Geodesic Lines on Free-Form Surfaces - Optimized Grids for Timber Rib Shells

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Summary

In order to optimize grids of timber rib shells with regard to the bending stress of the boards due to initial curvature, the software GEOS has been developed at the Swiss Federal Institute of Technology in Lausanne (EPFL) between 2002 and 2004. The construction of a prototype in the summer of 2005 proved the reliability of the assumptions, the program is based upon.

The following article describes the steps which were undertaken starting from the generation of the form and the design of the optimized grid to the final construction of the prototype. Loading tests have been carried out to evaluate the structural calculation model. Finally, the comparison between calculated and measured deformations is briefly discussed.

1. Introduction

Although concrete and steel are the most common materials for the construction of lightweight space structures in modern architecture, timber has recently regained a well-deserved renaissance. In addition to glue-laminated timber, screw-laminated timber has increasingly been applied to rib shells. In the past two decades several spatial structures of this type were constructed ([1],[2]). The transparency of their bearing behavior and their attractive architecture do fascinate both experts and laymen alike.

The ribs are made from laminated timber boards, which are joined together with the aid of pin-like fasteners such as screws or nails. In contrast to other rib shell structures which were made from squared timber sections ([3],[4]), relatively thin boards with a thickness from 16 mm to 35 mm are used. These laths are inexpensive and construction can be executed in a relatively simple way without sophisticated techniques. The extra cost due to manual labor for assembly can be compensated for by rationalization methods during the planning and manufacturing process. For geometrically demanding structures this method of construction is an interesting alternative compared to glue-laminated timber rib shells.

In order to reduce the stress due to initial curvature, the engineer tends to arrange the ribs on the surface according to geodesic lines. By this means, bending about the strong axis of the boards, which causes unfavorable stress, can be avoided. Ideally the boards are subjected only to bending about their weak axis and to torsion. In addition, this approach allows the use of straight boards. A geodesic line on a surface is defined as a curve, where the normal vector of both curve and surface are parallel or antiparallel at each point. The shortest distance between two points on a surface is always a geodesic line. In the plan a geodesic line represents a straight line. The name “geodesic” is derived from the Greek ge (earth) and daiesthai (to divide).

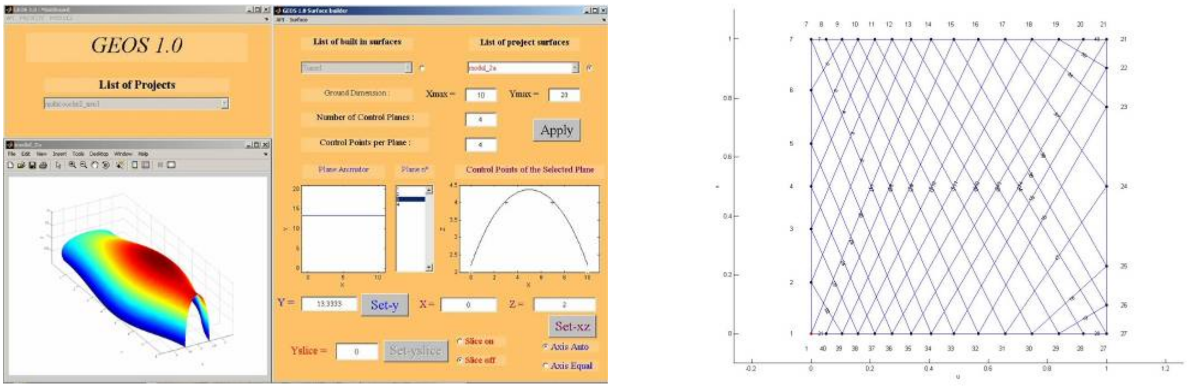
Geodesic lines on simple, regular shaped surfaces can be determined by analytic means. On cylindrical surfaces the geodesic lines correspond to helices, and on spheres to the great circles. On free-form surfaces, currently enjoying great popularity in contemporary architecture, the determination of geodesic lines is much more complex. In order to satisfy this demand and to improve automation of the production process, the software GEOS was developed in close collaboration with the Laboratory of Timber Construction (EPFL/IBOIS) and the chair of Geometry (EPFL/GEOM). This software calculates grids of geodesic lines on free-form surfaces and provides all geometrical data necessary for computer-controlled sawing. The project was financed by the Swiss National Science Foundation (SNF).

