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HEATING AND CONFINEMENT OF A PLASMA BY ALTERNATING MAGNETIC FIELDS

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Abstract

A method is proposed in which a plasma is confined by an alternating magnetic field. The plasma is field free except in a thin boundary layer. No diffusion takes place since this boundary is periodically reconstituted by the field, maintaining the penetration depth at the fixed value c/ω_p. The confining field heats at the same time both electrons and ions, at high temperatures preferentially the latter. This heating is very efficient. About half of the total applied power is absorbed by the plasma, the rest being copper losses. The technical feasibility is examined and numerical examples are given.

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1) Introduction

A survey of conventional methods of plasma heating and confinement shows that progress has been limited during the past four years. It appears that temperatures and confinement times have reached a ceiling. The confinement times reported at the Conference on Plasma Physics at Salzburg 1961 are of the order of 10 to 20 μsec for high density devices and 30 to 50 μsec for low density devices. Compared to the values presented at the Geneva Conference 1958, the new ones have hardly doubled. The serious nature of the difficulties encountered in the present programmes designed to achieve fusion finds a dramatic expression in the ratio of energy applied to the plasma to the energy converted to heat. This ratio is never less than fifty and appears to be increasing with new machines. In view of this fact it appears unrealistic to hope to achieve an eventual success by pursuing further the conventional approach to the thermonuclear problem.

In all conventional methods magnetic fields and plasma are deliberately mixed to achieve stable configurations. These configurations are, however, at best macroscopically stable. The trapped fields favour density gradients and anisotropic velocity distributions, both of which lead to microinstabilities [1) 2) 3)]. These in turn cause plasma loss by enhanced diffusion or enhanced scattering into loss angles. Sometimes (in pinches) the original macroscopically stable field configuration is altered by enhanced diffusion into an unstable one [4)].

The difficulties mentioned are avoided in the proposed method by using a magnetic field which changes direction periodically and thereby reconstitutes continuously the desired configuration.

Such a field penetrates the plasma to a finite depth only, which is independent of time. Hence no diffusion of the plasma through the field takes place. The depth of penetration depends on the density, the temperature of the plasma and the applied frequency. As the temperature rises, the thickness of the boundary layer decreases until it has reached a limiting value which equals the wave length of light at the plasma frequency (section 3). This is a very small length, it is equal to the thickness of the thinnest possible boundary of an ideal collision free plasma confined by a static magnetic field.

A thin boundary layer brings a number of advantages. First, no cyclotron radiation is emitted from the bulk of the plasma. This is important since cyclotron radiation represents a serious loss mechanism, which influences profoundly the energy balance of a thermonuclear reactor [5]. Second, the interior of the plasma being field free, and thus force free has an isotropic velocity distribution. Hence no instability of any type can develop in the interior. Third, concerning macroscopic instabilities, the boundary can be considered as sharp which permits easily the selection of stable plasma field configurations (section 2). Moreover such a configuration will be maintained because the boundary remains thin (no diffusion).

No analysis exists of the microscopic stability of the boundary layer itself and at present the difficulties of such an analysis seem overwhelming. However qualitative arguments indicate that no instabilities are likely to occur.

These arguments can be summarized as follows : For the cool plasma in which the boundary is collision controlled microinstabilities merely increase the resistivity which causes the boundary layer to be somewhat thicker. Even if the boundary layer were turbulent the apparent resistivity would not increase by a large factor, so that the thin boundary is maintained. For the hot plasma, in which the boundary is inertia controlled (section 3) each particle spends but a fraction of a plasma oscillation period in the layer as it comes from and returns to the interior of the plasma. Hence no cooperative effects can develop in the boundary layer.

The plasma is heated by two mechanisms : One of these is simply ohmic heating (section 4) of the electrons. The other affects the ions and is due to the periodic motion of the boundary under the influence of the confining field which applies a pressure of the form $\cos^2(\omega t)$ (section 5). As luck would have it the ion heating mechanism becomes much more important at high temperatures, where the transfer of heat from electrons to ions is slow.

As the temperature rises the applied field amplitude must be increased to maintain the pressure balance. At some temperature, depending on plasma density and geometry, the pressure becomes too large to be balanced by practically obtainable fields. It will then be necessary to slowly switch on a quasi static field to take up the excess pressure. A number of possible combinations of fast and slow alternating fields exist, which will keep the plasma confined and heated without allowing field penetration. Since the heating rate depends in detail on the arrangement, it will not be calculated here for these mixed cases.

Since the feasibility of the proposed method has been questioned repeatedly, special attention is given to the power and voltage requirements (section 6).

Compared to the very high rate of heating, the losses by heat conduction (section 9) and by bremsstrahlung are unimportant during the heating. Radiation by impurities may change this situation, it has however not been considered.

This report presents the results of calculations of the heating and confinement assuming that the temperature of the plasma is uniform. For high temperatures this is a reasonable assumption since the heat conductivity is high. At low temperatures gradients will be present in the plasma since it is heated in the boundary layer only. In this range of temperature the results are estimates only.

