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A SIMPLE STATIC MODEL FOR ECONOMIC COMPETITION

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A b s t r a c t

We construct a model which describes how a consumer allocates his income to the purchase of various products and services. Different forms of competition between producers are examined. The results are in qualitative agreement with observations.

Lausanne

1. Consumers Income Allocation

Every consumer is confronted with a market which offers various products and services. We wish to examine how such a consumer allocates fractions of his income to the purchase of products and services for the satisfaction of his needs. Products and services shall be labelled by the integer j . The quantity of any product shall at first be measured in arbitrary units. The quantity of a product bought by the consumer per unit time shall be designated by the acquisition rate σ_j . The unit price of product j shall be x_j . Different needs shall be identified by the integer α . A particular consumer may consider certain products, $j \in J_\alpha$, as interchangeable in the sense that they satisfy the same need. This interchangeability can be represented by a product substitution matrix $\bar{q}_{\alpha j}$, where the indices α , j label needs and products. The matrix elements are so chosen that, for a given positive value $\bar{s}_\alpha > 0$, every solution $\sigma_j^{(\alpha)} > 0$ of the equations

$$\sigma_j^{(\alpha)} = 0 \qquad , \qquad j \notin J_\alpha \tag{1}$$

$$\sum_{j \notin J_\alpha} \bar{q}_{\alpha j} \; \sigma_j^{(\alpha)} = \bar{s}_\alpha \; , \tag{2}$$

represents the acquisition of a product mix which satisfies the need α to the same extent. Clearly $\bar{q}_{\alpha j}$, can be multiplied by arbitrary dimensionless factors λ_α without changing the meaning of (2),

$$\lambda_\alpha \bar{q}_{\alpha j} = q_{\alpha j} \quad , \quad \lambda_\alpha \bar{s}_\alpha = s_\alpha$$

and

$$\sum_{j \in J_\alpha} q_{\alpha j} \; \sigma_j^{(\alpha)} = s_\alpha \tag{3}$$

s_α can be called the acquisition rate of the product mix satisfying the need α. The value of λ_α simply changes the units in which this acquisition rate is measured. Let $\sigma_j^{(\alpha)}$ be a solution of (1,3). These acquisition rates entail the spending rate

$$\phi_\alpha = \sum_{j \in J_\alpha} \sigma_j^{(\alpha)} x_j \tag{4}$$

where ϕ_α is a flow of money measured in units of money per unit time.

We assume in this section, that all solutions $\sigma_j^{(\alpha)}$ of equation (1) and (3) lead to the same flow ϕ_α. This can only be so if

$$x_j = \text{const} \cdot q_{\alpha j} \quad , \quad j \in J_\alpha \tag{5}$$

This is of course always approximately true, otherwise the products within J_α would not be considered interchangeable. However, it is never exactly true. This question is related to competition in the market which will be treated in the following sections. Here we simply assume (5) to hold and regard

$$p_\alpha = \phi_\alpha / s_\alpha$$

as the price for a unit of an abstract "product" satisfying the need α. The rate of consumption of this product is s_α.

We now wish to introduce a measure of satisfaction. The satisfaction of the need α must be a function of the acquisition rate s_α, which is zero for $s_\alpha = 0$, rises monotonically with s_α and reaches a maximum for some value $s_\alpha^{(sat)}$ for which the need is completely satisfied.

It is convenient although not necessary to assume that the shape of the satisfaction functions is the same independent of the need and that they differ only in scale factors. This means that the satisfaction S_α derived from the acquisition rate $\bar{s}_\alpha$ (measured in arbitrary units*) of the product α has the form

$$S_\alpha = a_\alpha \, S(\tau_\alpha \bar{s}_\alpha)$$

The parameter a_α measures the demand by the consumer of the product α. It is convenient to normalize the satisfaction function $S(x)$ in the following way

$$S(x) \quad \text{defined in} \quad 0 \le x \le 1$$

$$S(0) = 0 \;, \quad S'(0) = 1 \;, \quad S'(1) = 0$$

$$S'(x) > 0 \quad \text{if} \quad 0 \le x < 1$$

It is difficult to guess what the detailed shape of S should be, Fig. 1. We shall frequently use the simple form $S(x) = x - x^{3/2}$, Fig. 1b.

