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TRANSVERSE DYNAMIC STABILIZATION OF A SCREW PINCH

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Abstract

The influence of a small, steady axial current on the stability of a straight plasma column confined by a static axial magnetic field and a small oscillating transverse magnetic field is discussed. Stability conditions are derived and compared with those previously obtained in the absence of the steady current. The possibility of obtaining a toroidal equilibrium by using a steady current which is above the Kruskal-Shavranov limit and dynamically stabilizing the resultant instability is also discussed.

Lausanne

I. Introduction

In an earlier paper[1), hereafter referred to as I, we have examined the possibility of dynamically stabilizing a θ-pinch using a low frequency oscillating axial current. It was shown that the stabilizing effect could be used to provide a dynamic equilibrium for a toroidal θ-pinch. However, in order to remain below the threshold of excitation of parametric instabilities, the aspect ratio of the torus must be very large or the applied frequency must be chosen close to ν_p, the sound transit frequency across a plasma radius. For a torus of major radius R and minor radius b the conditions are

$$\frac{\tilde{B}_\theta}{B_z} \lesssim \frac{1}{2\eta_1^{3/2}} \left(\frac{\omega}{\nu_p}\right)^2$$

$$\frac{R}{b+a} = \frac{b-a}{2\Delta} \left(\frac{B_z}{\tilde{B}_\theta}\right)^2 \qquad (1)$$

in the most optimistic case, where a is the plasma radius, Δ the mean toroidal shift, ω the applied frequency of the oscillating current, B_z the steady axial field (B = 0 in the plasma), $\tilde{B}_\theta$ the oscillating field at the plasma surface (rms value) and

$$\eta_n = \frac{1 + (\frac{a}{b})^{2n}}{1 - (a/b)^{2n}} \quad .$$

Combining the two relations (1) gives the stringent condition

$$\frac{R}{b+a} \gtrsim 4 \quad \eta_1^3 \left(\frac{b-a}{2\Delta}\right) \left(\frac{\nu_p}{\omega}\right)^4 \tag{2}$$

In this paper we consider the case of a dynamically stabilized screw-pinch with a small steady axial current. We wish to examine the possibility of obtaining a toroidal equilibrium using the steady current (which is then above the Kruskal-Shafranov limit[2,3]) and dynamically stabilizing the resulting instability with the oscillating current. We may briefly summarize the results as follows:

In a straight geometry the stability conditions are

$$\frac{\tilde{B}_\theta}{B_z} \lesssim \left(\frac{\gamma}{\eta_1}\right)^{3/2} \left(\frac{\omega}{\nu_p}\right)^2 \left(\frac{\lambda}{4a}\right)$$

$$\frac{B_{\theta o}}{\tilde{B}_\theta} \lesssim \frac{\sqrt{2}}{\sqrt{b^2/a^2 - 1}} \tag{3}$$

where $B_{\theta o} << B_z$ is the value of the steady azimuthal field at the plasma surface and λ the ion collision length. The second condition in (3) shows that the maximum size of $B_{\theta o}$ is of the order of $\sqrt{2}\ \frac{a}{b}\ \tilde{B}_\theta$ for a<<b. Therefore the steady component can only be of importance at small compression ratios.

In a torus the relations (3) have to be completed by the equilibrium condition

$$\frac{R}{b+a} = \left(\frac{b-a}{2\Delta}\right) \frac{B_z^2}{B_{\theta o}^2 + \tilde{B}_\theta^2}$$

Replacing $\tilde{B}_\theta$ and $\bar{B}_{\theta o}$ by their maximum values, we get the condition, in the most optimistic case $\lambda \sim a$,

$$\frac{R}{a+b} \tilde{>} 4\ \eta_1^2 \left(\frac{b-a}{2\ \Delta}\right)\left(\frac{\nu_p}{\omega}\right)^4 \qquad (4)$$

This result differs only by a factor η_1^{-1} from (2). Clearly the steady current is only important if $\eta_1 >> 1$. However, the most favourable condition for stabilization of parametric instabilities is $\eta_1 \approx 1$, in which case the effect of the steady current is negligible. It appears that further investigation should be directed towards either a suppression of the parametric instability of the n = 1 mode, or towards finding schemes where these instabilities cannot occur.

II. The Basic Equations

As in work previously reported[1,7], we assume the plasma is field free ($\beta = 1$) surrounded by vacuum. We are interested in the case $\omega \ll \nu_p$, and since it was shown in I that the most dangerous instabilities occur for long wavelengths, we shall consider only the limit $ak \ll 1$. A further assumption, justified in I, is that the interaction between the forced motion of the surface and the perturbation may be neglected, since this interaction introduces a term at most of order $O\left(\frac{\omega^2}{\nu_p^2}\varepsilon^2\right)$ into the equations and the fastest growing parametric instability is of order ε.

For convenience we write

$$B_{\theta o} = \varepsilon B_z b_c \ , \quad \tilde{B}_\theta = \varepsilon B_z b_v \qquad (5)$$

where

$$b_c^2 + b_v^2 = 1.$$

Following standard perturbation theory, we suppose that the plasma surface experiences a small radial perturbation ξ, given by

$$\xi(\theta,z,t) = \xi_n(k,t)\, e^{ikz + in\theta}$$

The resultant perturbation of the magnetic pressure at the deformed plasma surface, p_m, may be written in the form

$$p_m = p_o \frac{\xi_n(k,t)}{a} \left\{ X_n(k) + A_n(k)\cos\omega t + B_n(k)\cos 2\omega t \right\} \quad (6)$$

where p_o is the average plasma pressure ($p_o = \frac{1}{2}(1 + \varepsilon^2)B_z^2$) and, for the present problem (to order ε^2), in the limit ak << 1,

