Airline Disruptions: Aircraft Recovery with Maintenance Constraints

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In collaboration with APM Technologies
Funded by CTI Switzerland
Prof. Michel Bierlaire
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Airline Scheduling Approach

1. Route Choice
2. Fleet Assignment
3. Tail Assignment
4. Crew Pairing
5. Crew Roistering
6. Passenger Routing (catering)
Disrupted Schedule and Recovery

The Airplane Recovery Problem (ARP)

**Input**
- Planes’ States
- Initial Schedule
- Maintenances
- Cancelation Costs
- Delay Cost

**Output**
- $T$
- New schedule up to $T$
- Recovery cost
Multi-objective optimization:

Minimize both $T$ and recovery costs

Strategy: for fixed $T$ find optimal recovery plan

Give several recovery plans for different values of $T$ (decision aid)
Definitions:

**PLANES:**
Initial State: position, initial time, initial resource consumption
Final State: position, expected time, expected resource consumption
Feasible Flight Set: coverable flights
Feasible Final State Set: coverable final states

**AIRPORTS:**
Activity Slots: periods when take-off/landings are permitted
Maintenance Slots: periods when given plane type can perform maintenance
Definitions (2):

**Flights:**
- Origin and Destination
- Scheduled Departure Time (SDT)
- Flight Duration
- Flight Cost
- Cancelation Cost
Solution to the ARP:

A recovery scheme for each plane:

- Initial State
- Flights and Maintenance
- Expected Final State
Example

\[ f_1 \] GVA to AMS
\[ f_2 \] AMS to BCN
\[ f_3 \] BCN to GVA
\[ f_4 \] MIL to BUD
\[ f_5 \] BUD to MIL
\[ f_6 \] BCN to MIL
Example (2)

• flights:
  \[ F = \{ f_1, f_2, f_3, f_4, f_5, f_6 \} \]

• final states:
  \[ S = \{ S^{GVA}, S^{MIL} \} \]

• planes:
  \[ P = \{ p_1, p_2 \} \]

• initial states:
  \[ p_1 \ (GVA, 0, 0) \]
  \[ p_2 \ (MIL, 0, 0) \]
Feasible Solution
Column Generation Approach

Find out optimal solution by combining individual recovery schemes \( r \in R' \) (master problem) on a subset \( R' \subseteq R \) of all feasible recovery schemes.

Generate potentially improving recovery schemes \( r \in R-R' \) dynamically for each plane (pricing problem).
Master Problem: MIP formulation

$$\begin{align*}
\text{min} \quad z_{MP} &= \sum_{r \in R} c_r x_r + \sum_{f \in F} c_f y_f + \sum_{s \in S} c_s z_s \\
\text{s. c.} \quad \sum_{r \in R} b^f_r x_r + y_f &= 1 \quad \forall f \in F \quad (\lambda_f) \\
\sum_{r \in R} b^s_r x_r + z_s &= 1 \quad \forall s \in S \quad (\eta_s) \\
\sum_{r \in R} b^p_r x_r &\leq 1 \quad \forall p \in P \quad (\mu_p) \\
x_r &\in \{0,1\} \quad \forall r \in R \\
y_f &\in \{0,1\} \quad \forall f \in F \\
z_s &\in \{0,1\} \quad \forall s \in S
\end{align*}$$
What is a column?

- cost
- vector

\[ b_r = \left( b_r^f, b_r^s, b_r^p \right)^T \]

Where

- \[ b_r^f = 1 \text{ if flight } f \text{ is covered by column } r \]
- \[ b_r^s = 1 \text{ if final state } s \text{ is covered by } r \]
- \[ b_r^p = 1 \text{ if column } r \text{ is affected to plane } p \]
Column examples

\[ b_1 = (0,0,0,0,0,0,1,0,1,0)^T \]
\[ b_2 = (1,1,1,0,0,0,1,0,1,0)^T \]
\[ b_3 = (0,0,0,1,1,0,0,1,0,1)^T \]
The Pricing Problem

Find new columns minimizing the reduced cost \( \bar{c}^p_r \):

\[
\min_{r \in R} \quad \bar{c}^p_r = c^p_r - \sum_{f \in F} b^f_r \lambda_f - \sum_{s \in S} b^s_r \eta_s - b^p_r \mu_p \quad \forall \ p \in P
\]
Recovery Networks  (Argüello et al. 97)

