# IFS-Modeling for Feasible Freeform Timber Constructions

**Ivo STOTZ**
**Dipl. Arch. EPFL, PHD-Student**
**Laboratory for Timber Constructions, IBOIS, Ecole Polytechnique Fédérale (EPFL)**
**Lausanne, Switzerland**
**Yves WEINAND**
**Prof. Dr Engineer EPFL, Architect ISA**
**Laboratory for Timber Constructions, IBOIS, Ecole Polytechnique Fédérale (EPFL)**
**Lausanne, Switzerland**

## Summary

Within the last years, architectural design has shown great interest in the design of new and complex geometries, commonly known as 'free-form' architecture. The lack of constructive feasibility of the designed architectural objects led us to consider discrete IFS-Modeling for architectural use. The aim is to develop a powerful tool for the generation and the fabrication of freeform architecture. Based on the findings of BARNSLEY, it uses controlled iterated function systems (IFS). The generated geometric figures are further treated by a set of post processing procedures - a set of specific geometric transformations, which allow converting the geometric data directly into construction elements. The method is tested and verified by the construction of prototypes, which prove the efficiency of the proposed method. The developed tool shows high potential to produce accurate free-form timber constructions at reduced design and production costs.

## 1. Framework of the research

This paper presents some results and findings of the research project 'Fractals and it's applications in the field of timber construction'. This project is currently being carried out at the IBOIS-EPFL [Laboratory for Timber Construction, Ecole polytechnique fédérale de Lausanne, Switzerland] in straight collaboration with the LIRIS, Université Lyon I in France. It presents interdisciplinary aspects that combine the domain of construction with the field of mathematics and computer science. The project has started in 2005 and is supported by a grant from the Swiss National Fund since 2006.

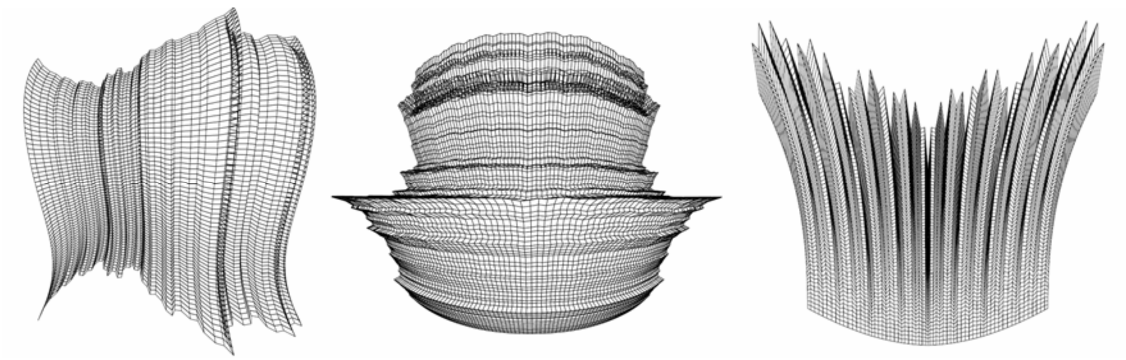

*Figure 1: Three IFS-surfaces of free-form shapes presenting different aspects, smooth and rough.*

## 2. Context

The creation of free-form architecture has recently been explored by a great number of contemporary architects. While the design of such new shapes is relatively easy - via classical CAD-Software – it's physical realization turns out to be rather difficult. The established traditional detailing is often not applicable to such 'new shapes'. Therefore, great effort is used to adopt the designed NURBS-surfaces to a constructible form.

Generally, the result is a set of constructive elements - each of different shape and size. Integrated production processes allow the fabrication of such a set of elements at relatively reasonable costs. Nonetheless, the effort put into the adoption of complex free-form geometries into a coherent set of constructive elements remains often tremendous. From the perspective of efficient architectural production - from design to production - several issues still need to be worked out, and a lot of questions remain unanswered.

## 3. Goals

The present work seeks new solutions, which will allow a more direct and efficient way to bring free-form geometries down to the level of construction and detailing. The method proposes to have a closer look at the geometrical and mathematical methods used to construct free-form objects. While the term 'free-form' is more or less common in the field of architecture, it is not precise enough to work on in the field of geometry. Therefore, the geometrical methods of our focus needed to be specified more precisely. We choose to work on iterated function systems (IFS), which is one among several methods used to produce free-form shapes. On the one hand, IFS-modeling allows the construction of smooth shapes such as Beziers or Splines. On the other hand, it offers the possibility to build less common shapes, like e.g. fractals (cf. figure 1).

