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COLLISIONLESS ION ACOUSTIC SHOCK WAVES IN A TWO COMPONENT PLASMA

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Abstract

The structure of stationary ion acoustic shocks in a two component plasma has been studied. Unlike to the situation in a one component plasma there exists an oscillatory structure of the trailing edge even for zero ion temperature due to the reflection of light ions. The wavelength and amplitude of these oscillations have been calculated in dependence of the Mach number and the concentration of light ions.

Lausanne

Introduction

Recently WHITE et al. (1972) studied the effect of light ions on the shock or soliton structure in a two component plasma. Using the fluid equations for a plasma model of cold ions and hot isothermal electrons they showed that, according to whether the light ions are reflected or not, there exist two types of stationary solutions in the parameter space of Mach number and light ion concentration within limited ranges separated by a region of turbulence. On the basis of a VLASOV equation treatment they also found that thermal ion motion causes only quantitative changes, so long as the ion to electron temperature ratio T_i/T_e is not too large to prevent shock or soliton solutions at all.

We have studied again the kind of stationary solutions and find that the analysis of WHITE et al. (1972) does not describe the down stream part of the shocks in cases, where the light ions are reflected but reflection of the heavy ions is negligible. According to our study based on the cold ion-hot isothermal electron-model there exist periodic potential oscillations in the trailing edge of the shock under these conditions. These oscillations must be distinguished from the MOISEEV/SAGDEEV type of oscillations resulting from a reflection of the heavy ions (WHITE et al. 1972).

In section I we derive the equations describing the shock structure and calculate the wavelength and amplitude of the oscillations in the trailing edge in dependence of the Mach number and the concentration of light ions. The nature of the oscillations is discussed in section II.

I. Analysis

For our plasma model and stationary conditions the conservation laws in the shock rest frame are

$$N_j(x)\, V_j(x) = N_{oj}\, V_o \tag{1}$$

$$\frac{1}{2} m_j\, V_j^2(x) + e\,\Phi = \frac{1}{2} m_j\, V_o^2 \tag{2}$$

Assuming that the electron thermal velocity is large compared to the drift velocities in the plasma, the convective term in the momentum equation for the electrons can be neglected and one obtains

$$N_e(x) = N_{oe}\, \exp\left\{ e\,\Phi(x)/kT_e \right\} \tag{3}$$

The system is closed by Poisson's equation

$$\frac{d^2\Phi}{dx^2} = \frac{e}{\varepsilon_o}\left(N_e - N_1 - N_2\right) \tag{4}$$

In equs. (1) to (4) m , N , V , and Φ represent the particle mass, density, flow velocity and electrostatic potential, respectively, with e and j = 1,2 labeling the electron and ion species. Requiring charge neutrality ahead of the shock at $x \to +\infty$, we assume $N_{oe} = N_{o1} + N_{o2}$.

Introducing $\mu = m_1/m_2 > 1$, $\psi = e\Phi/kT_e$, the light ion concentration $\alpha = N_{o2}/(N_{o1} + N_{o2})$, the Mach number $M = V_o\left(kT_e/m_1\right)^{-1/2}$ and the Debye length $\lambda_D = \left[kT_e\varepsilon_o/N_{eo}e^2\right]^{1/2}$ and using equs. (1) to (3), Poisson's equation takes the dimensionless form

$$\frac{d^2\psi}{d\tilde{x}^2} = -\left[\frac{\alpha}{\sqrt{1-\mu\frac{2\psi}{M^2}}} + \frac{1-\alpha}{\sqrt{1-\frac{2\psi}{M^2}}} - \exp\psi\right] = -\frac{dU}{d\psi} \tag{5}$$

where $\tilde{x} = x/\lambda_D$ and $U(\psi)$ is the SAGDEEV potential, showing the analogy of equation (5) to the equation of motion for a particle in a potential well.

Integrating equ. (5) once, yields

$$\frac{1}{2}\psi'^2 + U(\psi) = \text{const} \qquad (6)$$

with $\psi' = \frac{d\psi}{d\tilde{x}}$.

Physically, one has to distinguish two cases:

1) No reflection of light ions $\left(\psi \leq \frac{1}{\mu}\,\frac{M^2}{2}\right)$.

The well-known soliton solution

$$U(\psi, M, \alpha) = -\left\{\frac{M^2}{\mu}\alpha\left[\sqrt{1-\frac{2\mu\psi}{M^2}}-1\right]+M^2(1-\alpha)\left[\sqrt{1-\frac{2\psi}{M^2}}-1\right] + \exp\psi - 1\right\} \qquad (7)$$

is obtained by choosing the constant of integration in equ. (6) equal to zero and imposing $U_{(\psi=0)}=0$. The solitary pulse passes through the two component plasma without changing the state or the concentration.