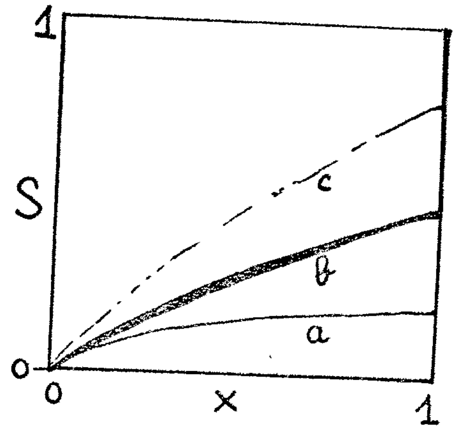

Fig. 1 Different possible shapes of the satisfaction function $S(x) = x - x^{\gamma}/\gamma$, $\gamma > 1$. a: $\gamma = 3/2$, b: $\gamma = 2$, c: $\gamma = 3$

In addition we assume that total satisfaction is the sum of the satisfactions of independent needs. Thus we write for the total satisfaction

$$S_{tot} = \sum_\alpha a_\alpha \, S(\tau_\alpha \bar{s}_\alpha) \tag{6}$$

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*For instance: 15 kg of sugar per year, or 0.2 Volks-Wagens per year.

We now choose the scaling factors λ_α such that

$$\lambda_\alpha = \tau_\alpha / \tau$$

where τ can be chosen freely*. Thus the satisfaction becomes

$$S_{tot} = \sum_\alpha a_\alpha \, S(\tau \, s_\alpha) \tag{7}$$

By this transformation the acquisition rate s_α has been normalized. This simply means that the quantity of a product is measured in a unit which represents a fixed fraction the quantity that gives complete satisfaction (which saturates the demand). As a consequence the unit prices of the various products are also defined in a unique way, apart from a constant factor. It is therefore possible to order all the products according to ascending prices. Actually, as we shall see shortly it is more useful to order the products according to ascending price demand ratio

$$p_\alpha / a_\alpha \leq p_{\alpha+1} / a_{\alpha+1} \tag{8}$$

The consumer may of course choose to save a fraction of his income. He may save in units of $\bar{p}_\beta$ at the rate of $\bar{s}_\beta$, which results in a flow of money ϕ_β into savings

$$\phi_\beta = \bar{p}_\beta \, \bar{s}_\beta$$

leading to a partial satisfaction from savings

$$S_\beta = a_\beta \, S(\tau_\beta \, \bar{s}_\beta)$$

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* τ could be set equat to 1. However, τ has the dimension of time, and we wish to postpone the normalization of the time scale.

or properly normalized

$$S_\beta = a_\beta \; S(\tau s_\beta)$$

this normalization determines the unit of savings p_β. Thus savings can be represented, in the present, context in the same way as consumption.

The consumer of course maximizes his satisfaction subject to the constraint that the out flow of money

$$\sum_\alpha s_\alpha p_\alpha = \phi \tag{9}$$

is equal to his income. Using Lagrange's method and introducing the multiplier $1/\mu$ we write the maximizing condition as

$$\frac{\partial}{\partial s_\alpha} \sum_\beta \left\{ a_\beta \, S(\tau s_\beta) - \frac{1}{\mu} p_\beta s_\beta \right\} = 0$$

whence

$$\tau a_\beta \, S'(\tau s_\beta) = p_\beta/\mu \qquad , \quad S'(x) = dS/dx$$

Introducing the function $E(x)$, defined as the inverse of $S'(x)$ we can write the result as

$$s_\alpha = \frac{1}{\tau} E\left(\frac{p_\alpha}{\tau \mu a_\alpha}\right) \tag{10}$$

The interval of definition of $E(x)$ is also $0 \leq x \leq 1$. (If we use $S(x) = x - x^2/2$ then $E(x) = 1 - x$).