The closer study of the geometrical method used to construct free-form objects should allow to find coherent ways to derivate construction elements directly from the geometrical data. The aim is to find ways to produce free form architecture more efficiently, what basically means at a lower price in terms of design and production time. The geometrical data, which is obtained by IFS-modeling, is expressed in a set of finite elements. Basically, we try to convert the geometric parts as directly as possible into constructible elements.

## 4. Brief presentation of IFS-Modeling

The principles of IFS-modeling found on the findings of Barnsley [1]. IFS-modeling is a modeling technique that constructs geometric objects step by step. The Iterative Function System (IFS) is defined by a set of functions. Each function represents a geometric operation, which describes a combination of rotations, displacements and scalings. A geometric operation is here called transformation. To start the construction of an object using IFS-modeling, each transformation is applied on a simple element (commonly called germ or primitive). As result we obtain a set of duplicates of the initial germ, exactly one duplicate per transformation. The first construction step of the geometric object is herewith done. Step two consists in the application of the same set of transformations to each of the duplicates, which have been obtained in the construction step before. In other words: in step two, the transformations are applied to the result of step one. In step three, the transformations are applied to the result of step two, and so on.

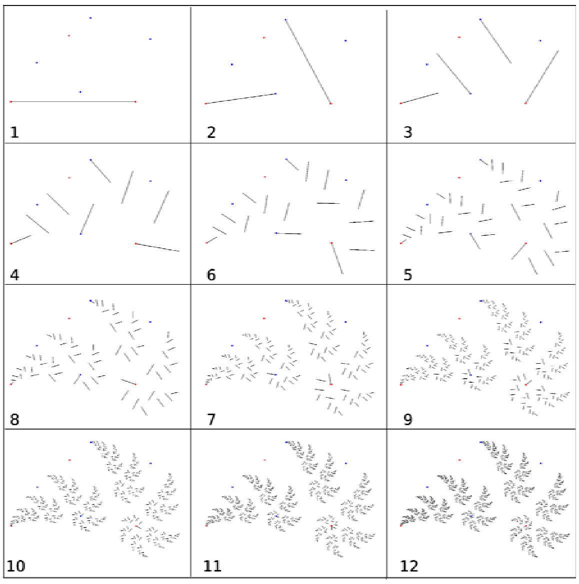

*Figure 2: After a few construction steps, this IFS Barnsley fern appears*

More Detailed information of the mathematical model that is used in our research can be found in [2] and [3]. To have a better understanding of IFS-modeling, we propose to look at the example of a Beziers curve. Within the following, the IFS and the construction steps are explained in detail.

### 4.1 Iterative Geometrical Construction of Bezier Curves

For the construction of Bezier curves De Casteljau's method is considered. In fact, this method is perfectly compatible to the IFS-model. It uses a set of two transformations, which operate on a certain number of entry points. In the case of classical CAD-software, the points are called 'control points'. The global shape of the curve can be manipulated by its control points. De Casteljau's algorithm operates iteratively on these points. The resulting object is an object constituted by edges and vertices. By increasing of the level of iteration (the number of construction steps) the obtained curve fits more and more the analytical model of the curve, always being represented in a discrete form.

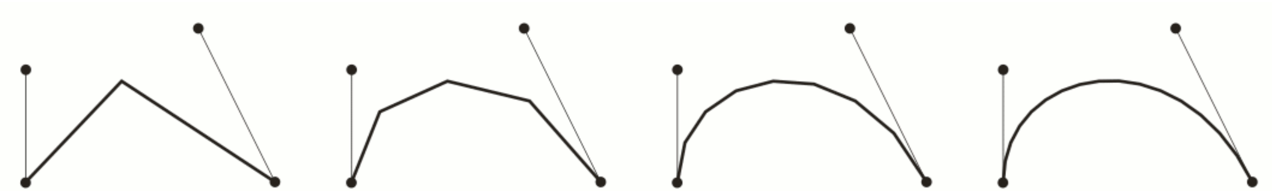

*Figure 3: The first four construction steps of a Bezier curve – using De Casteljau's method.*

