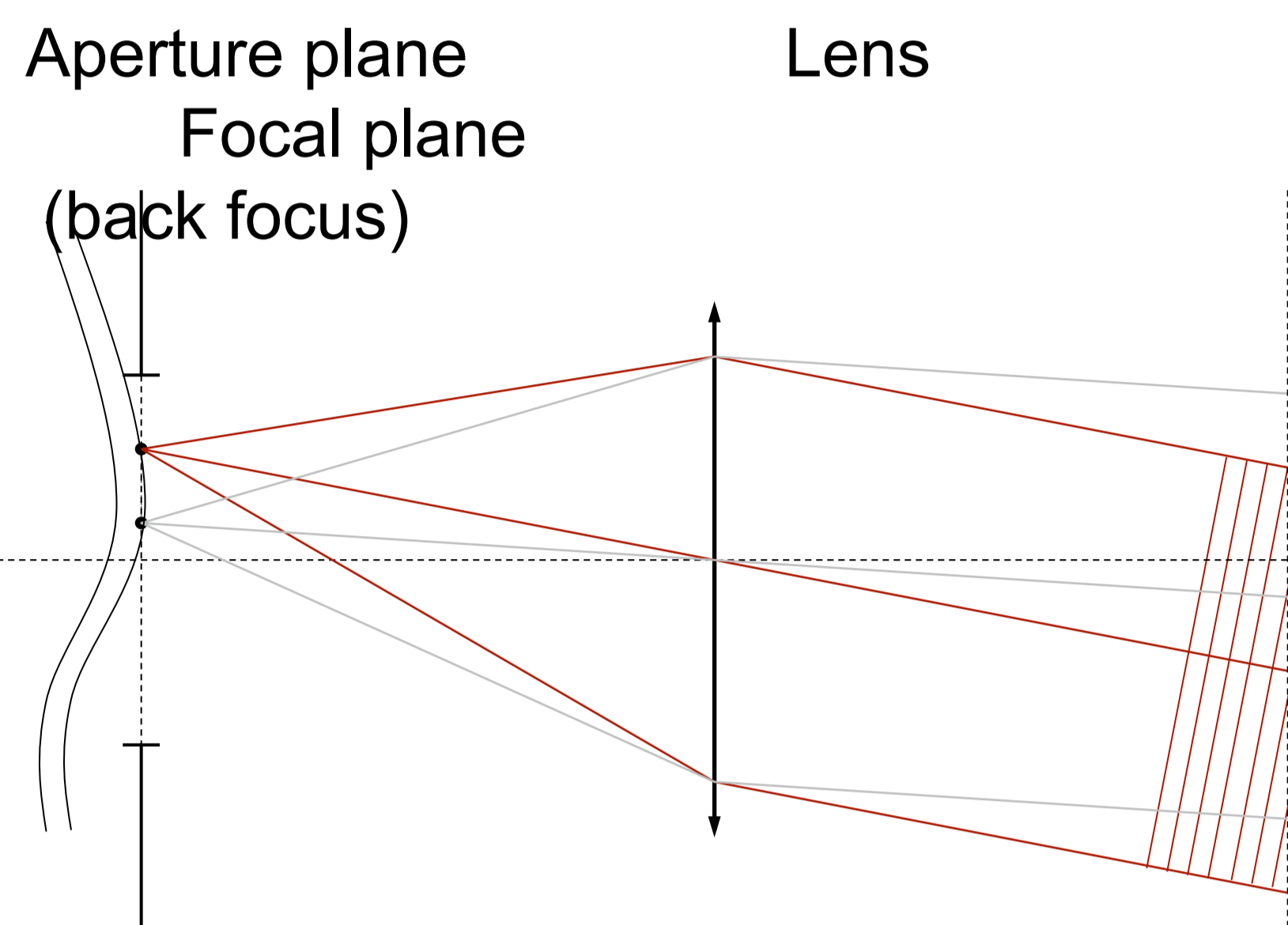


Abstract

We present a method for fast calculation of the electromagnetic field near the focus of an objective with a high numerical aperture (NA). Instead of direct integration, the vectorial Debye diffraction integral [1-3] is evaluated with the fast Fourier transform for calculating the electromagnetic field in the entire focal region. We generalize this concept with the chirp z transform for obtaining a flexible sampling grid and an additional gain in computation speed [4-7]. Under the conditions for the validity of the Debye integral representation, our method yields the amplitude, phase and polarization of the focus field for an arbitrary paraxial input field in the aperture of the objective. Our fast calculation method is particularly useful for engineering the point-spread function or for fast image deconvolution.