2) Reflection of the light ions $\left(\frac{M^2}{2\mu} < \psi_{max} < \frac{M^2}{2}\right)$.

There exist stationary solutions describing shocks that move through the plasma like a semi-permeable membrane allowing only the heavy ions to flow into the down stream region. The part $(0 \leq \psi < M^2/2\mu)$ of the leading edge is still described by equation (7), if α is replaced by $\alpha' = 2\alpha/(1+\alpha)$ and the whole expression on the right side multiplied by $(1+\alpha)$, in order to keep the same Debye length and normalization, whereas the remaining part of the shock is described by a solution of equ. (6) for the one component (heavy ion) plasma with an appropriate matching at $\psi = M^2/2\mu$, where ψ and ψ' are continuous:

$$U(\psi, M, \alpha) = -(1+\alpha)\left\{M^2\left[(1-\alpha')\left(\sqrt{1-\frac{2\psi}{M^2}}-1\right)-\frac{\alpha'}{\mu}\right]+\exp\psi - 1\right\} \qquad (8)$$

Fig. 1a presents examples of the SAGDEEV potentials, for both types of solutions, the solitary pulse (no reflection of light ions, curve I) and the shock wave (reflection of the light ions, curve II).

Due to the reflection of the light ions, curve II shows a cusp at $\psi = M^2/2\mu$. The dashed part of curve II is the prolongation of the potential given by equ. (8). This part is missing in corresponding curves of WHITE et al. (1972) although it determines the trailing edge of the shock*, as may be seen by utilizing the analogy to the particle motion in a potential well: A particle starting at $\psi = 0$ with zero velocity moves over the cusp at $\psi = M^2/2\mu$ into the potential well described by equ. (8) up to ψ_{max} with $U(\psi_{max}) = 0$ (end of the leading edge of the shock). Then it returns to the cusp and subsequently, since there are no light ions in the down stream region, follows the prolongation of the heavy ion potential (dashed curve) up to ψ_{min} with $U(\psi_{min}) = 0$, from where it returns, thus performing periodic oscillations between ψ_{max} and ψ_{min}.

In the following we study the wavelength and amplitude of these oscillations in dependence of the parameters M, α and μ.

Using equation (6) the shock structure $\psi(x)$ can be obtained by numerical integration of

$$x = \int_0^{\psi} \frac{d\psi}{\psi'(\psi)} = \int_0^{\psi} \frac{d\psi}{\pm\sqrt{-2U(\psi) + const}}$$

or by use of the computation method of RUNGE and KUTTA.

Fig. 1b shows as an example the soliton/shock structure for the two potential wells in fig. 1a. One can see from fig. 1b that the oscillations in the trailing

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*Omission of the dashed part of curve II would give the symmetric soliton solution.

edge of the shock are not sinusoidal, as the maxima and minima are of different width. The wavelength of the oscillations is about a few Debye lengths, which is of the same order of magnitude as the width of a solitary pulse.

The dependence of the wavelength and oscillation amplitude on M, α and μ is shown in fig. 2a and fig. 2b, respectively. The wavelength is rather insensitive to a variation of the parameters. The oscillation amplitude is roughly proportional to M^2 and depends only little on α and μ.

II. Discussion

It is already known from a one component plasma that a reflection of ions breaks the symmetry of the potential $\Psi(x)$ and leads to undamped oscillations in the down stream region (MOISEEV and SAGDEEV, 1963). The oscillations considered are due to the same mechanism, however, one must notice the following difference:

In a one component plasma a reflection of ions occurs only, if there is a sufficiently large spread in the ion velocities $\left(V(kT_e/m)^{-1/2} > M - \sqrt{2\Psi_{max}}\right)$, and a symmetric soliton structure is obtained for zero ion temperature or T_i/T_e - values so small, that the amount of reflected ions is negligible; whereas in a two component plasma an oscillatory structure of $\Psi(x)$ can exist even for $T_i = 0$ due to a reflection of the light ions.

The oscillatory shock structure, which we derived under the assumption of zero ion temperature, is still a valid description for the stationary shocks in a two component plasma with finite T_i/T_e - values, so long as practically all the light ions are reflected but a reflection of the heavy ions is negligible.