In figure 4, the input is a set four control points [P0, P1, P2, P3]. They are the same as the ones of the example shown in figure 3. The two transformations output (after the first iteration) two sets of points: [P0', P1', P2', P3'] and [P0", P1", P2", P3"]. The transformations consist in a set of simple midpoint calculations. Each of the generated points is the midpoint of two already existing points. All the computed points are part of the resulting output except Ph, which is a helper point. Once all points have been calculated, the output sets serve as input for the next level of construction. In that way, [P0', P1', P2', P3'] will become [P0, P1, P2, P3] for the left part of the curve as well as [P0", P1", P2", P3"] will become [P0, P1, P2, P3] for the right part of the curve.

$P_0, P_1, P_2, P_3$ ->>

$P_h = (P_1 + P_2) / 2$
$P_h = (P_2 + P_3) / 2$
$P_1' = (P_0 + P_1) / 2$
$P_2' = (P_1' + P_h) / 2$
$P_2'' = (P_2 + P_3) / 2$
$P_1'' = (P_h + P_2'')/2$
$P_3' = (P_2' + P_1'')/2$
$P_0' = P_0$
$P_0'' = P_3'$
$P_3'' = P_3$

->> $P_0', P_1', P_2', P_3'$
->> $P_0'', P_1'', P_2'', P_3''$

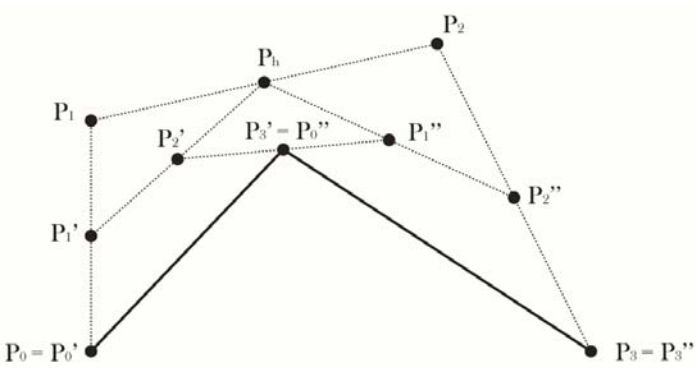


*Figure 4: The first iteration of the construction of a Bezier curve, using De Casteljau's method.*

## 5. Constraints from the field of construction

In order to construct physically the geometric figures, which have been built using the aforementioned IFS-formalism, additional geometric constraints will have to be added. First, the geometric elements will have to be converted into construction elements. Let's take for example an IFS-curve: It has no body, which means that its components, which are lines and points, have virtually no thickness. While converting the lines of an IFS-curve into its constructive pendants - linear construction elements like bars or beams - two new geometric attributes will be added: width and depth. Further, the geometric components will have to be subjected to a size control. On the one hand, the size of the geometric components must not exceed the size of a given construction material. On the other hand, too small elements would lead into complications in terms of handling and assembling of the actual building.

While the conversion problem of linear elements is relatively easy, the construction of IFS-surfaces implies a more complex conditioning of the geometric figures. One considered application is to build a shell structure out of planar elements such derived timber panels as e.g. plywood boards. The raw material is a planar construction material of a given thickness. This implies following problems:

The components of the modeled geometries, its faces, are generally not planar. Therefore, following questions arise: How to convert non planar geometric elements into planar ones, without affecting the global aspect of the initial design? Which are the methods which would allow the construction of geometric figures entirely composed by perfectly planar faces? Is it possible to bend planar construction material onto non planar geometries? If yes, to what extend? These questions show the importance of the problem of planarity. Point 7 will discuss the question of planarity respectively quasi-planarity more in detail.

## 6. Applications of an IFS-curve – Bezier vault

Hohler shows a larger spectrum of possible applications of fractal curves in [4]. In the present paper, an example of an iteratively modeled Bezier curve is discussed, which is applied to a vault structure. The utilization of an IFS-curve for the construction of a vault structure is of greater interest. The iteratively constructed geometric object is expressed in a discrete number of elements. The obtained geometric data consists of a set of ordered vertices and edges, which are part of **R2**.

