases of a linearly polarized, a travelling, and a standing circularly polarized pump. As will be shown, the coupling process depends essentially on the symmetry of the pump field.

2. The Coupled Equations

Let us consider the interaction between transverse and longitudinal waves, all propagating along the z-axis, in an homogeneous, unbounded plasma. Starting from the well-known fluid equations (namely the continuity, the momentum and Maxwell's equations, KRALL and TRIVELPIECE, 1973), one can show that the coupling between electrostatic and electromagnetic electron waves is governed by the following equations:

$$\text{(1a)} \qquad \frac{\partial^2 n}{\partial t^2} + 2\Gamma_1 \frac{\partial n}{\partial t} - 3\bar{v}_e^2 \frac{\partial^2 n}{\partial z^2} + \omega_p^2 n = n_0 \frac{\partial}{\partial z}\left(\sum_j V_j \frac{\partial V_j}{\partial z}\right)$$

$$\text{(1b)} \qquad \frac{\partial^2 V_j}{\partial t^2} + 2\Gamma_2 \frac{\partial V_j}{\partial t} - c^2 \frac{\partial^2 V_j}{\partial z^2} + \omega_p^2 V_j = -\frac{\omega_p^2}{n_0}\, n V_j \quad , \quad (V_j = V_x, V_y)$$

where n, $\vec{V}$, $\bar{v}_e$ and ω_p are the electron density, the transverse velocity, the thermal velocity and the plasma frequency, respectively. Γ_1 and Γ_2 are the linear (collisional and/or Landau) damping rates of the longitudinal and transverse waves.

From Eq.(1), it is easily seen that the coupling between two transverse waves induces a longitudinal wave through the non-linear Lorentz force $V_j \frac{\partial V_j}{\partial z}$, while the coupling between a transverse and a longitudinal wave induces another transverse wave through the non-linear current $-enV_j$.

After a Fourier transformation with respect to space and time, Eq.(1) becomes

(2a) $$\mathcal{D}(\omega,k)\, n(\omega,k) = \frac{n_0}{2} k^2 \sum_j \int V_j(\omega',k')\, V_j(\omega-\omega', k-k')\, d\omega' dk'$$

(2b) $$D(\omega,k)\, V_j(\omega,k) = \frac{\omega_p^2}{n_0} \int V_j(\omega',k')\, n(\omega-\omega', k-k')\, d\omega' dk'$$

with

(2c) $$\mathcal{D}(\omega,k) \equiv \omega^2 + 2i\Gamma_1 \omega - \omega_k^2 \equiv \omega^2 + 2i\Gamma_1 \omega - 3\bar{v}_e^2 k^2 - \omega_p^2$$

and

(2d) $$D(\omega,k) \equiv \omega^2 + 2i\Gamma_2 \omega - \Omega_k^2 \equiv \omega^2 + 2i\Gamma_2 \omega - c^2 k^2 - \omega_p^2 \; .$$

Let us now consider the case of parametric coupling where one has a strong, externally-imposed wave acting as a pump to excite other plasma waves through the mechanism previously described.

By assuming that $V_j = V_{oj} + v_j$, with $v_j \ll V_{oj}$,

and keeping only terms of first order in V_{oj} , Eq.(2) then reduces to the following linearized equations:

(3a) $$\mathcal{D}(\omega,k)\, n(\omega,k) = n_0 k^2 \sum_j \int V_{oj}(\omega',k')\, v_j(\omega-\omega', k-k')\, d\omega' dk'$$

(3b) $$D(\omega,k)\, v_j(\omega,k) = \frac{\omega_p^2}{n_0} \int V_{oj}(\omega',k')\, n(\omega-\omega', k-k')\, d\omega' dk' ,$$

which can be readily solved in a variety of situations.

3. The Linearly Polarized Pump

For a pump wave polarized along the x-axis (with unit vector $\vec{e}_x$) such that

(4a) $$\vec{E}_0 = 2\,\mathcal{E}_0\,\vec{e}_x \sin(\omega_0 t - k_0 z) \,,$$

the Fourier transform of the pump-induced velocity is

(4b) $$V_{0x} = V_0 \left[\delta(\omega-\omega_0)\,\delta(k-k_0) + \delta(\omega+\omega_0)\,\delta(k+k_0) \right]$$

with

(4c) $$V_0 = e\,\mathcal{E}_0 / m\omega_0 \,.$$

Substituting (4b) into (3) yields

(5a) $$\mathcal{D}(\omega,k)\; n(\omega,k) = n_0 V_0 k^2 \left[v_x(\omega-\omega_0, k-k_0) + v_x(\omega+\omega_0, k+k_0) \right]$$

(5b) $$D(\omega,k)\; v_x(\omega,k) = \frac{\omega_p^2}{n_0} V_0 \left[n(\omega-\omega_0, k-k_0) + n(\omega+\omega_0, k+k_0) \right],$$

which show the well-known properties of the parametric coupling process: each component $n(\omega,k)$ is coupled to $v_x(\omega \pm \omega_0, k \pm k_0)$, which in turn couple with $n(\omega,k)$ and $n(\omega \pm 2\omega_0, k \pm 2k_0)$. Consequently, one has to deal with an infinite set of linear equations, whose solutions can be obtained by the so-called resonant approximation (NISHIKAWA, 1968).