The multiplier μ must be determined from the condition (9)

$$\phi = \frac{1}{\tau} \sum_{\alpha} p_\alpha \, E\left(\frac{p_\alpha}{\tau\mu a_\alpha}\right)$$

It is important to note that $S(x)$ and $E(x)$ are defined only in finite intervals. As the argument of E increases, s decreases. If x exceeds 1 the corresponding acquisition rate must be set equal to zero. Thus we are led to define an integer α_* by the inequalities

$$\frac{p_{\alpha_*}}{a_{\alpha_*}} < \tau\mu \leq \frac{p_{\alpha_*+1}}{a_{\alpha_*+1}} \tag{11}$$

Hence

$$\phi = \frac{1}{\tau} \sum_{\alpha=0}^{\alpha_*} p_\alpha \, E\left(\frac{p_\alpha}{\tau\mu a_\alpha}\right) \tag{12}$$

Assuming a large number of different products we may approximate the sum (12) by an integral

$$\phi = \frac{1}{\tau} \int_0^{\alpha_*} p_\alpha \, E\left(\frac{p_\alpha}{\tau\mu a_\alpha}\right) d\alpha \tag{13}$$

$$\frac{p_{\alpha^*}}{a_{\alpha^*}} = \tau\mu \tag{14}$$

It is obvious that α_* is a monotonically increasing function of μ. We now show that ϕ also is increasing with μ : Indeed, varying μ one obtains

$$\frac{d\phi}{d\mu} = p_{\alpha_*} \, E\left(\frac{p_{\alpha_*}}{\tau\mu a_{\alpha_*}}\right) \frac{d\alpha^*}{d\mu} - \frac{1}{\mu^2} \int_0^{\alpha_*} p_\alpha \, E'\left(\frac{p_\alpha}{\tau\mu a_\alpha}\right) d\alpha$$

The first term is zero, the second positive. Hence

$$d\phi/d\mu > 0 \tag{15}$$

Thus if more income is available more needs can be satisfied, and to a greater extent. It will sometimes be convenient to regard μ as the independent variable and to consider $\phi(\mu)$ and $\alpha_*(\mu)$ as functions of μ*.

If we look at our result (10) we see at once that our "consumer" cannot represent an individual person. Indeed this equation predicts that a rich individual buys both Volks-Wagens and Rolls Royces, the former at a higher rate ! This is true for large groups of consumers but almost never for an individual. Our "consumer" thus represents an average or a group of consumers.

The size of a consumer group can easily be taken into account by multiplying the demand a_α and the rate of acquisition which saturates this demand by the weight g of the group:

$$S_\alpha = g a_\alpha \, S(\tau s_\alpha / g)$$

For a group of individual consumers g would be their number. For the sake of simplicity we shall henceforth consider only a single group of consumers, and normalize g to one.

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*As an illustration we take the case in which all $a_\alpha = 1$ and we assume that p_α is represented by $p_\alpha = p_0(\exp \alpha - 1)$. Hence $\alpha_* = \lg(1+\tau\mu/p_0)$ and a simple integration gives the relations

$$\tau\phi = \tau\mu/2 + p_0 - p_0\left(1+\frac{p_0}{\tau\mu}\right)\lg\left(1+\tau\mu/p_0\right)$$
$$= \frac{p_0}{2}\left(1 + e^{\alpha_*} - \frac{2\alpha_*}{1-e^{\alpha_*}}\right)$$

expressing ϕ as a function of μ or of α .

If we describe the entire economy by means of a single consumer group then α_* is no longer a variable, but simply the number of all types of products. There is no $\alpha > \alpha_*$ since a product that is not sold is not part of the economy. Equation (12) or (13) then establishes a definite relation between the total flow of money and the prices, assuming of course a steady state and fixed properties of the consumer a_α. This result could easily be generalized for several consumers groups of different characteristics.