The size of the geometric elements depends on the level of iteration. By choosing the adequate level of iteration, it is possible to adjust the size of the geometric elements to available sizes of construction elements. The linear segments of the geometric object (the edges) are translated into linear construction elements, which are raw sawn timber planks in the present case. The lengths of the timber planks are completely defined by the geometric model. In addition to the length, the angle data for the chamfer cut of the planks is also extracted from the modeled object. Knowing the length as well as the two cutting angles, the planks are ready for integrated manufacturing (cf. figure 5). By this method, the entire design process is reduced to two steps which are:

1) The control of the Bezier curve via its control points (shape control)

2) The choice of the level of iteration (size control)

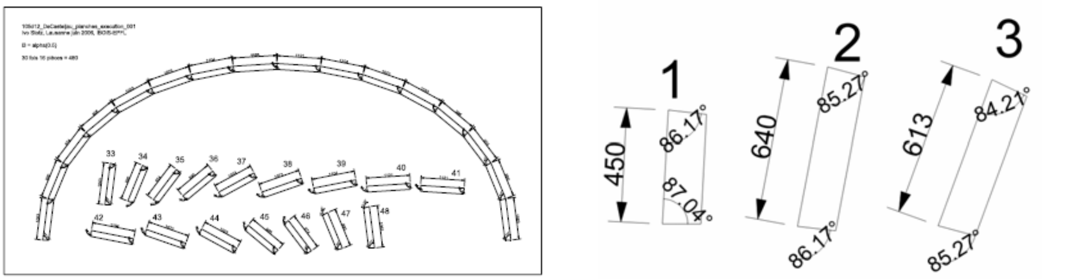


*Figure 5: The fourth level of iteration of the construction of a Bezier curve is transformed into construction elements. Lengths and cutting angles are extracted from the geometric model. Automatic generation of the construction elements. One CNC-file is written for each plank.*

The question of the construction grid and the construction rhythm, which is generally a big issue for the construction of complex shapes, becomes obsolete as it is directly induced by the geometric modeling method. The construction of the vault structure, which is shown in this example (cf. figure 6), consists of a series of five different Bezier-curves. Each of the five curves has a slightly different set of transformations, a slightly different IFS. The choice of the IFS assures a minimal overlapping of the planks from one curve to the other, which is necessary to stiffen the structure.

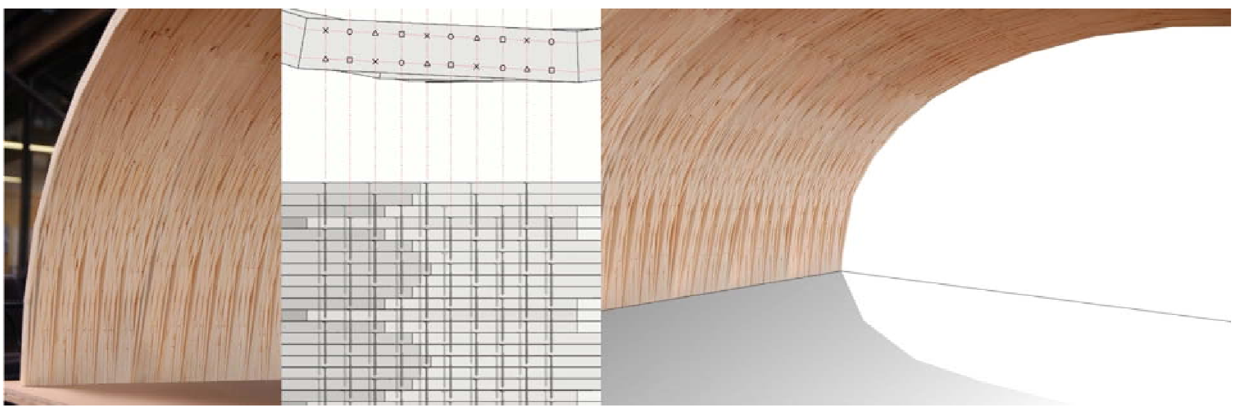

*Figure 6: Reduced scale model and assembling detail of a Bezier vault construction realized out of screwed timber planks.*

## 7. IFS-Surface design

The following example utilizes tensor products as method for the geometric design of IFS-surfaces. In figure 7, the geometrically modeled surfaces are entirely composed of quadrilateral elements, called faces or quads. These faces, which are situated in the three dimensional space, are defined by four points called vertices. Generally, four points are not part of one common plane, which means that the modeled faces are not constrained to be coplanar. The construction material that we use for the physical construction is a planar timber panel. This puts up the following two questions:

1. How to build using non-planar elements
2. How to improve planarity of non-planar geometries

Ideally, it would be far preferable to model directly geometric objects, which are entirely composed of planar faces. This is actually the main focus of the ongoing research. That would imply the definition of additional geometric constraints which guarantee/verify the planarity of each face. On the one hand, these constraints might restrict the design possibilities of the proposed modeling technique. On the other hand, to dispose figures of completely planar faces allows bypassing the geometric transformations, which actually try to improve planarity of non planar faces.