In order to examine the reliability of this program and its precision concerning the assumptions that were made, a free-form shaped timber rib shell prototype has been designed with GEOS and constructed at the IBOIS. The realization of this prototype was financed by the “holz21”-fund, a research program of the Swiss Federal Environmental Agency (BAFU).

2. Form Design and Calculation of the Geodesic Lines

The form is defined by manipulating control points of cubic Bézier-polynomials within a certain user-defined number of parallel cutting planes along the surface. Based on these polynomials a Bézier surface is calculated. Starting points and end points of each geodesic line and the connectivity of the lines have to be defined for the calculation of the grid (Fig 1). The iterative calculation of the geodesic lines is based on an L-BFGS-B algorithm. The tangential vector is used as control variable. This quasi-Newtonian targeting turned out to be the most efficient method for the calculation of geodesic lines on mono-patched free-form surfaces [5].

The user has no direct influence on the topology of the resulting grid but by manipulating the starting points and end points and by defining the linking of the geodesic lines. Thus, the result is not always compatible to structural and architectural demands like the regularity of the grid or the minimum radius of curvature of a single board. It has to be adapted by an iterative process. In general it can be stated that the spacing of the meshes tends to widen in significant convex areas, whereas it narrows in concave hollows. However, the existence of geodesic grids with homogeneously spaced meshes on complex free form surfaces is not always evident.



Ground Dimension : Xmax = 10 Ymax = 20

Number of Control Planes : 4

Control Points per Plane : 4

Y = 13.3333 X = 0 Z = 2

Yslice = 0

Fig 1 Software GEOS; design of the free form surface by means of control polynomials (left); definition of starting and end points of the geodesic lines and their connectivity in u,v-parameter range (right)

Based on the calculated grid of geodesic lines a multilayered model is developed according to the number and the dimension of the laths (Fig 2). This is done by extrapolating the layers normal to the surface inwards and outwards. The midlines of such extrapolated layers do not correspond to geodesic lines any longer. They are neither part of the initial surface nor do they represent geodesic lines for the enveloping surface to which they belong. However, the error resulting from this inaccuracy seems to be negligible with regard to the relatively large ratio of the effective radius of curvature R_0 and the thickness d of one single board board, usually applied in practice. The smaller this ratio is, the more important becomes the inaccuracy, with the risk that the boards of a rib are no longer parallel and that the predrilled holes at the intersections do not fit accurately. Therefore, the front part of the prototype has been designed with a relatively small radius of curvature, with a minimum ratio R_0/d of 100.

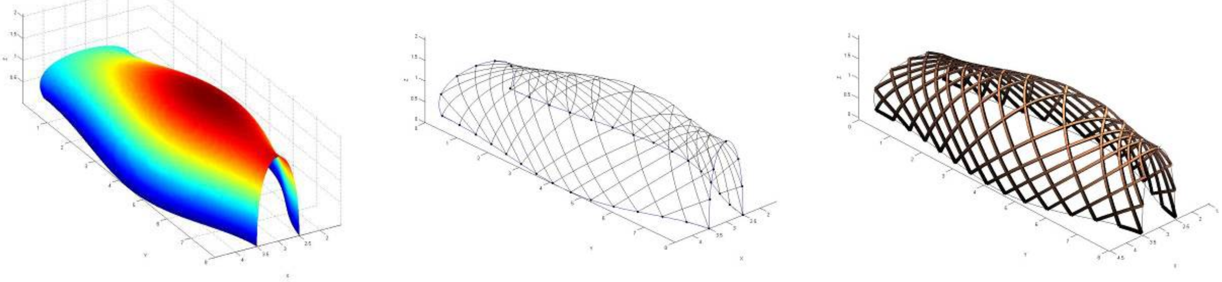


Fig 2 Design of the timber rib shell prototype; generated mono-patched free form surface (left); geodesic line model (center); multilayered model (right)

3. Construction of the Prototype

Thin timber laths with a rectangular section of 12/60 mm, made out of Swiss spruce wood (*Picea abies*), were used for the construction. Due to the tight curvature of the front part of the shell, good quality wood for the laths is very important. Careful visual grading guaranteed an excellent strength class with only a few knotholes. The modulus of elasticity was determined for five random samples. The average value is 14800 MPa. The ribs consist of four laths, of which two are continuous and the two remaining are considered as intermediate in-fill layers, interrupted at each intersection point by the continuous layers of the crossing direction. Both directions are offset by the thickness of one lath. No finger joints were necessary. The longest lath has a length of 6740 mm, the shortest is about 160 mm long. In all, 792 pieces were sawn automatically by means of a five-axis computer-controlled saw (type Créno). The data was exported from GEOS in dxf-format.