Low NA focus fields – Fraunhofer approximation



Focus field in overlap region
(Fraunhofer diffraction integral)

$$\vec{E}(\vec{r}) = -\frac{if}{\lambda_0 k^2} \mathbf{F}(\vec{E}_A \exp(ik_z z))$$

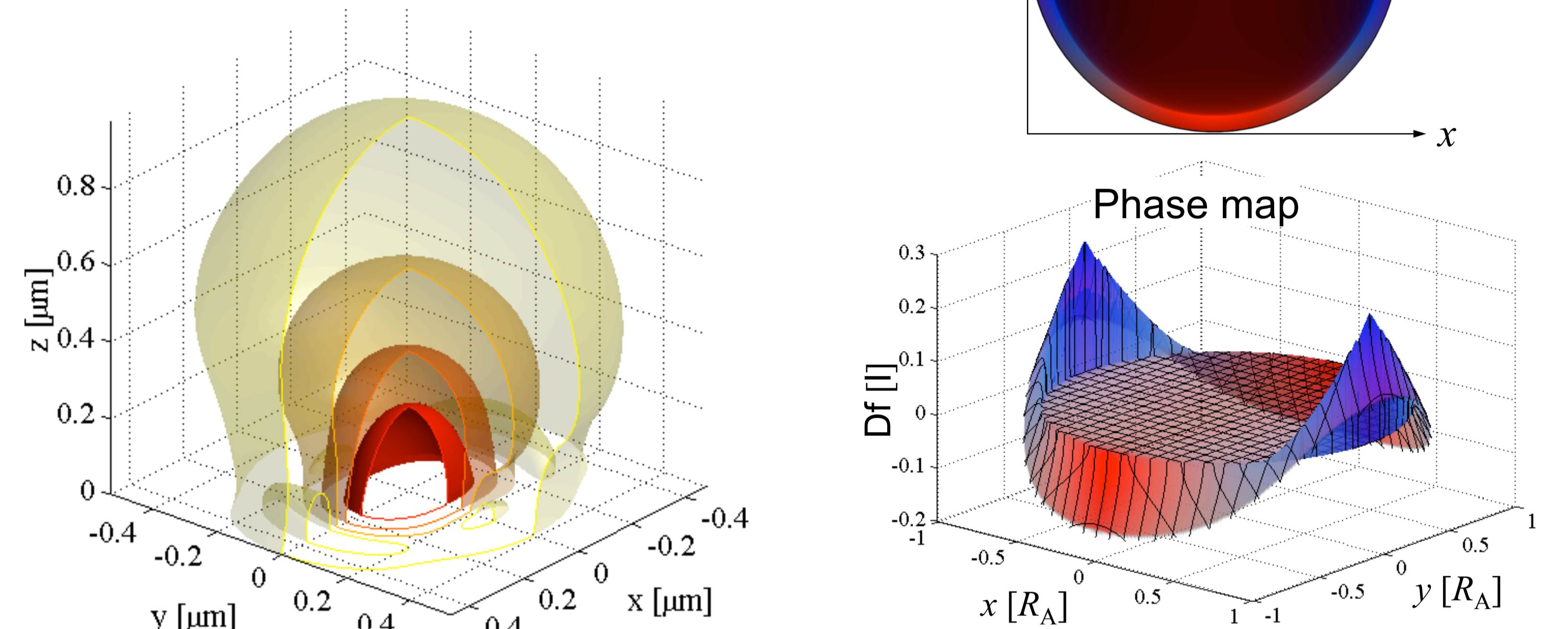
Key properties

- Paraxial description
- Simple calculation with Fourier transform of *incident* field
- Input polarization conserved, i.e. scalar calculation justified