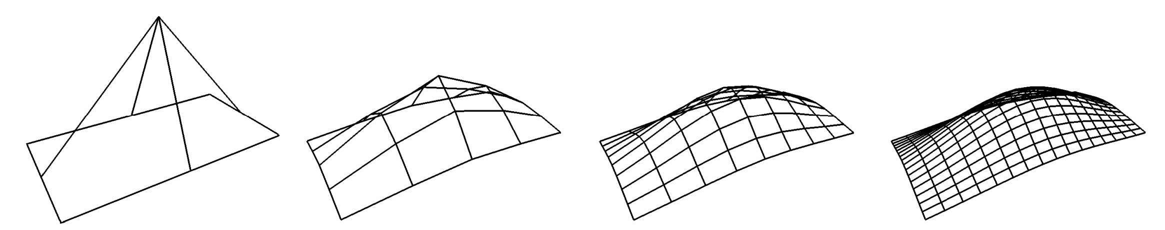

*Figure 7: Four construction steps of a B-Spline-surface.*

### 7.1 Bending planar construction material onto non-planar geometries

We propose to unroll each face in order to get a planar cutting pattern, which then will be automatically manufactured using CNC-machines. The produced pieces will then be constrained (bent) in order to reproduce the non-planar geometry of the initial faces. The limits of this procedure are directly linked to the material properties of the employed timer panel. In order to limit the initial stress due to the bending of the wooden panel, minimizing the bending curvature is of greatest interest. Exceeding the limits of the material properties will result in fractured and therefore unusable construction panels. In order to minimize the curvature of the faces, a perturbation method is employed. This method acts on the IFS-Figure with the goal of reducing the local curvature of each face. It will be discussed later on.

Two ways to unroll a non-planar face are considered. Each of them is triangulating the quad into two triangles. The first triangulation gives us the triangles ***ABC*** and ***ACD*** while the second one gives the triangles ***ABD*** and ***BCD***. Assuming that each triangle forms a plane having a normal vector, we are able to measure the angle between the two normal vectors. In general, the angle between the planes ***ABD*** and ***BCD*** are different form the angle between the planes ***ABC*** and ***ACD***. This property allows choosing the way of triangulation in function of the curvature value. Aiming to reduce the initial stress of the timber panels, the smaller angle between the surfaces normal vectors will be preferred.

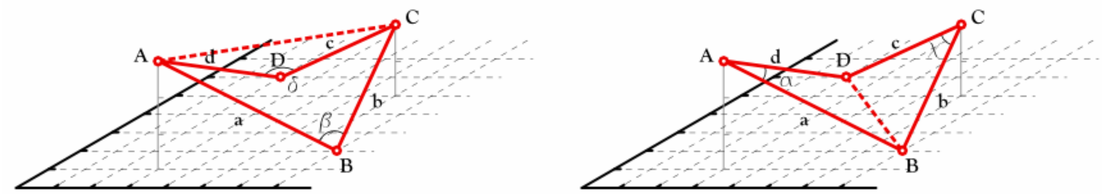


*Figure 8: Non planar quads present two ways of triangulation. Depending on the angle value between the triangular planes the case presenting the lower curvature is chosen. This allows lessening the initial stress of the bent timber board.*

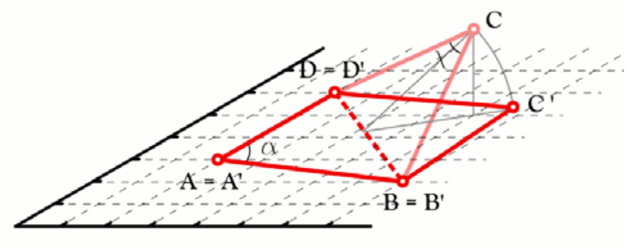


*Figure 9: Quadrilateral faces are generally non planar. To unroll the shown quad ABCD, point C is transformed by a rotation along the axis BD, in order to be part of the plane ABD.*