They suffice however to give the data which are necessary for planning an experiment.

Most aspects of the problem have been treated only summarily and will require deeper understanding. Further theoretical work should concentrate on the following problems :

A) The equations of motion and heat transfer for the plasma cylinder should be integrated numerically using the two fluid equations and the simple approximations for the boundary layer given here, as well as the usual transport coefficients.

B) The boundary layer must be investigated in detail :
   a) the macroscopic motion according to fluid dynamics,
   b) the velocity distribution based on Bolzmann's equation,
   c) transport coefficients for rapidly alternating and for large amplitude fields.

2. Stable plasma field configurations

Since the boundary layer is very thin in comparison with the dimensions of the plasma, it can be considered as sharp. It has been shown previously [6] for a special case that stability is assured if two conditions are satisfied. First, the field must provide on the average a positive restoring force for any surface deformation. Second the applied frequency must satisfy

$$\omega = \frac{\alpha}{a}\sqrt{\frac{k(T_e+T_i)}{m_i}} \qquad (1)$$

where a is a linear dimension of the plasma and α is a number estimated to be not larger than ten. This second condition is necessary to avoid any destabilizing effect due to the oscillating part of the confining force. It appears reasonable to assume that the same conditions suffice for other geometries. The frequency $f = \omega / 2\pi$ is plotted on Fig. 5 for a = .05 m , α = 10 against $T = T_e = T_i$.

Two examples of configurations which satisfy the first condition are the cusp [7] and the toroidal plasma with combined z and φ fields [6].

Fig. 1a, 1b, 1c, show cusped fields schematically, which can be excited by one turn coils. Fig. 1d shows an arrangement for the production of a highly symmetrical cusped field, which can be excited in push pull from two coaxial lines. Compared to current experiments with cusped fields, the particle losses will be substantially reduced due to the small depth of penetration.

Fig. 2a, 2b, show schematically the arrangement for the production of a toroidal plasma. The conducting shell which surrounds it is slotted in two directions to permit the application of many oscillators for the excitation of the z and φ fields. It should be noted that no voltage larger than V_z or V_φ appears anywhere even though the oscillators appear to be connected in series. Fig. 2c shows a chain of cusps arranged in a torus which requires only one set of oscillators (B_z) and is suitable for the switch over to a quasi static field at high temperatures, without change in geometry.

To maintain the pressure balance it is necessary that the field amplitude rises with the temperature according to

$$B^2 = 4\mu_0 k(T_e + T_i)n \tag{2}$$

which requires the programmation of the field strength. Feedback may be necessary but the problems attending it shall not be considered here.

If the frequency ω is to satisfy (1) it would also have to be programmed. In the calculations that follow, it has always been assumed that (1) is satisfied, that is that the frequency is programmed. The corresponding curves on all graphs are shown in solid lines.

Frequency programming may be difficult to achieve practically; fortunately it is not necessary since it suffices that

$$\omega \geq \frac{\alpha}{a}\sqrt{\frac{k(T_e+T_i)}{m_i}} \tag{3}$$

Thus may fix the frequency for the highest temperature T_m to be attained. This can be taken into account by making the substitution

$$\alpha = \alpha_m \sqrt{\frac{T_m}{T}} \qquad (4)$$

in all formulas that follow.

On the various graphs the curves corresponding to this choice with $T_m = 2\ 10^6\ °K$, $a = .05$ m, $\alpha = 10$, giving f = 2.7 Mc are traced with dotted lines.

3. The boundary layer

The structure of a plane boundary layer will be examined. All quantities shall depend only on x and t. The magnetic field is chosen parallel to z while E and j will be parallel to y. Thus

$$-\frac{dB}{dx} = \mu_0 j \qquad \frac{dE}{dx} = -i\omega B$$

where

$$j = \frac{e^2 n}{m_e(i\omega + \tau_e^{-1})} E$$

and *

$$\tau_e = \frac{93}{\lg\Lambda} \frac{\varepsilon_0^2 m_e^{1/2} (kT_e)^{3/2}}{e^4 n}$$

We require equilibrium of the plasma with the time averaged force

$$-\frac{d}{dx} k(T_e + T_i)n = -\langle j B \rangle_t \qquad (5)$$

Since the differentiation and averaging can be interchanged (6) is equivalent to

$$\frac{d}{dx}\left[k(T_e + T_i)n + \frac{1}{2\mu_0} \langle B^2 \rangle_t \right] \qquad (6)$$

Two limiting cases will be considered.