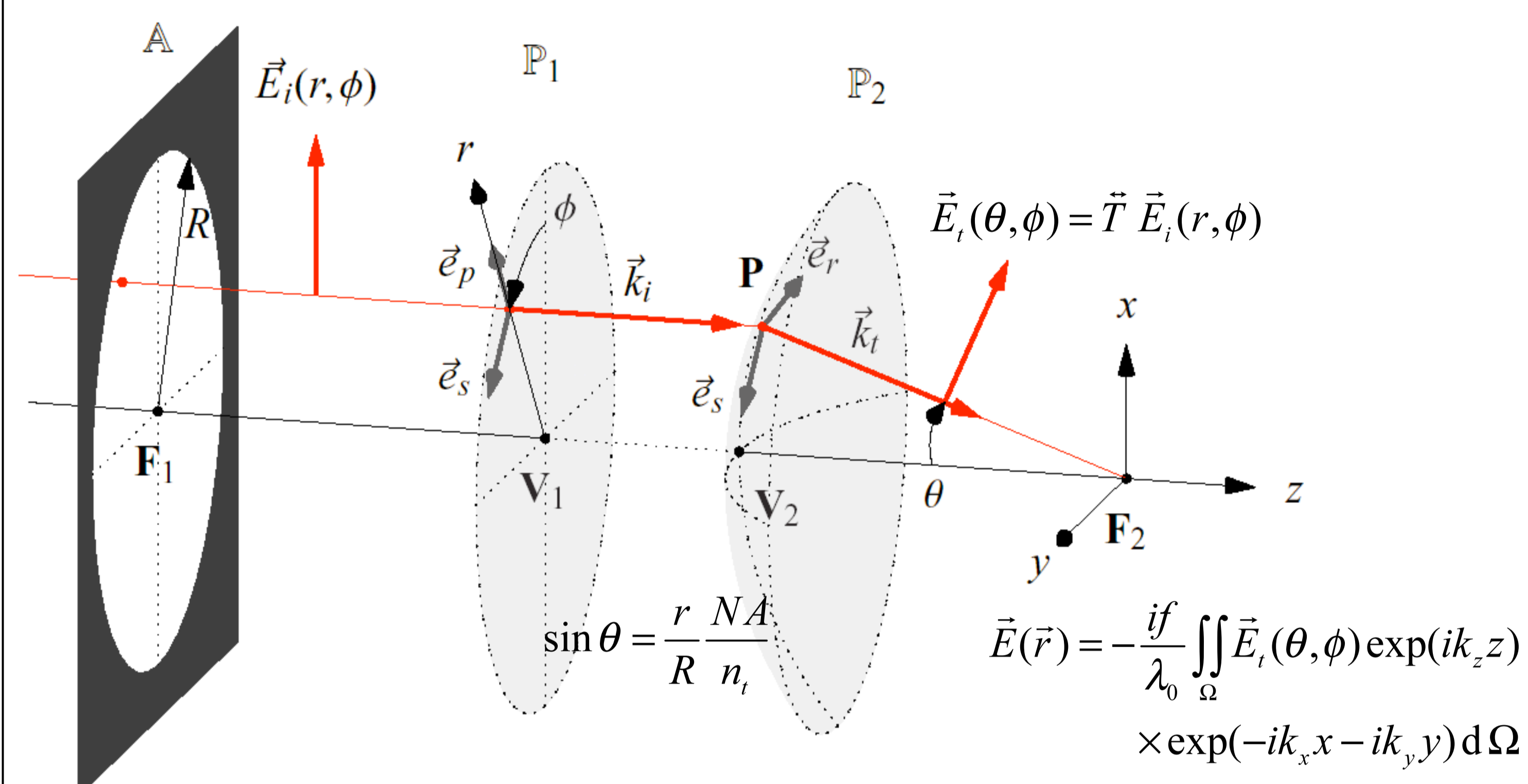
Focus field with 1.45 NA oil immersion objective

- x-polarized laser beam, $\lambda = 633\text{nm}$
- Overfilled aperture A
- Aqueous sample ($n_s = 1.33$)