Beginning with the inner layer the laths are first fixed at their starting points, then bended across scaffolds and connected to the correspondent end points. The five transversal scaffolds have been planed in order to ensure a maximum of fixed points in space. In general, these supports are not necessary because the expected form is obtained automatically by successively connecting the laths at their intersection points by means of bolts (diameter 10 mm). The continuous inner layers in both directions could thus be realized in a relatively short time followed by the installation of the two outer continuous layers (Fig 3). In general, it can be stated that even after assembly of the fourth layer the predrilled holes at the intersections fitted perfectly and precisely about 85% of the time. This confirms the reliability of the calculation and shows that the demanded precision is kept even for significant curvatures.

After placing all continuous layers in both directions, the intermediate in-fill layers were mounted. Screws with a diameter of 4.0 mm were used to connect the layers. The effective distance of the doubled-rowed fasteners is about 50 mm. A detail before and after the assembly of the intermediate in-fill layers is shown in Fig 4. The in-fill layers are, at both ends, about 2 mm shorter then the clear distance between the continuous laths.

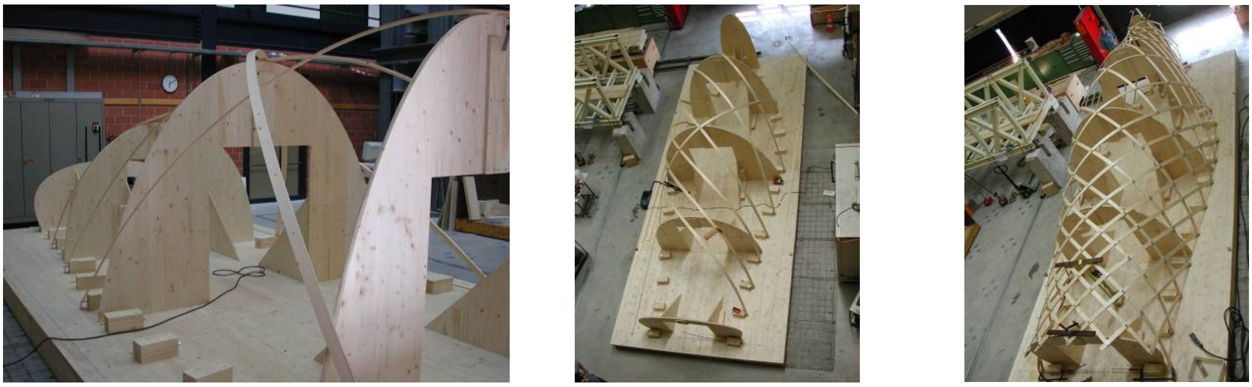


Fig 3 Construction of the prototype

Fig 4 Intersection points before (left) and after (right) assembly of the intermediate in-fill layers

4. Data

Lath’s rectangular section: 12/60 mm (Swiss spruce of very good quality, 602 running meters)
Number of pieces: 792 including the intermediate in-fill layers
Timber volume: 0.434 m³
Dimension of the shell: 8000 mm x 3000 mm x 2060 mm
Developed surface: ≈ 36.0 m²
Base: ≈ 18.6 m²
Number of bolts: 202 M10
Number of screws: ≈ 3000 ∅ 4.0 mm

5. Load tests

For a better understanding of the load-bearing behavior of the structure load tests were carried out before and after the assembly of the intermediate in-fill layers (construction phases C1 and C2). For both phases three symmetric load cases with single forces on different intersection points were examined. The forces, introduced by means of a cable winch, were applied normal to the surface. Uniaxial deformation of a total of seven intersections around the charged points was measured (Fig 5). The displacement transducers were adjusted so as they measured deformation normal to the shell’s surface and parallel to the introduced load in the case of the charged intersections. The test results show that, for relatively low charges, the structure hardly suffers plastic deformation at all and reacts perfectly symmetrically. A distinct increase in stiffness could be shown after mounting the intermediate in-fill layers. This is not only true for those load cases, where the structure shows flexional behavior, but also for load cases, which are essentially transmitted by normal forces. Certainly, the flexional stiffness of the ribs increases considerably mounting and screwing the intermediate in-fill layers. However, due to a favorably double-curved surface the effect of the intermediate layers on the stiffness of the structure was estimated far less significant. In most of the cases examined, the deformations were halved by assembling the intermediate in-fill layers.