2. Distribution of purchases among different sales channels and among similar products

Products are rarely exactly interchangeable in the sense defined in section 1. Substitution will lead to slightly different prices of the product mix, so that (5) is not exactly satisfied. Furthermore, the price of a given product varies from one supplier to the other. A given need α can be satisfied to the extent s_α at different prices according to the product and the supplier chosen. This fact can be represented by the price spectrum $p_{\alpha k}$ where α labels the need and k the channel through which the product is acquired. We number the different supply channels of the product α from 1 to N_α. If the same producer operates several sales organizations each is counted separately. Experience shows that a consumer does not simply buy through the least expensive channel. The decision to buy here or there is, when many buyers are involved, a matter of chance. However, as we have seen, the maximization of satisfaction by each consumer leads to a definite flow of money ϕ_α destined to satisfy the need α. The most probable distribution of this flow of money among the different channels can be obtained by arguments which are familiar to every physicist.

A consumer who has allocated the flow ϕ_α (of money per unit time) to the satisfaction of the need α makes $\phi_\alpha / p_\alpha = s_\alpha$ purchases per unit time, each time he decides through which channel he wants to buy. In the time t he may buy $s_{\alpha j}\, t$ units through channel j. The number W_α of different decision sequences leading to the same buying pattern is given by

$$W_\alpha = \frac{(t s_\alpha)!}{\prod_j (t s_{\alpha j})!}$$

On the average we expect the most probable sequence to be realized. With respect to all needs the number of possible sequences is

$$W = \prod_\alpha W_\alpha$$

To find the most probable sequences we have to maximize W subject to the constraints

$$\sum_{j=1}^{N_\alpha} s_{\alpha j} = s_\alpha$$

and

$$\sum_{j=1}^{N_\alpha} s_{\alpha j}\, p_{\alpha j} = \phi_\alpha \tag{16}$$

As is well-known, the result is

$$s_{\alpha j} = s_\alpha \, \frac{e^{-\beta_\alpha p_{\alpha j}}}{Z_\alpha} \tag{17}$$

with the partition function

$$Z_\alpha = \sum_{j=1}^{N_\alpha} e^{-\beta_\alpha p_{\alpha j}} \tag{18}$$

This result fortunately does not depend either on t nor on the units with which the quantity of a product is measured.

The condition (16) takes the form

$$\phi_\alpha = - s_\alpha \frac{\partial}{\partial \beta_\alpha} \lg z_\alpha \tag{19}$$

The average price is

$$p_\alpha = - \frac{\partial}{\partial \beta_\alpha} \lg z_\alpha \tag{20}$$

Obviously:

$$\min(p_{\alpha j}) \leq p_\alpha \leq \max(p_{\alpha j})$$

If the price spectrum is fixed the average price is a function of β_α. This quantity shall be called the consumers' selectivity with respect to the product α. The selectivity in general is positive, however, in some exceptional cases (products with "snob appeal") it may be negative. Equation (20) renders precise the meaning of the average price for the satisfaction of a unit of need α, which was used in section 1. Sales $s_{\alpha j}$ increase with demand a_α through s_α, and they increase with decreasing prices; this agrees with common sense.

We have established a system of equations (10, 11, 12, 17,18, 20) which describes the consumers'response to the offers of the market. These equations are valid and make sense only if the quantity of each product is measured in terms of units which have been normalized against the saturation rate of acquisition. That is the rate of acquisition which provides the maximum of satisfaction. These units therefore implicitely reflect a property of the consumer. The consumer is further characterized explicitely by his income ϕ his demand a_α and selectivity β_α with respect to each product. The market is described by the price spectrum $p_{\alpha j}$.

The system of equations then determines uniquely how the consumer distributes his income for the purchase of goods through different channels: that is ϕ_α and $\sigma_{\alpha j}$.