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*) Subsequently the factor $93/\lg\Lambda$ was set equal to 8.

a) $\omega\tau_e \ll 1$ This is the low temperature, low frequency and high density case. The boundary is"collision-controlled". One obtains

$$B'' = -i\omega\mu_0\sigma B \quad , \quad \sigma = \frac{e^2 n \tau_e}{m_e}$$

which has the solution

$$B = B_0 \exp\left[(-1+i)\sqrt{\tfrac{1}{2}\mu_0\sigma\omega}\, x + i\omega t\right]$$

since σ is independent of n and hence of x. The physical field is given by the real part

$$B_r = B_0 \exp\left(-\sqrt{\frac{\mu_0\sigma\omega}{2}}\, x\right)\cos\left(\omega t + \sqrt{\frac{\mu_0\sigma\omega}{2}}\, x\right)$$

Substituting this expression into (6) one obtains

$$\begin{aligned} n &= n_0\left(1 - \exp\left(-\sqrt{2\mu_0\sigma\omega}\, x\right)\right), && x > 0 \\ n &= 0 , && x < 0 \end{aligned} \tag{7}$$

where

$$n_0 = B_0^2 / 4\mu_0 k(T_e + T_i)$$

This result is shown in Fig. 3a. The boundary layer has the depth

$$\delta = \sqrt{\frac{2}{\mu_0\sigma\omega}} \quad , \quad \sigma = \frac{93}{\lg\Lambda}\,\frac{\varepsilon_0^2 (kT_e)^{3/2}}{m_e^{1/2}} \tag{8}$$

and which is the ordinary skin depth. According to (7) the density drops to zero at a finite distance. In reality this is not so, since with the density approaching zero the inequality $\omega\tau_e \ll 1$ no longer holds. This brings us to the case

b) $\omega\tau_e \gg 1$ In this case the boundary layer is controlled by the inertia of the electrons. One has

$$-\frac{dB}{dx} = \frac{\mu_0 e^2 n}{i\omega m_e} E$$

and hence

$$\frac{d^2E}{dx^2} = \frac{\mu_0 e^2 n}{m_e} E$$

which has the solution

$$E(x,t) = E_0(x)\, e^{i\omega t}$$

$$B(x,t) = B_0(x)\, e^{i\omega t} = \frac{i}{\omega} E_0(x)\, e^{i\omega t}$$

To obtain the force on the plasma the real parts of B and are again required. $E_0(x)$ can be chosen real as equation (9) will show.

$$B_r = -\frac{E_0'(x)}{\omega} \sin \omega t$$

$$j_r = \frac{e^2 n}{m_e \omega} E_0(x) \sin \omega t$$

The magnetic time averaged force becomes now

$$\langle j_r B_r \rangle_t = -\frac{e^2 n}{4 m_e \omega^2} \frac{d}{dx} E_0^2(x)$$

Substituting this expression into (6) yields after integration

$$n = n_0 \exp\left[-\frac{e^2 E_0^2(x)}{4 m \omega^2 k(T_e+T_i)}\right]$$

The equation for $E_0(x)$ now becomes

$$E_0'' = \frac{\mu_0 e^2 n_0}{m_e} \exp\left[-\frac{e^2 E_0^2(x)}{4 m \omega^2 k(T_e+T_i)}\right] E_0(x)$$

The last two equations describe a boundary of thickness

$$\delta = c/\omega_p \tag{9}$$

or

$$\delta = \sqrt{\frac{m_e}{\mu_0 e^2 n_0}} \tag{10}$$

In contrast to the collision controlled case the density decays exponentially (Fig. 3b). It is interesting to note that according to (10) δ is just the width of the thinnest boundary for a collision free plasma confined by a static magnetic field [8].

The solutions given for the boundary layer are only approximate since the oscillating part of the confining pressure has been neglected. In reality the boundary will execute a periodic motion. An approximate (linearized) solution of this motion has been given earlier, but the problem remains essentially unsolved.

In Fig. 7 the skin depth δ according to (8) and (10) is plotted as a function of temperature assuming $T_e = T_i = T$, for a plasma of density $n = 10^{21}\ m^{-3}$. If the frequency is programmed according to (1) with a = .05 m and $\alpha = 10$ then the solid line is obtained. For fixed frequency according to (4) with $T_m = 2\ 10^6\ {}^\circ K$ the dashed line results.

4. Ohmic dissipation

To calculate the ohmic dissipation in the boundary layer we shall further simplify the problem by taking n to be constant inside the plasma and zero outside. This introduces but a small error and permits the two cases $(\omega\tau_e \gtrless 1)$ to be handled at once. Setting

$$B = B_0\, e^{-kx + i\omega t}$$

one finds

$$c^2 k^2 = \frac{\omega_p^2}{1 + (i\omega\tau_e)^{-1}}$$

and

$$E = \frac{i\omega}{k} B \quad , \quad j = \frac{1}{\mu_0} k B$$

Hence the average power dissipated in a surface of the plasma of area S becomes

$$P_e = S \int_0^\infty \mathcal{Re}\ \tfrac{1}{2}\, j \bar{E}\, dx$$

$$= \frac{c}{2\mu_0} \frac{B_0^2 S}{\omega_p \tau_e \left[\sqrt{1+(i\omega\tau_e)^{-1}} + \sqrt{1-i\omega\tau_e)^{-1}}\right]}$$

In the two limiting cases one obtains

$$P_c = \frac{c}{\mu_o}\sqrt{\frac{\omega}{8\,\omega_p^2\,\tau_e}}\;B_o^2 S \tag{11}$$

$$P_c = \frac{c}{\mu_o}\,\frac{B_o^2 S}{4\,\omega_p\,\tau_e} \tag{12}$$

In these expressions B_o is the field amplitude and P_c the average power. This power is delivered to the electrons.