A study of Eq.(5) shows that the coupling process is most effective under the decay conditions

(6a) $$\omega_0 \approx \omega_k + \Omega_{k-k_0} \,,$$

which can be satisfied by

(6b) $$k \approx k_0 \pm \frac{\omega_0}{c}\sqrt{1 - 2\omega_p/\omega_0} \equiv k_0 \pm k_1 .$$

Within the resonant approximation, one then obtains the following threshold value for the pump intensity:

(7a) $$V_{om}^2 = \frac{4\,\omega_k\,\Omega_{k-k_0}\,\Gamma_1\Gamma_2}{\omega_p^2 k^2} .$$

For a pump intensity above this threshold value, the maximum growth rate of the excited waves is given by

(7b) $$\gamma_M = \frac{1}{2}\left[-(\Gamma_1+\Gamma_2) + \sqrt{(\Gamma_1+\Gamma_2)^2 + \omega_p^2 k^2 V_0^2/\omega_k\Omega_{k-k_0}}\right] .$$

Note that in Eq. (7), k must take the values given by Eq. (6b).

4. The Travelling Circularly Polarized Pump

For a pump wave with a circular polarization such that

(8a) $$\vec{E}_0 = 2\mathcal{E}_0\left[\vec{e}_x \sin(\omega_0 t - k_0 z) + \vec{e}_y \sin(\omega_0 t - k_0 z - \tfrac{\pi}{2})\right] ,$$

the Fourier transform of the induced velocity V_{oj} is

(8b)
$$V_{ox} = V_0\left[\delta(\omega-\omega_0)\,\delta(k-k_0) + \delta(\omega+\omega_0)\,\delta(k+k_0)\right]$$
$$V_{oy} = -iV_0\left[\delta(\omega-\omega_0)\,\delta(k-k_0) - \delta(\omega+\omega_0)\,\delta(k+k_0)\right] .$$

Upon substituting (8b) into (3) and introducing the new variables

(9a)
$$v_- = v_x - i\,v_y$$
$$v_+ = v_x + i\,v_y ,$$

we obtain the following coupled system:

$$\mathcal{D}(\omega,k)\, n(\omega,k) = n_0 V_0 k^2 \left[v_-(\omega-\omega_0, k-k_0) + v_+(\omega+\omega_0, k+k_0) \right]$$

(9b) $$D(\omega,k)\, v_-(\omega,k) = \frac{2\omega_p^2 V_0}{n_0}\, n(\omega+\omega_0, k+k_0)$$

$$D(\omega,k)\, v_+(\omega,k) = \frac{2\omega_p^2 V_0}{n_0}\, n(\omega-\omega_0, k-k_0)\ .$$

Eq.(9b) shows that each component $n(\omega,k)$ is coupled to $v_-(\omega-\omega_0, k-k_0)$ and $v_+(\omega+\omega_0, k+k_0)$, which in turn couple only with $n(\omega,k)$. Consequently, we have, in this case, a closed system of 3 equations for a three-wave coupling process.

From Eq.(9b) we easily deduce the following dispersion relation for the electrostatic waves:

(10) $$\mathcal{D}(\omega,k) = 2k^2 V_0^2 \omega_p^2 \left[\frac{1}{D(\omega+\omega_0, k+k_0)} + \frac{1}{D(\omega-\omega_0, k-k_0)} \right],$$

which is valid for a pump field of arbitrary intensity.

Under the decay conditions (6), we now obtain:

- the threshold pump intensity

(11a) $$V_{om}^2 = \frac{2\omega_k \Omega_{k-k_0} \Gamma_1 \Gamma_2}{\omega_p^2 k^2}$$

- and the maximum growth rate of the excited waves

(11b) $$\gamma_M = \frac{1}{2}\left[-(\Gamma_1+\Gamma_2) + \sqrt{(\Gamma_1-\Gamma_2)^2 + 2\omega_p^2 k^2 V_0^2/\omega_k \Omega_{k-k_0}} \right].$$

Note that the factor 2 on the RHS of Eq.(11) is only an apparent advantage of the circular pump over the linear pump. In fact, one has to apply two linearized fields to obtain a circular one. The real advantage of a circular pump wave lies in the fact that it only induces a three-wave coupling process, and therefore can feed its energy more efficiently into these modes.