In contrast to solitons, shock waves of the type discussed here can exist only in a plasma with small (1 % - 3 %) concentration of light ions. There exists a critical concentration depending on μ and M, above which the shock solution changes discontinuously to the soliton solution (WHITE et al. 1972). For concentrations below this critical value both, the soliton and the shock solution, are possible, however, the solitons are strongly reduced in amplitude, especially for large values of μ (TRAN and HIRT, 1973).

This is important for the design of experiments:

Shock wave experiments should be best done in a heavy ion plasma (for example: Argon), since under usual working conditions (Argon pressure: 10^{-4} Hg, residual gas pressure: 10^{-6} Hg) the contamination of the plasma by Hydrogen, Nitrogen and Oxygen is then of light ions. On the other hand experiments on solitons will be better done in a light ion plasma (for example: Helium), in order to deal with solitons of observable amplitude.

Figure Captions

Fig. 1a shows the SAGDEEV potential $\mathcal{U}(\psi)$ for the soliton (α = 1 %, μ = 4, M^2 = 1,1 ; curve I) and shock (α = 1 %, μ = 4, M^2 = 1,5 ; curve II) solutions.

Fig. 1b presents the corresponding soliton (curve I) and shock (curve II) structures.

Fig. 2: Wavelength and amplitude of the oscillations as a function of M^2 for different concentration of light ions (α = .5 %, 1 %, 1.5 %) and mass ratios (μ = 4, 10).

References

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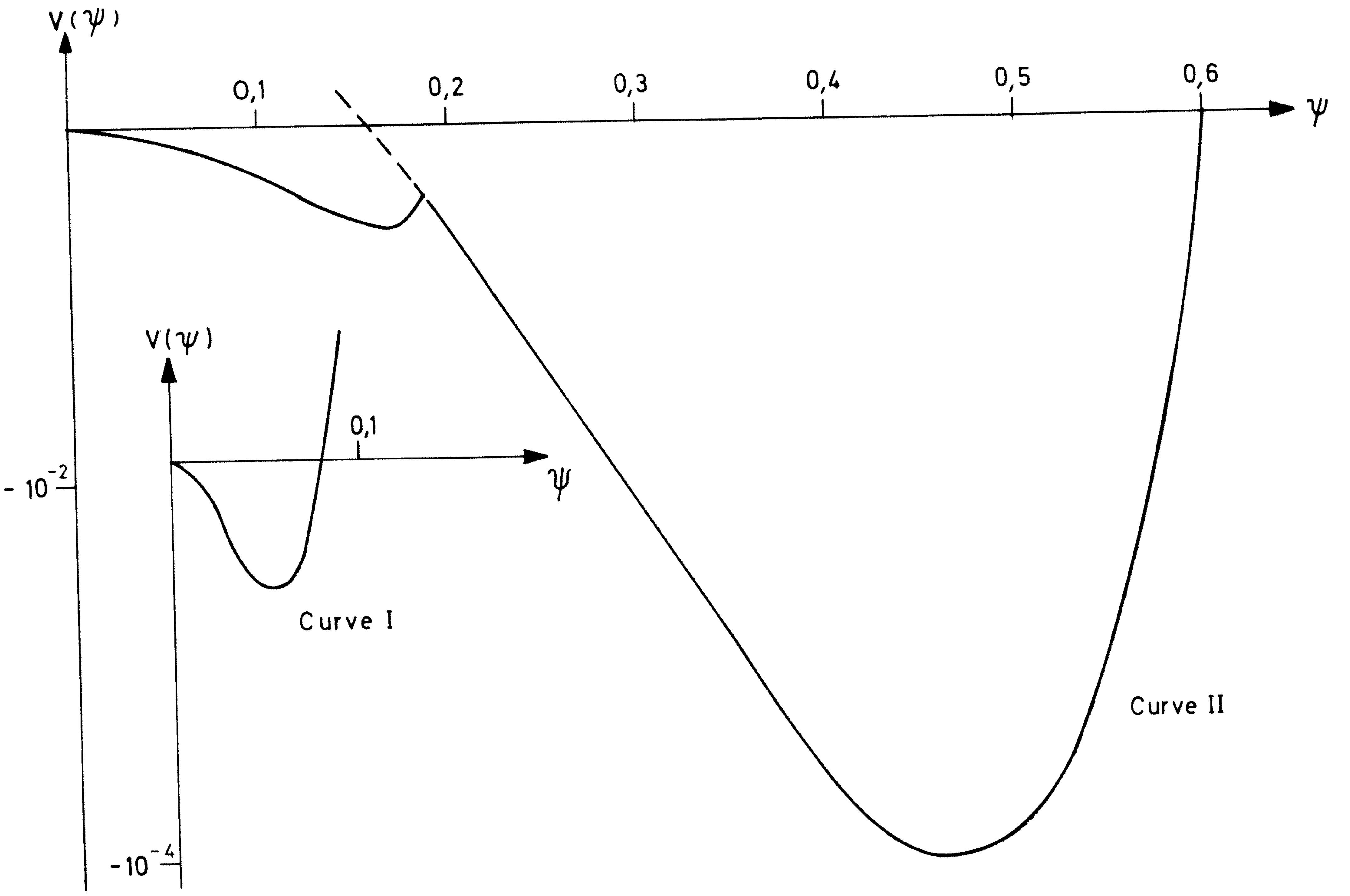