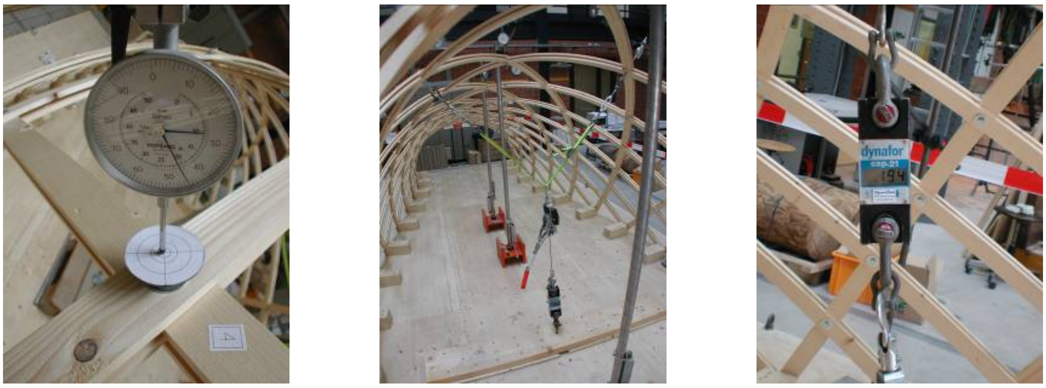


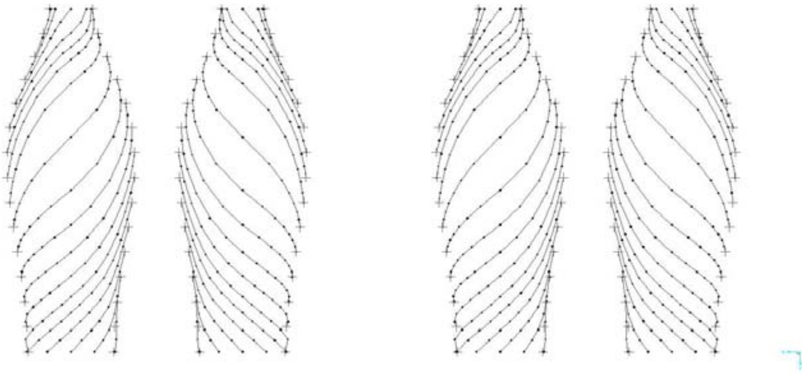
Fig 5 Load tests; uniaxial displacement transducer (left); loading with two single loads (center); load measuring (right)

6. Structural analysis

The comparison of the measured deformations w_{mes} with the calculated deformations w_{cal} enabled us to evaluate the calculation model, based on a framed load-bearing system. The structural analysis was carried out with the software SAP2000 V9.16 NL from CSI-Berkeley. Based on the geometrical data exported as dxf-file from GEOS the structure was programmed in an external text file and then imported. The generation by means of external text files allows one to carry out modifications in a relatively simple way and ensures a more general overview of the data.

The load-bearing behavior of the structure is quite complex and the engineer has to take into consideration some effects, of which the extend on the structure is more or less unknown. This makes modeling quite difficult. Apart from the influence of the rotational stiffness of the rib's intersections (distortion of the diamonds in their plane), the initial slip modulus of the screwed connection is largely unknown. Due to different displacement behavior of the latter, the characteristic values of this modulus, empirically determined for standard connections and given in the technical specifications, cannot be applied without modification [6]. Preliminary tests were carried out to determine the initial slip modulus in this specific connection. Furthermore, the initial bending of the laths has a pre-stressing effect on the structure's stiffness. This effect, which can ideally be favorable remains disregarded.