After having decided about the sense of triangulation, the non planar quad face is ready to be unrolled. Three points ***ABD*** will be considered as fix points, which define the reference plane. As ***ABCD*** form a non planar quadrilateral face, point ***C*** is not part of the reference plane ***ABD***. The transformation which will bring point ***C*** into the reference plane is a rotation along the axis ***BD*** (cf. figure 9). This way, the lengths of the four sides of the quad faces will remain unchanged. The only geometric data, which is actually affected, are the two angles next to point ***B*** respectively point ***D***. Choosing the rotation rather than an orthogonal projection, has the advantage to conserve the length of the sides. This is highly important for the later assembly of the pieces, in order to guarantee perfectly closed joints in-between the quad elements. Finally we can state that we obtain a planar image (***A'B'C'D'***) of the initial quad (***ABCD***) by one transformation on point ***C***.

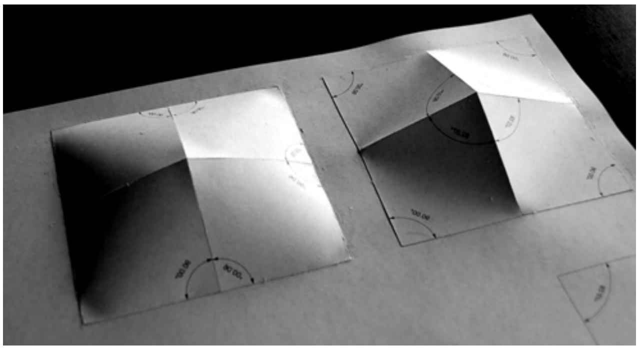

*Figure 10: Non planar quads present two ways of triangulation. The rendering of the shape is a more or less accurate representation of the initially modeled geometric object.*

Figure 10 illustrates a simple case of four non planer quad faces. The way the faces are triangulated does not depend on the criterion of the smallest angle, which is not always the most accurate way to represent the initial geometry. The different appearance of the two solutions shows the importance of the sense of triangulation. In the first case, the rendering of the object is much smoother than in the second case, where you can clearly identify the edges between the single faces. The sense of triangulation acts on the angles between the faces. The given example shows an extreme situation presenting a very important curvature angle. Further on, smaller curvature angles will be used for the physical construction out of timber panels.

### 7.2 Perturbation method to lessen the local curvature of non-planar quad meshes

In the following example, the curvature analysis of an IFS-surface shows that the greatest curvature is situated near the four corners of the initial geometric object, which is a cubic B-Spline surface (cf. figure 11). A perturbation algorithm acts on the vertices of the object tending to minimize the curvature then of the elements constituting object.

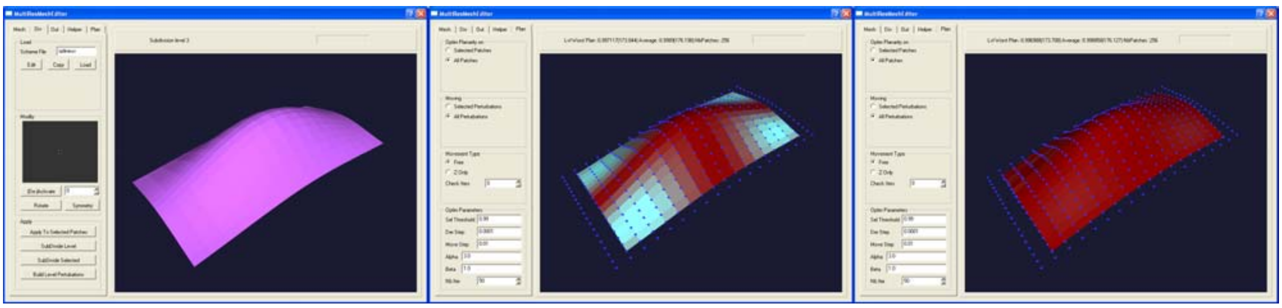

*Figure 11: [Left] Initial geometric object (tensor product of two cubic B-splines). [Middle] Curvature analysis showing the greatest curvature near the corners of the shape. [Right] Optimized geometric object after planarization.*

While existing methods for the optimization of planarity presented in [5] and [6] work well for smooth 'free-form' surfaces (***C1***), the optimization of fractal surfaces turned out to be more delicate. Applied on fractal surfaces, the method presented by Pottmann et al. provided unsuitable results. Therefore a new optimization method needed to be worked out.