3. Prices and Profits

In this section we examine the market from the point of view of the producer j of product α. We assume here that he produces exactly as many units $s_{\alpha j}$ as he can sell. The production process entails fixed basic costs $B_{\alpha j}$ per unit time and costs which are proportional to the number of units produced, the unit costs being $x_{\alpha j}$. The product unit is priced at $p_{\alpha j}$. The rate of profit per unit time $P_{\alpha j}$ which is realized is therefore

$$P_{\alpha j} = -B_{\alpha j} + (p_{\alpha j} - x_{\alpha j})\, s_{\alpha j}$$

where $s_{\alpha j}$ is given by

$$s_{\alpha j} = s_\alpha \frac{e^{-\beta_\alpha p_{\alpha j}}}{z_\alpha}, \quad z_\alpha = \sum_{j=1}^{N_\alpha} e^{-\beta_\alpha p_{\alpha j}}$$

and

$$s_\alpha = \frac{1}{\tau} E\left(\frac{p_\alpha}{\tau \mu a_\alpha}\right)$$

The producer tends to maximize his profit. He can try to influence the consumer by advertising in an attempt to increase the parameter a_α which measures the consumers demand of the product. He may also reduce his fixed and unit costs. Finally he may adjust his price so as to maximize his profit. This is the variable that he controls easily and without delay. Putting $dP_{\alpha j}/dp_{\alpha j}$ we obtain the condition

$$1 + (p_{\alpha j} - x_{\alpha j})\left\{-\beta_\alpha + \left[\beta_\alpha + \frac{\beta_\alpha (p_\alpha - p_{\alpha j}) + 1}{\tau \mu a_\alpha} \cdot \frac{E(x)}{E'(x)}\right] \frac{e^{-\beta_\alpha p_{\alpha j}}}{z_\alpha}\right\} = 0 \qquad (21)$$

In this expression p_α is the average price of the product α and $x = p_\alpha / \tau \mu a_\alpha$. We have assumed that the number of channels N_α is so large that changes of the price in one of them has a negligible effect on μ, which we have kept constant. The condition (21) determines in principle the price $p_{\alpha j}$ which maximizes $P_{\alpha j}$. However, this equation may have no solution, or it may have no acceptable solution: Having achieved the maximum possible profit is of small comfort to the business man, if this maximum is negative, as it might be if the fixed costs are large.

The optimum price for a given channel is a function of the characteristics of the buyer $a_\alpha \, \beta_\alpha$, but in general also of all other prices. Hence analytic solutions are difficult, except in special cases.

In real life a producer does not have an analytic expression for his profit. He attempts to maximize it by trial and error, so that the actual prices are scattered about the optimal ones. But this is a different problem which is of no concern to us here.

In the following section we examine several limiting cases representing individual competing producers, a monopoly, and a cartel against one individual producer.

4. Large number of competitions

We assume that a very large number of competitors produce the product α. In this case $\exp(-\beta_\alpha \, p_{\alpha j})/z_\alpha$ is a very small number and we obtain the approximate solution

$$p_{\alpha j} = x_{\alpha j} + \frac{1}{\beta_\alpha} \tag{22}$$

The price variations from one to the other producer reflect simply the variations of unit costs. This approximate solution is valid only if $E(x) = E(p_\alpha/\tau\mu a_\alpha$ is not too small. If x_α represents the average unit cost of all producers then the average price is

$$p_\alpha = x_\alpha + \frac{1}{\beta_\alpha} \tag{23}$$

Thus, the validity of (23) is assured only if

$$x_\alpha \ll \tau\mu a_\alpha - \frac{1}{\beta_\alpha} \tag{24}$$

The profit margin, $p_{\alpha j} - x_{\alpha j} = 1/\beta_\alpha$, which is the same for each producer can be quite small if the consumers selectivity is large. Total profits can vary appreciably depending on production costs:

$$P_{\alpha j max} = - B_{\alpha j} + \frac{\sigma_\alpha}{\beta_\alpha} \; \frac{e^{-\beta_\alpha x_{\alpha j}}}{\sum_i e^{-\beta_\alpha x_{\alpha i}}}$$

Particularly if consumers selectivity is high $\beta_\alpha x_{\alpha j} \gg 1$, the success of a producer depends critically on small variations of his unit costs. Free competition then incites the producer to increase his efficiency, and forces the prices in all channels to be nearly the same*.