As the intersections of two ribs are not perfectly rigid, both directions are modeled separately in form of two substructures. The degrees of freedom are subsequently coupled by means of kinematic linking ("local-constraint" option). This allows the release of single rotation about the local axis normal to the surface at each intersection and thus the simulation of the distortion of the diamonds in their plane. According to the method of shear analogy ([6],[7],[8],[9]) two subsystems are introduced. They are called subsystem A and B. This method is applied for the modeling of the construction phase C2 in order to take into consideration of the flexible compound cross section of the ribs. Subsystem C, introduced for stability problem by Scholz in [9] is neglected due to minor normal forces. Therefore, the model consists of four substructures, two for each subsystem (Fig 6).



substructures B1 and B2

substructures A1 and A2

Fig 6 Four substructures used for modeling of the construction phase C2 in the floor plan. Linking by means of kinematic constraints (not in view)

The following assumptions were made:

- Isotrope and ideal elastic (Hook) behavior of timber
- Geometrical non-linearity (P-delta effect)
- Linear slip behavior of the mechanical fasteners with an initial slip modulus $K_{\text{ser},1}$ of 500 N/mm, according to preliminary load-tests carried out at the IBOIS [6]
- Rotation about the frame 1-axis is considered to be constant between two intersections
- Initial state of bending stress is not taken into account
- No fixed restraints of the substructure B (slip between the laths is not hindered at the restraints)
- Torsion stiffness due to the build-up compound section is not activated
- Weakened sections at the intersections are not taken into account

For two exemplified load cases of the construction phases C1 and C2 the relative deviation between measured and calculated deformation is shown in Fig 7 (before and after fitting the intermediate layers). Observing the former, one can see that with rising load level, the deviation increases distinctly. This is due to non-linear effects, which were not taken into account in the structural analysis. However for small charges, good accordance between calculation and reality (with relative derivations mostly below 30%) could be noticed. Even the comparison of the final construction phase C2, much more complex in its structural behavior and with activation of additional compound stiffness, shows quite satisfactory accordance.

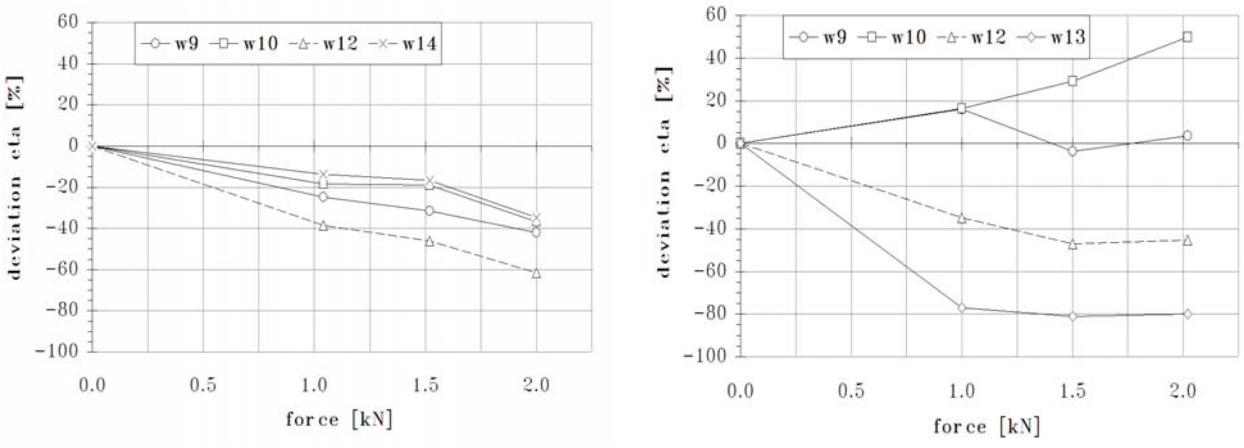


Fig 7 Comparison between measured and calculated deformations of two load cases of construction phase C1 (left) and C2 (right). Directly charged joints in dotted lines

7. Conclusions

The software GEOS, developed to calculate grids of geodesic lines on free-form surfaces was tested by the construction of a timber rib shell prototype. The precision of the calculated geodesic lines, of the computer controlled prefabrication of the laths and its assembly was found to be highly satisfactory and provokes an application for this type of structure at greater scale.

In order to evaluate the structural computer model load tests were carried out. For the observed load cases the tests show that the structure is approximately twice as stiff after assembly of the intermediate in-fill layers. Good accordance between the measurement and the calculation, at least for relevant loads, has been noted. This is all the more true regarding the complexity of the treated structure of which several parameters like the initial slip modulus of this specific connection or the level of rotation at the intersections (no in-plane shear stiffness) are not yet completely clarified. Further reflection on the influence of these parameters on the structure’s bearing behavior is necessary.

GEOS provides an important tool for the design and the realization of timber rib shells. It contributes to clarifying current uncertainties in the design planning process and will highly improve the confidence of engineers and architects in conceiving and realizing this type of challenging lightweight spatial structures.

8. Literature

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