5. Ion heating

Since the confining pressure

$$\frac{1}{2} B_o^2 \cos^2\omega t = \frac{1}{4} B_o^2 (1 + \cos 2\omega t)$$

depends on time, the plasma boundary cannot remain stationary. Large amplitude pressure waves are generated at the surface and propagated into the plasma. At present no complete treatment of these waves exists. However in the high temperature limit, where the mean free path of the ions is long $\lambda \gg a$ there exists a small amplitude theory [6]. It relates the pressure increment at the plasma surface to its displacement ε as follows :

$$-\delta p = \left[\sqrt{\frac{8}{\pi}}\,\frac{p}{u} + 2\,\frac{p}{a}\,\hat{K}_o\left(0, \frac{a\omega}{u}\right)\right] i\omega\varepsilon \tag{13}$$

$$p = kT_i n \qquad u = (kT_i/m_i)^{1/2} \tag{14}$$

In this expression $\hat{K}$ is the Fourier transform of the function $K_o(0,\xi)$ defined in the paper cited above [6]. This Fourier transform has been computed numerically for values of $a\omega/u$ up to 14. It turns out that it is negligeable in comparison with the first term for values of $a\omega/u$ larger than 8. Equation (1) requires

$$\frac{a\omega}{u} = \alpha = 10$$

so that (13) simplifies to

$$-\delta p = \sqrt{\frac{8}{\pi}}\ \frac{\rho}{u}\ i\omega\varepsilon \qquad (15)$$

This corresponds physically to the pressure due to particles which have been struck once by the moving surface. The pressure pulse due to the reflected particles is already sufficiently spread out so that it gives only rise to an increased average pressure, which is compensated by the rising average field pressure.

From equation (15) one obtains immediately the power delivered to the ions

$$P_i = \mathcal{R}\ \tfrac{1}{2}\left(-i\omega\varepsilon\ \overline{\delta p}\right) S$$

and since

$$\delta p = B^2/4\mu_0$$

one finds

$$P_i = \frac{1}{4}\sqrt{\frac{\pi}{2}}\ \frac{(B^2/4\mu_0)^2 S}{(m_i\, kT_i)^{1/2}\, n} \qquad (16)$$

which is valid for $\lambda \gg a$.

Strictly speaking equation (13) should be used only for small pressure amplitudes. However it is reasonable to expect that the dependence of P_i on the temperatures as given by (16) remains valid for large amplitudes.

6. Feasibility : Voltage amplitudes and power requirements

As will be shown in this section the voltage and power required to reduce the proposed method to practice are very large, but they are not beyond the present state of the art.

To calculate voltage and power, we choose for simplicity the cylindrical geometry (Fig. 4), which will give results that are applicable to the other cases with only minor modifications. A conducting shell surrounds the plasma at the radius b and carries the currents that produce the confining fields. This shell is subdivided into sections by two sets of orthogonal slots, each section having the length γa and the width $\beta(a+h)$. The driving voltages are applied to the slots as indicated in Fig. 4. Thus for a total length L of the plasma the number N of oscillators which is required will be $(L/\gamma a)\cdot(2\pi/\beta)\cdot 2$. In what follows the power and voltage are calculated per oscillator. The total power to be applied to a device is N times larger. But the voltage that appears anywhere on the device never exceeds that of a single oscillator.

The results will be applied to a standard example, which corresponds to a device that could be experimentally tested in the laboratory. Its characteristics are :

Plasma radius	a	$= .05$ m
Length	$2\pi a$	$= .31$ m
Radius of shell $b = a + h$		$= .08$ m
Density	n	$= 10^{21}\ \mathrm{m}^{-3}$
Frequency multiplier	α	$= 10$
Sections:	β	$= \gamma = 2\pi$

With $\beta = \gamma = 2\pi$ only one longitudinal slot is used which means one z and one φ oscillator per 31.4 cm. For a toroidal device this means still a considerable number of power sources. Thus it appears more practical for a first experiment to use the cusped geometry (Fig. 1a, b, d) which requires just two oscillators. For these our calculations are applicable with only slight changes in the numerical factors.

To determine voltage and power we return to Fig. 4. For the excitation of the B_φ field one slot represents the inductance

$$L = \mu_0 \frac{\gamma}{\beta} a \lg\left(\frac{a+h}{a}\right) \cong \mu_0 \frac{\gamma}{\beta} h$$

To produce the field amplitude B_φ a current amplitude

$$I_z = \frac{\beta a}{\mu_0} B_\varphi$$

is required which in turn gives rise to a voltage amplitude

$$V_z = I_z L \omega = \gamma h a \omega B_\varphi$$

Analogously the voltage for the generation of the B_z field becomes

$$V_\varphi = \beta h a \omega B_z$$

Substituting $a\omega$ from the stability condition (1) and B from the pressure balance equation (2) gives for the voltage amplitudes

$$(V_\varphi , V_z) = 2(\gamma , \beta) \alpha \sqrt{\frac{\mu_0}{m_i}} \, k(T_e + T_i) \, n^{1/2} h \tag{17}$$