5. Monopoly

We speak of a monopoly if a particular product α is sold by a single producer $N_\alpha = 1$, $p_\alpha = p_{\alpha 1}$. Total sales of the product are simply s_α units and total profit becomes

$$P_\alpha = - B_\alpha + (p_\alpha - x_\alpha)\, s_\alpha$$

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* Very high selectivity of buyers is found for instance in the international grain markets. Extremely low selectivity is exhibed by the tourists who shop for pottery as souvenirs.

B_α and $\varkappa_\alpha$ are the monopoly producer's fixed and unit costs. Since $\exp(-\beta_\alpha p_\alpha)/Z_\alpha = 1$ equ. (21) becomes

$$p_\alpha = \varkappa_\alpha - \tau\mu a_\alpha \frac{E(x)}{E'(x)} \quad , \quad x = \frac{p_\alpha}{\tau\mu a_\alpha} \leq 1 \tag{26}$$

The monopoly price may be higher or lower than the competitive price depending on whether the positive quantity

$$Q = -\tau\mu a_\alpha \beta_\alpha \frac{E(x)}{E'(x)}$$

is larger or smaller than one.

If we adopt the particular satisfaction function $S = x - x^2/2$ the equation (26) can be solved for the price

$$p_\alpha = \frac{1}{2}(\varkappa_\alpha + \tau\mu a_\alpha) \tag{27}$$

provided that

$$\varkappa_\alpha < \tau\mu a_\alpha \tag{28}$$

If the latter condition is violated the product cannot be sold for profit at any price. Whether or not a profit is actually possible depends of course on the fixed costs; (27) is only a necessary condition. Obviously the profit margin increases with affluence of the consumer $d\mu/d\phi > 0$ and with his demand a_α but is independent of his selectivity: he has in fact no choice.

We can compare the monopoly price to the competitive price in the special case considered above $\varkappa_\alpha \ll \tau\mu a_\alpha$, if we assume that the unit cost is about the same for both, and that the selectivity of the consumer is high. In this case the monopoly price is higher than the competitive price

$$p_\alpha^{mono} - p_\alpha^{comp} = \frac{1}{2}\left(\tau\mu a_\alpha - \varkappa_\alpha - \frac{2}{\beta_\alpha}\right) \approx \frac{1}{2}\tau\mu a_\alpha > 0$$

However the opposite can be true if the buyer's selectivity β_α or demand a_α is small.

6. Cartel versus single producer

A cartel exists if different producers of the same product conspire to charge the same price. This case can easily be represented by putting $p_{\alpha i} = p_{\alpha j}$ for all $i\ j \in C_A$. The partition function then becomes

$$Z_\alpha = \sum_A n_{\alpha A}\ e^{-\beta_\alpha p_{\alpha A}}$$

Where $n_{\alpha A}$ is the number of producers within the cartel A. Total sales of the cartel A then become

$$s_{\alpha A} = \sigma_\alpha \cdot \frac{n_{\alpha A} e^{-\beta_\alpha p_{\alpha A}}}{Z_\alpha}$$

The maximization of profit for cartel A then requires the condition

$$1 + (p_{\alpha A} - \varkappa_{\alpha A})\left\{-\beta_\alpha + \left[\beta_\alpha + \frac{1+\beta_\alpha(p_\alpha - p_{\alpha A})}{\tau\mu a_\alpha}\ \frac{E(x)}{E'(x)}\right]\frac{n_{\alpha A} e^{-\beta_\alpha p_{\alpha A}}}{Z_\alpha}\right\} = 0 \quad (29)$$

Assume now that only one large cartel $n_{\alpha 1} = n \gg 1$ and one single producer $n_{\alpha 0} = 1$ sell the product α . Then

$$Z_\alpha = n\, e^{-\beta_\alpha p_{\alpha 1}} + e^{-\beta_\alpha p_{\alpha 0}}$$

The average price will very nearly be equal to the cartel price $p_\alpha \cong p_{\alpha 1}$.
Thus for the cartel A = 1 equation (29) implies

$$p_{\alpha 1} = \varkappa_{\alpha 1} - \tau \mu a_\alpha \frac{E(x)}{E'(x)}$$

or

$$p_{\alpha 1} = \frac{1}{2}\left(\varkappa_{\alpha 1} + \tau \mu a_\alpha\right)$$

if we use $S(x) = x - x^2/2$. For the individual producer we obtain

$$p_{\alpha 0} = \varkappa_{\alpha 0} + \frac{1}{\beta_\alpha}$$

provided (24) holds. These results are formally the same as those obtained separately for monopoly and free competition.