V(ψ)
0,1
0,2
0,3
0,4
0,5
0,6
ψ
- 10^-2
-10^-4
V(ψ)
0,1
ψ
Curve I
Curve II

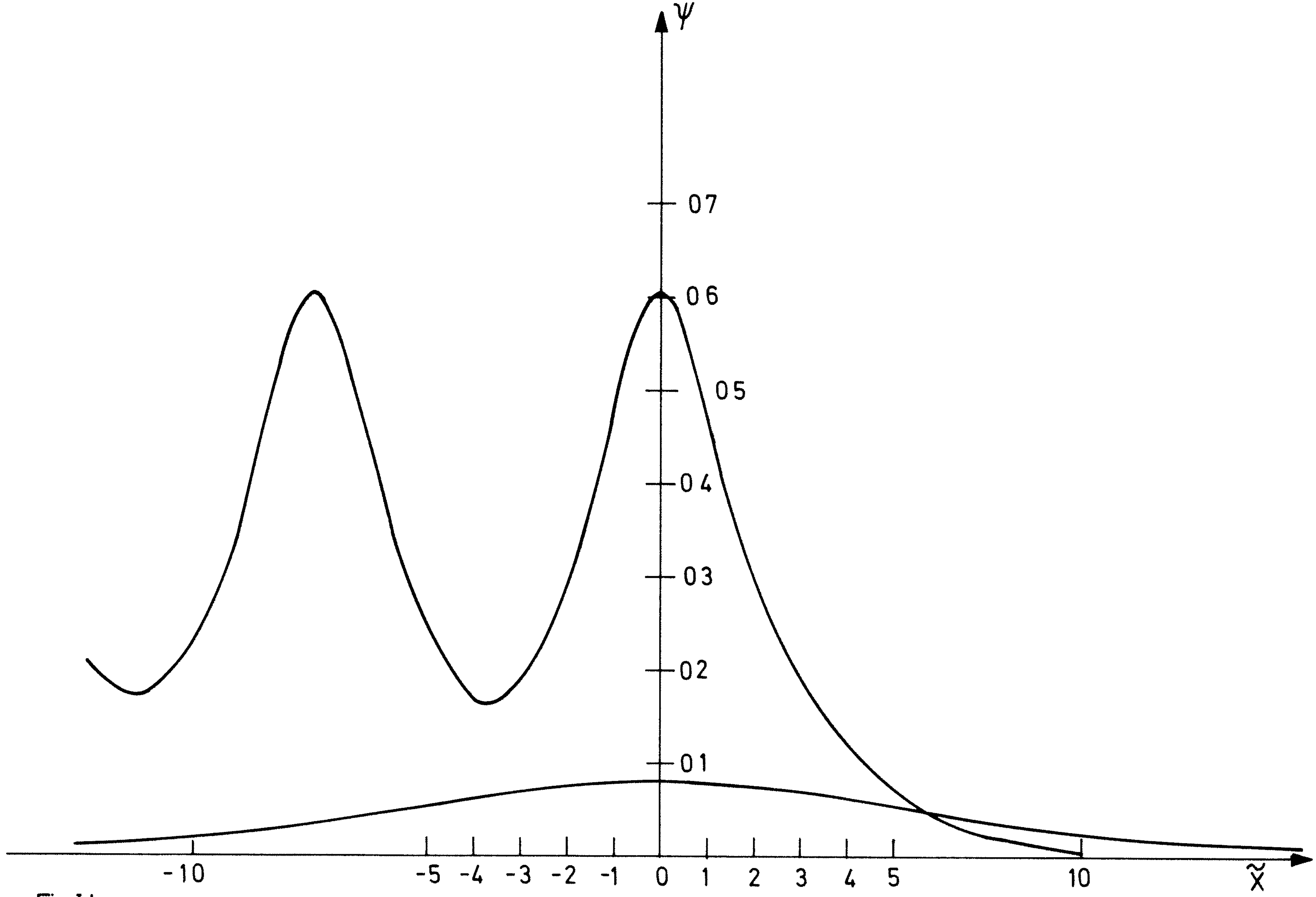
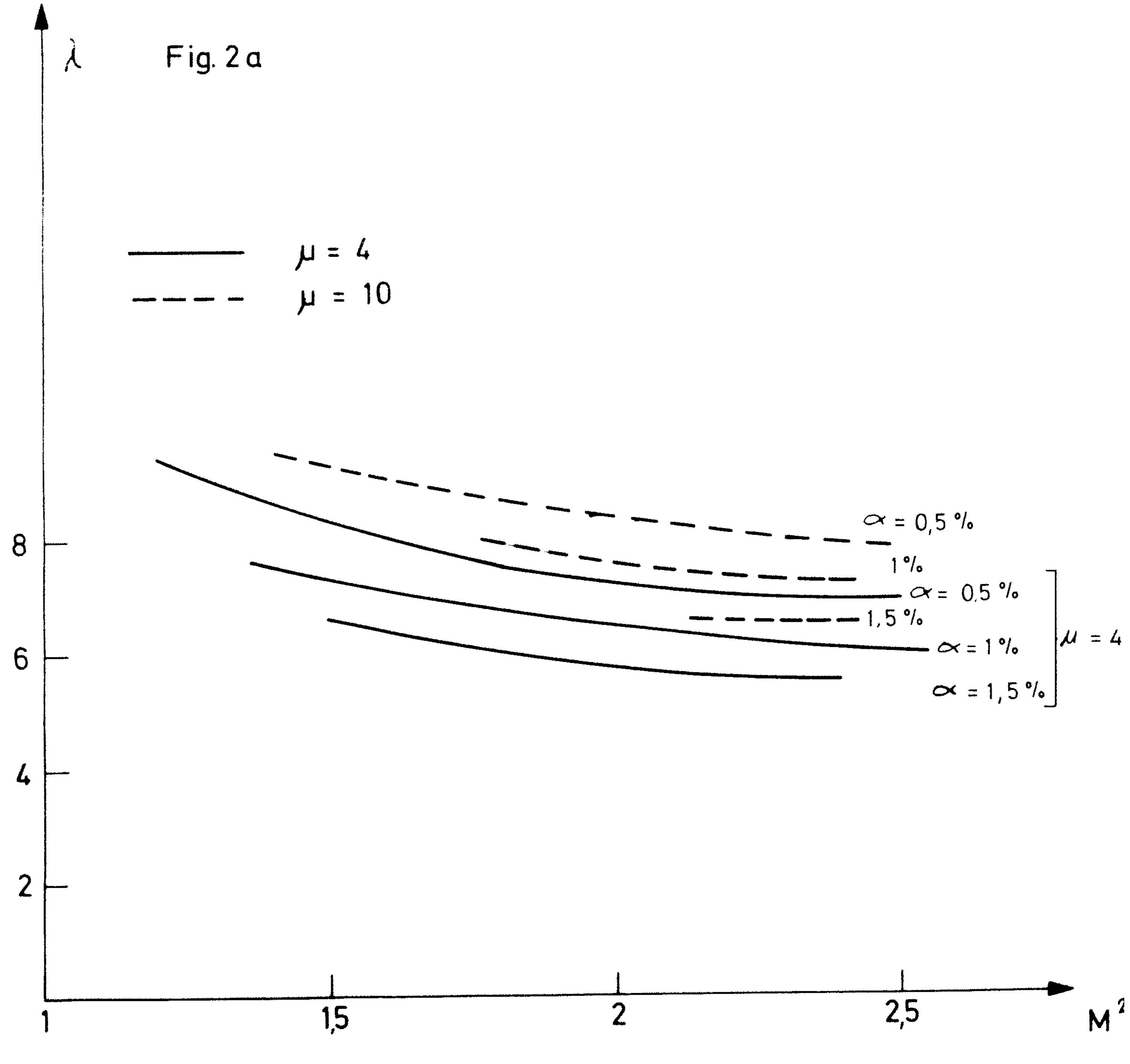


Fig.I b



λ
Fig. 2 a
μ = 4
μ = 10
α = 0,5 %
1 %
α = 0.5 %
1,5 %
α = 1 %
α = 1,5 %
μ = 4
8
6
4
2
1
1,5
2
2,5
M²

Fig. 2b