In details, the planarization method lets you make a post treatment on quad mesh surfaces. Each patch is made of several faces, made of four points. The goal of the planarity optimization is to get faces as planar as possible. The planarity of each face is defined by the cosines of the angles between the corners. The closer it is to 1, the more the planarity is verified. A planarity is considered correct when this cosine is above a given threshold.

Within the following, the principle parameters that define the planarity optimization are prsented:

It is possible to act on selected areas of the figure. A selection threshold lets you define the value of the cosine used to select the patches whose worst planarity is above this limit.  For calculating the gradient, a step determines the deepest descent direction. Further, we defined a move step, which is the initial step utilized to move points to make planarity. Two coefficients, Alpha and Beta, define the importance respectively of the worst planarity and the average planarity in the optimization calculus. For example if Alpha is set to 1 and Beta to 2, then optimizing the average planarity is twice more important than optimizing the worst planarity. Finally, the number of iterations made to improve planarity can be set. To improve planarity by moving the points can last a very long time. The number of iteration let's one define in some way the time spent on the planarization. By increasing the number of iterations the calculus will take more time to complete but its result will tend to be more accurate.

### 7.3 Testing the method on a reduced scale model

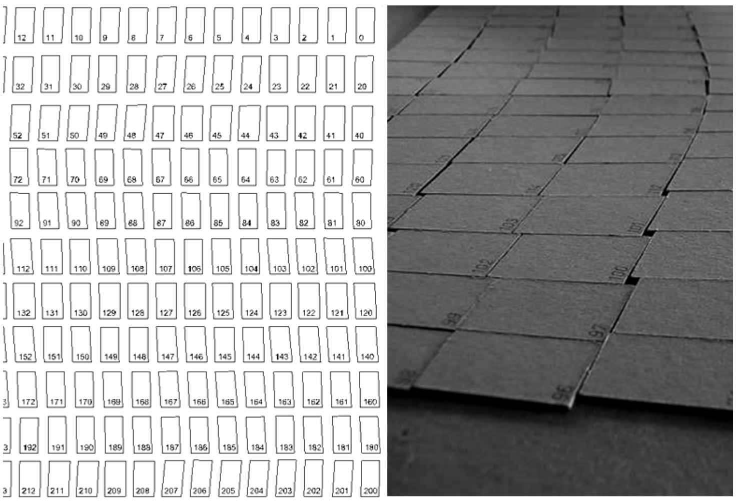

*Figure 12: Each board, different in size and shape, receives a unique address. Automated generation of the CNC-files are prepared for integrated production and produced by a 3 axis cnc-machine*

So far, the geometry of the cubic B-spline surface has been optimized. The quads that compose the surface have been planarized in order to minimize their local curvature values. Still, the quads are not completely planar jet. Therefore, each quad will have to be unrolled. This is done via a triangulation in function of the lowest angle criterion. Once the quads are divided into two triangles, they are unrolled respectively unfolded and drawn on a 2D plane. This gives us the basis of the manufacturing plans. In addition to the geometric data, each flattened quad receives a unique address, since all the elements are different in shape and size. The addressing (unique numbers) will allow the organization of the 'construction elements' in space, which is necessary for their assembly. The whole process of unrolling, addressing, and NC-file creation has fully been automated. It is done in one click.

The geometrical data of each board and its address are then written into single files for integrated production. As shown in the figures12 and 13, the method and the geometry are verified by the construction of a reduced scale prototype. The cut plates are first laid out on the ground according their address. The set of plates on the ground does not cover entirely the plane. There exist gaps in between the elements. The assembly of the elements has to assure a joint-less connection of the plates along their sides. This process was started in one corner of the B-Spline surface and ended in the opposite corner. The fact that all the plates are assembled joint-less will lead automatically to the generation/representation of the initially modeled geometry.