Setting $T_e = T_i = T$ one obtains

$$(V_\varphi , V_z) = (\gamma , \beta) \alpha \; 1.08 \; 10^{-12} \; T n^{1/2} h \tag{18}$$

To minimize the voltage at fixed T and n one can decrease γ and β, that is increase the number of oscillators per length a of the device. Practical considerations limit the number of oscillators for a given plasma radius a. However since V does not depend on the radius a it is in principle possible to decrease the voltage by increasing a and keeping γa, the distance between feeding slots constant. The distance between the plasma and the conducting shell, h, should also be kept at a minimum. Finally the voltage is proportional to α which fixes the frequency according to the stability condition (1). It is not yet clear what the minimum value for α is. To be on the safe side $\alpha = 10$ has been used in the numerical example illustrated in the various graphs.

The graph of Fig. 5 represents the voltage as a function of T for the standard example as described in the beginning of this section.

In this case the amplitude of 100 kV is reached for $T = 1.5 \; 10^6$ °K. If 100 kV are considered to be the maximum amplitude then one would have to begin superimposing a quasi static field at this temperature. At this temperature also the boundary layer has attained the smallest possible depth as seen from Fig. 7.

The reactive power P_r which gives a measure of the circulating energy and of the Q of the system becomes

$$P_r = \frac{1}{2} VI = 4\sqrt{2}\, \alpha\beta\gamma\, m_i^{-1/2} (kT)^{3/2}\, n a h \tag{19}$$

$$P_r = 5.05\ 10^{-21}\, \alpha\beta\gamma\, T^{3/2}\, n a h \tag{20}$$

A plot of P_r against temperature is shown in Fig. 6 for the standard example.

The power delivered to electrons and ions is obtained from equations (11), (12) and (15) by substituting

$$S = \beta\gamma a^2$$

Thus one obtains *) :

Electron heating for $\omega\tau_e \ll 1$

$$P_e = \frac{2^{1/4} c}{\varepsilon_0^{1/2}}\, \alpha^{1/2} \beta\gamma \left(\frac{m_e}{m_i}\right)^{1/4} e n \frac{k(T_e + T_i)/2}{(kT_e)^{1/2}}\, a^{3/2} \tag{21}$$

Electron heating for $\omega\tau_e \gg 1$

$$P_e = \frac{c}{4\varepsilon_0^{3/2}}\, \beta\gamma\, \frac{e^3\, k(T_e + T_i)/2}{(kT_e)^{3/2}}\, n^{3/2} a^2 \tag{22}$$

Ion heating

$$P_i = \frac{1}{4}\sqrt{\frac{\pi}{2}}\, \beta\gamma\, m_i^{-1/2} k^{3/2} \frac{(T_i + T_e)^{5/2}}{T_i}\, n a^2 \tag{23}$$

These powers are plotted against T on Fig. 6 for the standard example and setting $T_i = T_e = T$.

$$P_e = .920\ 10^{-17}\, \alpha^{1/2} \beta\gamma\, T^{1/2}\, n a^{3/2} \quad (\omega_e\tau \ll 1) \tag{24}$$

*) The factor $93/\ln\Lambda$ was set equal to 8 in all equations.

$$P_e = 3.12\ 10^{-21}\ \beta\gamma\ \frac{n^{3/2} a^2}{T^{1/2}} \qquad (\omega\tau_e \gg 1) \tag{25}$$

$$P_i = 2.81\ 10^{-22}\ \beta\gamma\ T^{3/2} n a^2 \tag{26}$$

The transition temperature T^* is defined by the equality of the two limiting expressions for P_e

$$T^* = \frac{2^{1/4}}{4\alpha^{1/2}} \left(\frac{m_i}{m_e}\right)^{1/4} \frac{e^2}{\varepsilon_0 k} (na)^{1/2} \tag{27}$$

$$T^* = 4.05\ 10^{-4} \left(\frac{na}{\alpha}\right)^{1/2} \tag{28}$$

For the standard example one obtains the transition temperature $T^* = 1.5\ 10^6$ °K.

Furthermore the copper losses need to be known. They are

$$P_c = 2^{11/4} (\mu_0 \sigma_{copper})^{-1/2}\ m_i^{-1/4}\ \alpha^{1/2}\ \beta\gamma \left(k(T_e+T_i)/2\right)^{5/4} \times n a^{3/2} \tag{29}$$

or

$$P_c = 8.8\ 10^{-23}\ \alpha^{1/2}\ \beta\gamma\ T^{5/4}\ n a^{3/2} \tag{30}$$

The copper losses also are plotted on Fig. 6. This figure indicates that the combined power dissipated in the device up to the temperature $T = 10^6$ °K never exceeds 25 MW. While this power is large it is only required during a few microseconds, since the total energy of the plasma

$$W = \frac{3\beta\gamma}{\tau}\ k(T_e+T_i)\, n a^2 \tag{31}$$

is only 62 joules at $T = 10^6$ °K.