7. Dumping

Maximizing one's profit is a relatively harmless strategy. A producer may instead lower his price in order to attract most of the sales. He forgoes profit temporarily in the hope of ruining his competitor. This is called dumping.

Let us examine, as in section 5, rates of sales of a cartel and an individual producer.

individual: $$s_{\alpha 0} = s_\alpha \frac{e^{-\beta_\alpha p_{\alpha 0}}}{e^{-\beta_\alpha p_{\alpha 0}} + n e^{-\beta_\alpha p_{\alpha 1}}}$$

cartel: $$s_{\alpha 1} = s_\alpha \frac{n e^{-\beta_\alpha p_{\alpha 1}}}{e^{-\beta_\alpha p_{\alpha 0}} + n e^{-\beta_\alpha p_{\alpha 1}}}$$

If the cartel and the individual practice the same price $p_{\alpha 0} = p_{\alpha 1}$ then the sales are distributed in the ratio of $1:n$. However, if the cartel reduces its price $p_{\alpha 1} = p_{\alpha 0} - \Delta$ then the ratio of sales drops to $1:n \exp(\beta_\alpha \Delta)$. If the selectivity is high, $\beta_\alpha p_{\alpha 0} \gg 1$, then a small fractional change $\Delta / p_{\alpha 0}$ can make this ratio large. Since a small business can rarely survive the loss of half of its sales the value of 2 or more for $n \exp(\beta_\alpha \Delta)$ is sufficient for its ruin.

On the other hand if the consumers selectivity is small with respect to the product α , then dumping is ineffective. It is equally obvious that the individual has no incentive to practice dumping against a large cartel since he would have to reduce his prices so as to make $\exp \beta \Delta \gg n$ in order to divert sales from the cartel.

8. What is the relation of this model to the real world ?

In the first three sections we have developped an economic model which is conceptionally rather simple. It only describes stationary states, which is a severe restriction. On the other hand, using only a very few parameters of obvious significance it describes qualitatively quite well several well known economic relations: the connection between prices and total flow of money, the distribution of a consumer's income in response to the market, and various cases economic of competition. The equations contain much more information which could be extracted by numerical computation. Such an effort, however, is justified only if the model makes not only qualitative but also quantitative sense. This brings us to the question of how to measure in a real economy the various quantities on which the model is based.

The flow of money spent for various products ϕ_α has been compiled in some countries for a long time. The flows $\phi_{\alpha j}$ representing the distribution of money among different sales channels is certainly in principle obtainable through a survey of the corresponding industries. Once it is known one could verify whether or not the exponential distribution of sales actually holds; if it does then this exercise will have determined the selectivity β_α.

It should be possible to estimate the saturation demand by observing the consumption of those who obviously suffer from no pecuniary constraints. This way determines the normalized units and prices of all products*. Knowing ϕ_α, p_α one can then find the demand a_α, using the relation (10).

There are of course some very serious problems with this model. What for instance is the precise definition of a separate sales channel ? What products are to be considered as interchangeable ? How does one estimate the weight of different consumer-groups if these include industries and businesses as well as individuals ? These difficulties however are of small importance in comparison with the major limitation of the model: it describes steady states only. Future developments, if they are possible at all, must concern the dynamics of this system. One such attempt shall be described in a subsequent paper.

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*One might find for instance that 15 kg of sugar per year saturates the demand of sugar. Thus the consumption of sugar will be measured in units of 15 kg/year. If a kg of sugar costs on the average two francs then $p_{sugar} = 30$ fr.