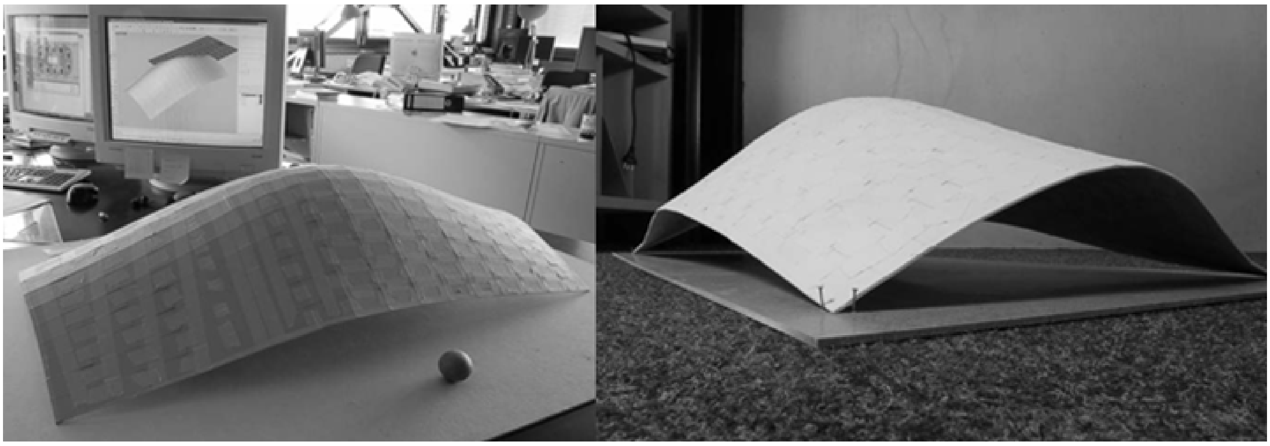

*Figure 13: The method and the geometry are tested and verified on a reduced scale prototype.*
Figure 13 shows the assembled reduced scale model. The visual comparison of the virtually modeled geometry, shown on the computer screen and its physical image, presented in the front on the table, match really well and therefore proof the high accuracy of the engaged procedure - from the geometric design down to its physically built form.

## 8. Discussion and perspectives

The present paper shows two specific design possibilities offered by IFS-modeling. The discussed applications reveal only a preview of a broader spectrum of design capabilities, which provide a powerful design tool to the architect. Hence, the design of more complex and uncommon figures gets possible. The presented modeling method has the advantage to produce a discrete expression of free-form shapes, which has proved to be an ideal basis for the construction grid. The design of the construction elements is herewith directly given by the geometric model – and this at an early stage of the design process. This saves design time and will help to achieve free-form timber constructions at relatively reasonable costs.

The conversion of IFS-surfaces into construction elements let to the development of relatively advanced methods of geometric conditioning. This geometric post treatment is achieved with a high degree of automation. The discussed case was based on a tensor product surface and therefore composed of non-planar elements. Even if the developed methods which improve planarity and unroll the quad faces (in order to get the production drawings) work fairly well, the actual research investigates on alternative surface design methods that provide directly planar construction elements (cf. figure 1). This will allow further optimisation of the production process of complex architecture. The perspectives for the use of IFS-modeling in the field of construction are very promising. Regarding the present development in terms of integrated production in the European timber industry, the proposed design method has a high potential to offer new efficient ways to produce innovative timber constructions.

## 9. References


[1] M. F. Barnsley, "Fractals Everywhere", *Academic Press*, 1988

[2] E. Tosan, I. Bailly-Salins, G. Gouaty, I. Stotz, Y. Weinand, P. Buser, "Modélisation itérative de courbes et surfaces : aspect multirésolution", *GTMG07*, 2007.

[3] E. Tosan, I. Bailly-Salins, G. Gouaty, I. Stotz, Y. Weinand, P. Buser, "Une modélisation géométrique itérative basée sur les automates", *GTMG06*, 2006.

[4] A. Hohler, "Modèles fractals dans la construction", *NIKE-Bulletin*, 1-2/2007, NIKE, p. 40ff.

[5] Y. Liu, H. Pottmann, J. Wwallner, W. Wang, Y.-L. Yang, "Geometric Modeling with Conical Meshes and Developable Surfaces" ACM Trans. Graphics, Vol. 25/3, *Proc. SIGGRAPH 2006*, p. 681-689

[6] H. Pottmann, J. Wallner, "The focal geometry of circular and conical meshes", *Geometry preprint 163*, Technische Universität Wien; 2006