While the attainment of thermonuclear temperatures by direct extrapolation of this method seems very difficult it will be possible by suitable combination of alternating and quasi static fields. This will be treated in a subsequent report. The important feature of the proposed method is the production of an uniformly heated plasma at some 10^6 - 10^7 °K which is free of trapped fields and has the thinnest possible boundary layer.

7. Heating time

To estimate the heating time we shall assume that the plasma is isothermal. This is of course not true since for a low temperature plasma the heat conduction is low while at high temperatures the equilibration time between ions and electrons is very long. Nevertheless the calculation gives an estimate of the heating time. Thus

$$\frac{dW}{dt} = P_e + P_i$$

leads to

$$kT = \left[\frac{c_1}{c_2} \, \mathrm{tg}\left(\frac{c_1 c_2 t}{2} \right) \right]^2 \tag{32}$$

where

$$c_1^2 = \frac{2^{5/4}}{3} \frac{c}{\varepsilon_0^{1/2}} \left(\frac{m_e}{m_i} \right)^{1/4} e \, a^{-3/2} \tag{33}$$

$$c_2^2 = \frac{1}{6} \sqrt{\frac{\pi}{2}} \, m_i^{-1/2} a^{-1} \tag{34}$$

In Fig. 8, the heating time is plotted according to (32) as a function of T. (The pole of the tangent has no influence in the interval shown). Thus while the power level is very high, the heating time is very short.

8. Heat conduction and mean free path

The extent to which the electron and ion temperatures can be considered uniform can be estimated from a consideration of the time constant for thermal conduction. In the field free interior the coefficient of thermal conduction is $\kappa T^{5/2}$ so that for electrons

$$t_{ce} = \frac{k n a^2}{\kappa T_e^{5/2}} = 8.62 \; 10^{-13} \, \frac{n a^2}{T_e^{5/2}} \tag{35}$$

and for ions

$$t_{ci} = (m_i / m_e)^{1/2}\ t_{ce}$$

and the mean free path which is the same for both electrons and ions

$$\lambda = \frac{16\sqrt{2}}{\sqrt{\pi}}\ \frac{\varepsilon_0^2 (kT)^2}{e^4 n} = 2.93\ T^2 n^{-1} \qquad (36)$$

The particles can be considered as isothermal if $t_{ce} \ll t(T)$, equ. (32).
For electrons in our example this is the case if T $\gtrsim$ 8 10^4 °K

The ions can be considered isothermal when $\lambda >$ a which is true for T $\geq 10^5$ °K. Both λ and t_{ce} are plotted in Fig. 7 and 8 respectively.

9. <u>No trapped magnetic field</u>

The question arises whether any magnetic field can become trapped in the interior of the plasma. As the temperature increases and the skin depths decreases rapidly some field might be left behind.

If λ is the wave length of the field inside the plasma then at most a layer of thickness $\lambda/2$ could contain a trapped unidirectional field. The time constant for the diffusion of such a field is t = $\mu_0 \sigma (\lambda/2)^2$
Since λ equals 2π times the skin depth δ one obtains t = π/f.
That is the "trapped" field disappears in about three periods.

The time constant for the diffusion of an externally applied quasi static field shall also be given. It is

$$t_d = \mu_0 \sigma a^2 = \frac{a^2 \omega_p^2 \tau_e}{c^2} \qquad (37)$$

$$= 1.65\ 10^{-9}\ T^{3/2}\ a^2 \qquad (38)$$

which is plotted in Fig. 8. Hence if after heating by an alternating field to 2 10^6°K the plasma is confined by a static field, it would have a life time of about 10 ms.

10. Energy losses

In comparison with the rate of heating and for the temperature range considered here (10^4 °K - $2\,10^6$ °K) losses by radiation are negligible.

Losses by thermal conduction shall be considered because they may be important. The thermal conductivity across a magnetic field is given [9] approximately by

$$K_B = \frac{K_0}{1 + (\omega_e \tau_e)^2} \quad , \qquad \omega_e = \frac{eB}{m_e} \tag{39}$$

where

$$K_0 = \frac{151.4}{\lg \Lambda} \; \frac{\varepsilon_0^2 \, k^{7/2} \, T_e^{5/2}}{m_e^{1/2} \, e^4}$$

No theory exists at present which gives the thermal conductivity in a rapidly oscillating magnetic field. The dependence of this conductivity on B is however of crucial importance. For it is the inhibition of the heat flux by the magnetic field which makes it possible to maintain the high temperature gradients between the wall and the plasma. Using the average value of B^2 in (39) (for lack of a better theory) one notices that in the tenuous plasma between boundary layer and wall the conductivity reduces to

$$K_B = 1.75 \; 10^{-2} \lg \Lambda \; \frac{m_e^{1/2} \, e^2 \, k n}{\varepsilon_0^2 \, B^2 (kT)^{1/2}} = 2.34 \; 10^{-42} \, \frac{n^2}{B^2 T^{1/2}} \tag{40}$$

Thus in the important heat insulating region K_B depends fortunately on temperature as $T^{-1/2}$ instead of $T^{5/2}$ and on the density as n^2. If K_B according to (40) is correct for alternating fields then heat conduction can be neglected as a loss mechanism during the heating phase. However, it is important to note that the theory of heat conduction must be reviewed for the present case of oscillating magnetic field.