Focus field ($e^{-1..4}$ iso-intensity surfaces)



High NA focus fields – Debye approximation



Key properties

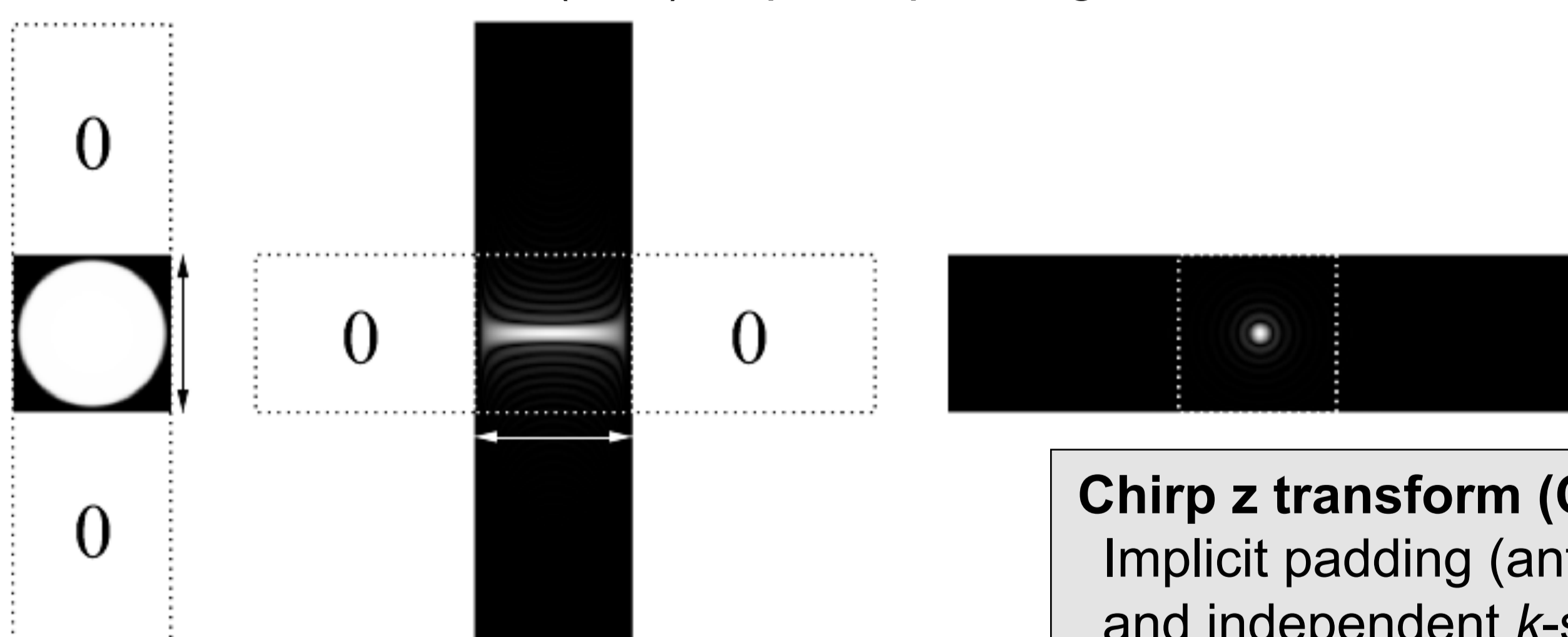
- Vectorial description
- Fourier transform of the *transmitted* and *apodized* field
- Polarization modified

Focus field near F_2
(Debye diffraction integral)

$$\vec{E}(\vec{r}) = -\frac{if}{\lambda_0 k_i^2} \mathbf{F}\left(\frac{\vec{E}_i(\theta, \phi) \exp(ik_z z)}{\cos \theta}\right)$$

High NA focus fields – Generalization

Fast Fourier transform (FFT) requires padding.