5. The Standing Circularly Polarized Pump

For a standing pump wave such that

(12) $$\vec{E}_0 = 2\mathcal{E}_0 \Big\{ \vec{e}_x \left[\sin(\omega_0 t - k_0 z) + \sin(\omega_0 t + k_0 z) \right] + \vec{e}_y \left[\sin(\omega_0 t - k_0 z - \tfrac{\pi}{2}) + \sin(\omega_0 t + k_0 z - \tfrac{\pi}{2}) \right] \Big\} ,$$

the coupled system (3) becomes

(13) $$\begin{aligned} \mathcal{D}(\omega,k)\, n(\omega,k) &= n_0 V_0 k^2 \Big[v_-(\omega-\omega_0, k-k_0) + v_-(\omega-\omega_0, k+k_0) + v_+(\omega+\omega_0, k+k_0) + v_+(\omega+\omega_0, k-k_0) \Big] \\ D(\omega,k)\, v_\pm(\omega,k) &= \frac{2\omega_p^2 V_0}{n_0} \Big[n(\omega\mp\omega_0, k\mp k_0) + n(\omega\mp\omega_0, k\pm k_0) \Big], \end{aligned}$$

which shows that one has to deal again with an infinite set of linear equations. One notes that, in this case, the coupled components $n(\omega,k)$ all have the same frequency, while their wave numbers differ from one another by integer numbers of k_0 . This is in contrast to the case of a linearly polarized pump where the coupled components differ from one another in frequency <u>and</u> in wave number by integer numbers of ω_0 and k_0 , respectively.

Under the decay conditions (6), Eq.(13) can be approximated as

(14) $$\begin{aligned} \mathcal{D}(\omega,k)\; n(\omega,k) &= n_0 V_0 k^2\; v_-(\omega-\omega_0, k-k_0) \\ \mathcal{D}(\omega, k-2k_0)\, n(\omega, k-2k_0) &= n_0 V_0 (k-2k_0)^2\; v_-(\omega-\omega_0, k-k_0) \\ D(\omega-\omega_0, k-k_0)\, v_-(\omega-\omega_0, k-k_0) &= \frac{2\omega_p^2 V_0}{n_0} \Big[n(\omega,k) + n(\omega, k-2k_0) \Big] , \end{aligned}$$

which yield the following dispersion relation:

(15) $$\mathcal{D}(\omega,k) = \frac{2\omega_p^2 V_0^2}{D(\omega-\omega_0, k-k_0)} \left[k^2 + (k-2k_0)^2 \frac{\mathcal{D}(\omega,k)}{\mathcal{D}(\omega,k-2k_0)} \right]$$

$$\approx \frac{4\omega_p^2 V_0^2 \,(k_0^2 + k_1^2)}{D(\omega-\omega_0, k-k_0)} .$$

The minimum threshold value for the pump intensity is given by

(16a) $$V_{om}^2 = \frac{\omega_k \, \Omega_{k_1} \, \Gamma_1 \Gamma_2}{\omega_p^2 \, (k_o^2 + k_1^2)} ,$$

while the maximum growth rate of the excited waves is

(16b) $$\gamma_M = \frac{1}{2}\left[-(\Gamma_1 + \Gamma_2) + \sqrt{(\Gamma_1 + \Gamma_2)^2 + 4\,\omega_p^2 V_o^2 (k_o^2 + k_1^2)/\omega_k \Omega_{k_1}}\,\right] .$$

Note that, due to the pump symmetry in this case, one observes the same phenomena in the forward and backward directions.

6. Conclusion

In the foregoing sections, we have shown how the parametric coupling process depends on the pump nature: A linearly polarized pump excites simultaneously an infinite number of plasma waves with frequencies and wave numbers $\omega \pm N\omega_o$, $k \pm Nk_o$, $(N = 0 \rightarrow \infty)$; a travelling circularly polarized pump can excite only one plasma wave while a standing circularly polarized pump excites an infinite set of plasma waves having the same frequency, but whose wave numbers differ from one another by integer numbers of k_o .

Finally, note that for the high frequency waves considered in this paper, effects of the ions have been neglected. Extension to the case of a magnetized plasma, and inclusion of the ion waves have also been considered and will be reported later.

Acknowledgement

The author wishes to thank Dr. F. Hofmann for the reading of this manuscript.

This work is performed under the auspices of the Swiss National Science Foundation.

References

DuBOIS D.F. (1972) Interaction of Intense Radiation with Plasmas. Lectures given at the University of Colorado, Department of Astro-Geophysics.

KRALL N.A. and TRIVELPIECE A.W. (1973) Principles of Plasma Physics. Mc Graw-Hill Book Company, New York.

NISHIKAWA K. (1968) J.Phys.Soc. Japan 24, 916 and 24, 1152.