References

1) M.N. Rosenbluth, Risö Report No. 18, 190 (1960)
2) W.B. Thompson, " " " 18, 237 (1960)
3) N.A. Krall, M.N. Rosenbluth, Phys. Fluids, 4, 163 (1961), 5, 1435 (1962) and 6, 254 (1963)
4) S.A. Colgate and H.P. Furth, Phys. Fluids, 3, 982 (1960)
5) W.E. Drummond and M.N. Rosenbluth, Phys. Fluids, 6, 276 (1963)
6) E.S. Weibel, Phys. Fluids, 3, 946 (1960)
7) T.K. Allen, R.W.P. McWhirter, I.J. Spalding, Nuclear Fusion, 1962 Supplement, Part 1, 67
8) H. Grad, Phys. Fluids, 4, 1366 (1961)
9) W. Marshall, Kinetic theory of an ionized gas, Atomic Energy Research Establishment, Report T/R 2419, Harwell Berkshire (1960).

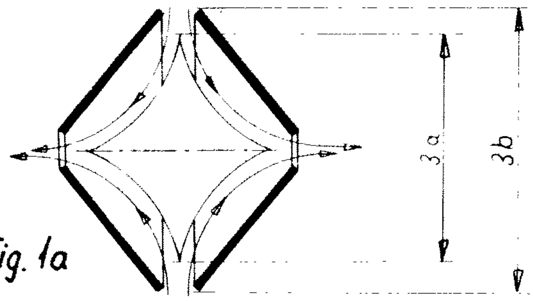

Fig. 1a

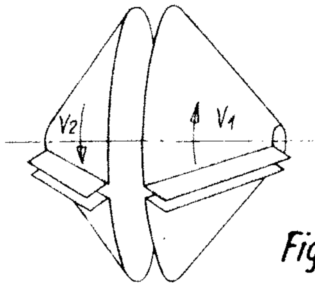

Fig. 1b

Cusp

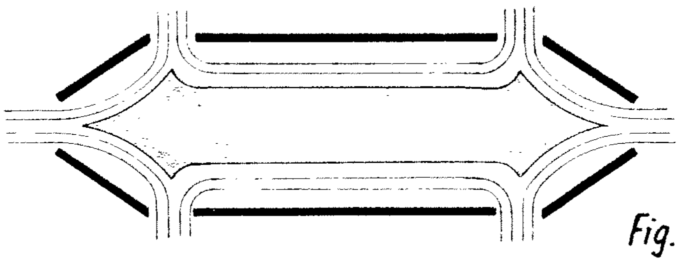

Fig. 1c

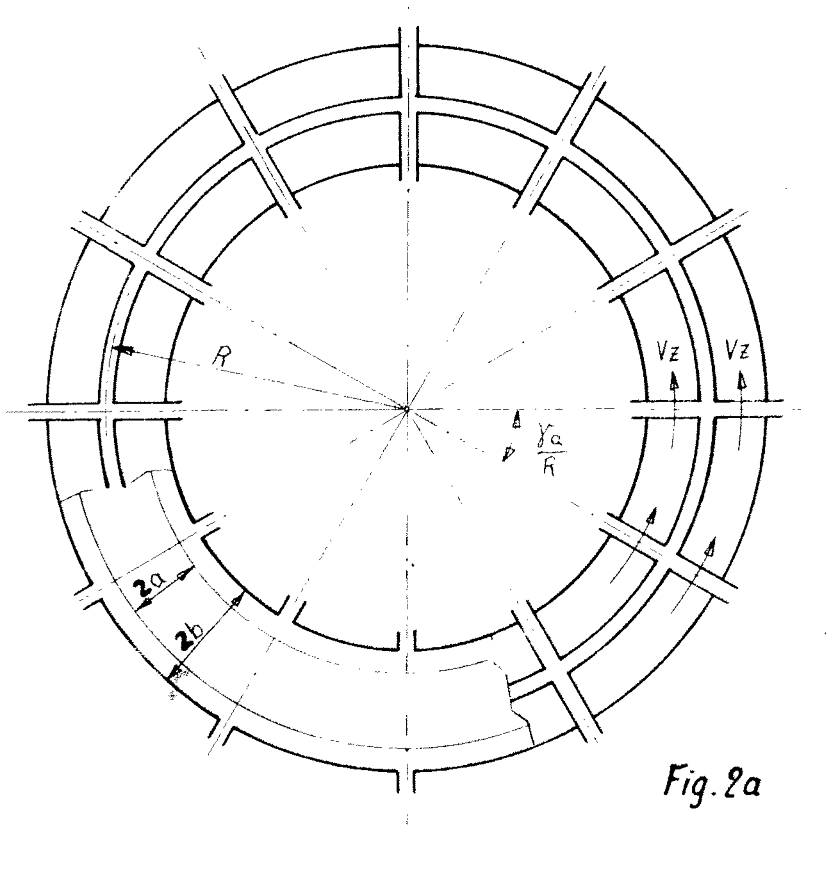

Fig. 2a

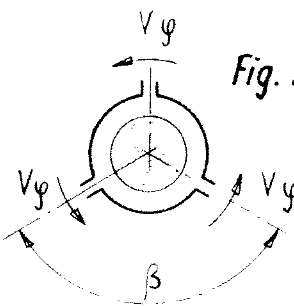

Fig. 2b

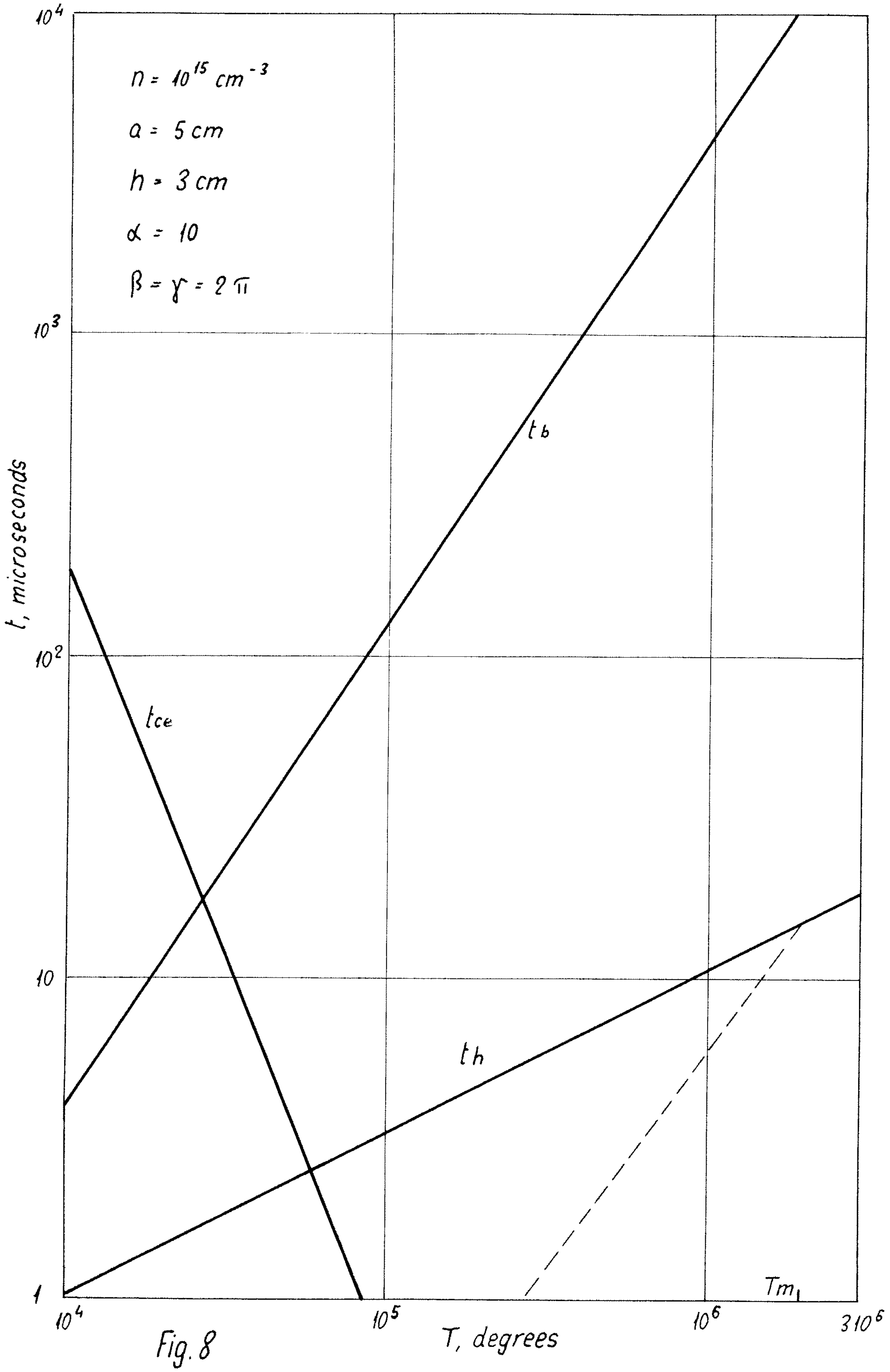
$n = 10^{15}\ cm^{-3}$
$a = 5\ cm$
$h = 3\ cm$
$\alpha = 10$
$\beta = \gamma = 2\pi$
t_b
t_{ce}
t_h
T_{m_1}
t, microseconds
T, degrees
10^4
10^3
10^2
10
1
10^4
10^5
10^6
$3\,10^6$

Fig. 8