Chirp z transform (CZT)

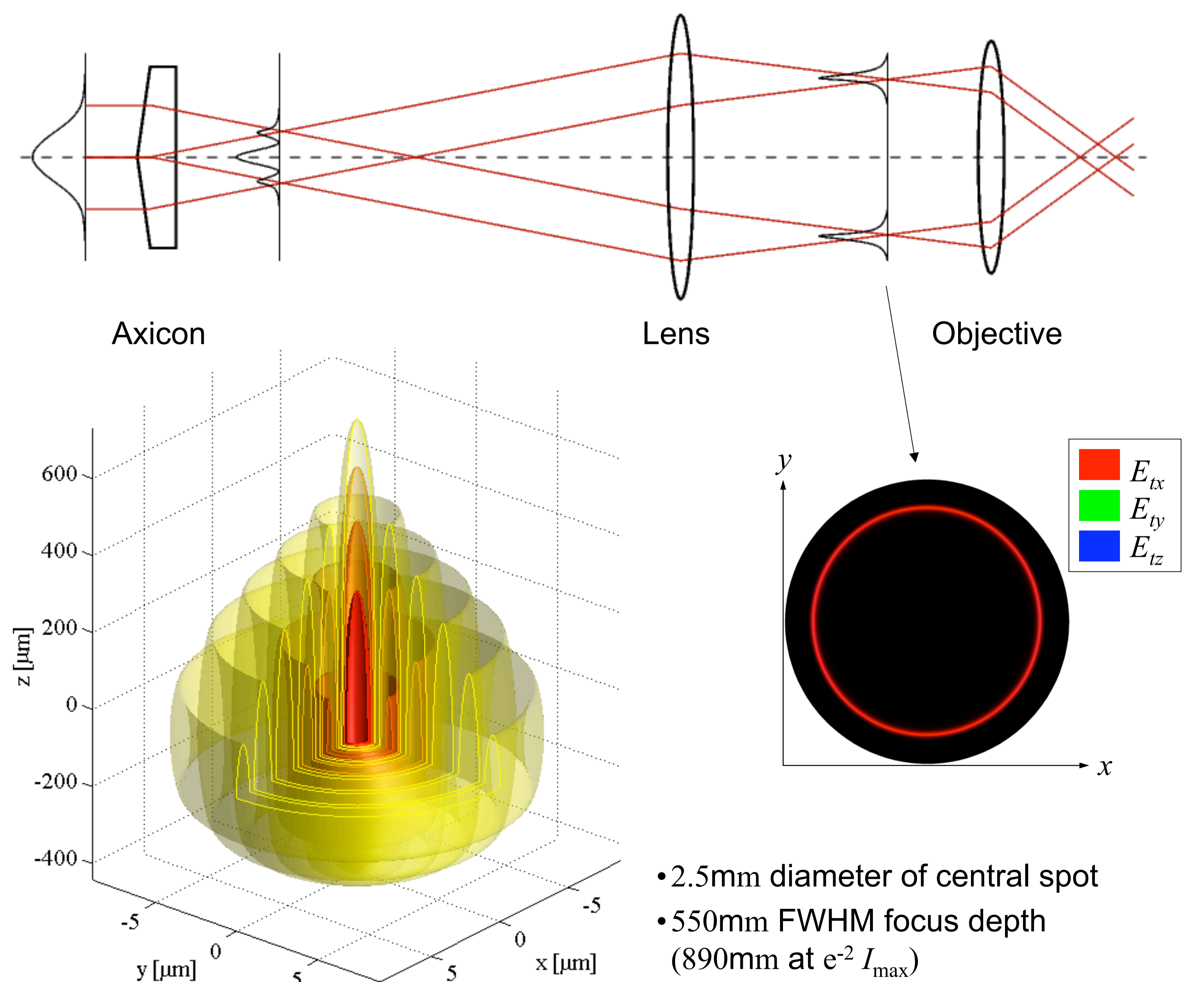
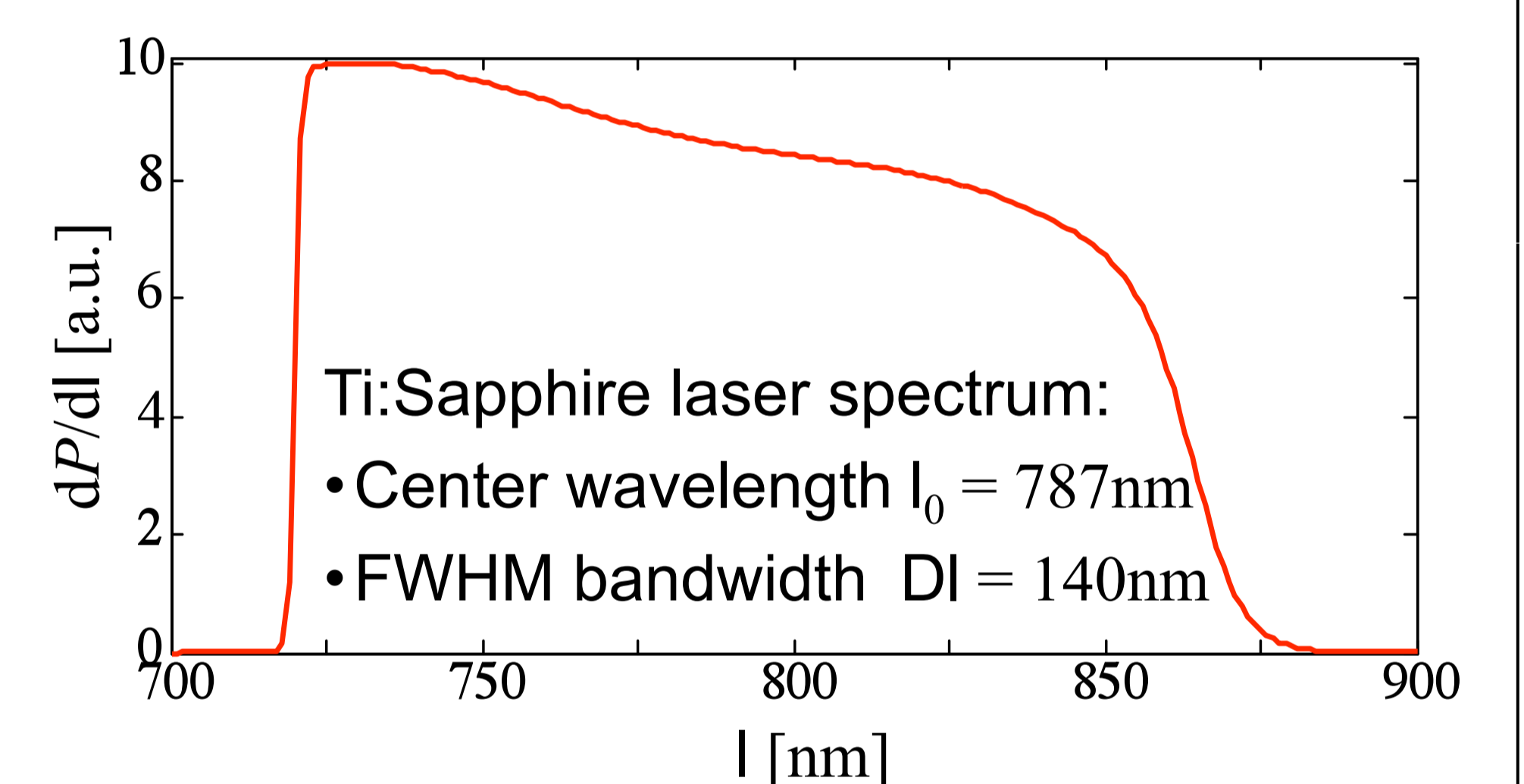
Implicit padding (anti-aliasing) and independent k -space and focus field sampling.

Conclusions

+ Full field, rapid and accurate calculation of illumination and detection point-spread functions (PSF). + Generalized concept using aperture rim smoothing, chirp z transform and effective NA at significant axial distances. + Low computational cost.

Large depth of focus with polychromatic Bessel beam

- Objective 10×0.30
- fs pulsed Ti:Sapphire laser
- x-polarized laser beam
- Underfilled aperture A
- Aqueous sample ($n_s = 1.33$)



- 2.5mm diameter of central spot
- 550mm FWHM focus depth (890mm at $e^{-2} I_{\max}$)

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