$$X_o(k) = \frac{2}{1+\varepsilon^2}\left[\frac{2a^2}{b^2-a^2} - \varepsilon^2\right] ,$$

$$A_o(k) = -4\sqrt{2}\,\varepsilon^2\, b_c b_v \left[\frac{\eta_1}{2\gamma(b^2/a^2-1)} + 1\right] ,$$

$$B_o(k) = -2\,\varepsilon^2\, b_v^2 \left[\frac{\eta_1}{2\gamma(b^2/a^2-1)} + 1\right] ;$$

$$\left.\begin{aligned}
X_n(k) &= \frac{2}{1+\varepsilon^2}\left[\frac{\eta_n}{n}(ak)^2 \pm 2\,\varepsilon b_c \eta_n (ak) + \varepsilon^2(\eta_n |n| - 1)\right], \\
A_n(k) &= 4\sqrt{2}\,\varepsilon b_v \left[\pm\, \eta_n ak + \varepsilon b_c (\eta_n |n| - 1)\right], \\
B_n(k) &= 2\,\varepsilon^2 b_v^2 \left[\eta_n |n| - 1\right].
\end{aligned}\right\} \quad n \neq 0$$

The Laplace transform of the perturbed pressure, p_G , is shown in I to be of the form

$$\tilde{p}_G = s\,\tilde{R}\,(n,k,s)\;\xi(s), \tag{8}$$

where, in the limit $ak \ll 1$,

$$\tilde{R}(0,k,s) = p_o \frac{2s\gamma}{(k^2u^2 + s^2)a},$$

$$\tilde{R}(n,k,s) = p_o \frac{s\gamma a}{u^2 n},$$

u is the sound speed in the plasma and γ is the ratio of specific heats. The superposed tilde has been used to denote the Laplace transform, defined through

$$\tilde{p}_G(s) = \int_0^\infty p_G(t)\, e^{-st}\, dt.$$

We may thus write

$$p_G(t) = -\int_0^t R(n,k,t-t')\, \dot{\tilde{\xi}}(t')\, dt' \,, \qquad (9)$$

where R(n,k,t) is defined through (8).

III. Influence of the Azimuthal Field

If $\varepsilon \neq 0$, the dispersion relation obtained by equating the magnetic and plasma pressures at the deformed interface, given by equations (6) and (8) respectively is

$$(X_n(k)+A_n(k)\cos(\omega t - \emptyset)+ B_n(k)\cos 2(\omega t - \emptyset))\, \frac{\xi_n(k,t)}{a}$$

$$+ \int_0^t R(n,k,t-t')\, \dot{\xi}_n(k,t')\, dt' = 0,$$

where $\emptyset$ represents the phase difference between the oscillating field and the perturbation, which is arbitrary, since the perturbation may be created at any time.

We look first at the $n = 0$ modes. The only difference brought by the steady current is an oscillating term $A_o(k)\cos\omega t$, the amplitude of which is of order ε^2. The contribution of this term to the non-resonant modes is of order ε^4 and is thus negligible. The stability condition for the non-resonant modes is then $X_o(k) > 0$. The possible parametric excitation due to the new term $A_o(k)$ is of the same order as the one due to $B_o(k)$ which has been shown to be of no importance in I.

For $n \neq 0$ the steady current has two effects: an additional term of order ε^2 in $A_n(k)$, which is negligible compared to the leading term in ε, and a destabilizing term in $X_n(k)$ or order ε. The condition on the oscillating field such that no parametric instabilities are excited is thus identical to that derived in I,

$$\varepsilon b_v < \left(\frac{\lambda}{4a}\right)\left(\frac{\gamma}{\eta_1}\right)^{3/2}\left(\frac{\omega}{\nu_p}\right)^2 \tag{10}$$

which is the first condition (3), where λ represents the ion collision length for the plasma if $\lambda < a$ and $\lambda \tilde{=} a$ if the plasma is collisionless. For the non-resonant modes the correction, of order ε^2, is only important for marginal modes. If $A_n(k) = 0$ the stability condition for these modes is

$$X_n(k) > 0.$$

Equation (26) of I shows that $A_n(k) \neq 0$ gives additional stability, but this effect is negligible here when (10) is satisfied (order $\frac{\varepsilon^2 \nu_p^2}{\omega^2} (ak)^2$). This is in sharp contrast to the Berge-Wolf scheme[4-6] where it is precisely this term which is used to stabilize the system. The condition $X_n(k) > 0$ is equivalent to

$$\frac{b_c}{b_v} < \frac{\sqrt{2}\ a}{\sqrt{b^2 - a^2}}$$

which is the second condition (3). Note that (10) implies $X_o(k) > 0$ as long as $\omega \tilde{<} \nu_p$.

IV. Dynamic Stabilization of a Toroidal Screw Pinch

Finally, we wish to consider the stability of a screw pinch if it were to be curved into a torus. The mean outward toroidal drift of the plasma column is given by

$$\Delta = \frac{(b^2 - a^2)}{2 \varepsilon^2 R} \quad ,$$

where we have assumed $\Delta \ll b - a$. The oscillating component of the restoring force causes an oscillation of the plasma column about the average position Δ, but this is negligible provided (10) is satisfied. In order that the plasma column be prevented from touching the wall and no parametric instabilities be excited, we arrive at the rather stringent condition (4),

$$\frac{R}{a+b} \gtrsim 4 \eta_1^2 \left(\frac{b-a}{2\Delta}\right)\left(\frac{\nu_p}{\omega}\right)^4$$

Thus we conclude that the introduction of a steady axial current will not provide any worthwhile contribution to a dynamically stabilized toroidal confinement.

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