1. Generate a recovery network for each plane
2. Update arc costs according to dual variables
3. Solve Resource Constrained Elementary Shortest Path (RCESPP)
4. Add Columns to $R'$
5. Resolve restricted LP until optimality and branch
Time – Space Network with

• source node $n_0 = [t, m, r]$
• node $n = [t, m, r]$
• sink $s = [t, m, r]$

• flight arc $[n, n']$
• maintenance arc $[n, n']$
• termination arc $[n, s]$
• maintenance termination arc $[n, s]$
Example (continued)

- flights: \( F = \{f_1, f_2, f_3, f_4, f_5, f_6\} \)
- final states: \( S = \{S^{GVA}, S^{MIL}\} \)
- planes: \( P = \{p_1, p_2\} \)
- initial states: \( p_1 \) (GVA, 0, 0)
  \( p_2 \) grounded for the day
- only maintenances at GVA
Recovery Network of $p_1$
Generating Recovery Networks

- Create Source node $n_0$ (initial time, location, resource cons.)
- $S = \{n_0\}$
- While $S \neq \emptyset$:
  - Select $n \in S$, $S \leftarrow S - \{n\}$
  - For all feasible flights:
    - create flight and maintenance arcs
    - create destinations node $n_f$ and $n_m$
    - $S = S \cup \{n_f, n_m\}$
- Clean network
Updating arc costs

- flight arcs: \[ c = c^f + c^d - \lambda_f \]
- maintenance arcs: \[ c = c^f + c^d + c^M - \lambda_f \]
- termination arcs: \[ c = -\eta_s \]
- maintenance term. arcs: \[ c = -\eta_s + c^M \]

Solve RCESPP on networks returns column minimizing the reduced cost!

Righini & Salani (2006), which is an extension of Desrochers et al. (1988)
Some References

• Argüello et al. (1997): recovery without maintenance
  up to 27 planes, 162 flights, 30 airports

• Desrosiers et al. (1997): daily scheduling NOT recovery
  up to 91 planes, 383 flights, 33 airports; max delay of 30 minutes

• Clarke (1997): maintenances requirements but no decision on them
  up to 177 planes, 612 flights, 37 airports; only 0 or 30 min delay

• Kohl et al. (2004): Descartes project, good survey of state of the art
  no instance size mentioned for DAR

• Barnhart and Bratu (2006): passenger oriented recovery algorithm
  up to 302 planes, 1032 flights, 74 airports
Implementation Issues

Implementation issues

- Implemented in C++ with COIN-OR BCP framework
- Used interior point methods to solve the LP
- Used linear time and logarithmical resource discretisation
- 2 phase pricing:
  - generation (keep also non optimal columns, heuristic pricing)
  - proving optimality (optimal column only, exact pricing)
Implementation Issues (2)

Linear Time Discretization

Logarithmic Resource Discretization
Real Instances

• Got real schedules from Thomas Cook Airlines (APM’s main customer)
• Solved original schedules up to 250 flights (algorithm validation)
• Generated disruption scenarios
  - delayed planes (initial states)
  - grounded planes (initial states)
  - airport closures (activity slots)
  - forced maintenances (initial resource consumption)
### Solved Instances (2): Problem Sizes

<table>
<thead>
<tr>
<th>Instance</th>
<th>2D_5AC</th>
<th>2D_5AC_1del</th>
<th>2D_10AC</th>
<th>2D_10AC_1del</th>
<th>2D_10AC_2del</th>
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<tbody>
<tr>
<td># planes</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10</td>
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<td># flights</td>
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<td>75</td>
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<td>0</td>
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<tr>
<td># cancelled flights</td>
<td>0</td>
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<td>0</td>
<td>2</td>
<td>2</td>
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<tr>
<td># delayed flights</td>
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<tr>
<td>total delay [min]</td>
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<td>969</td>
<td>0</td>
<td>969</td>
<td>989</td>
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<td>max delay [min]</td>
<td>0</td>
<td>370</td>
<td>0</td>
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<td>370</td>
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<td>cost</td>
<td>380(*)</td>
<td>21175(*)</td>
<td>750(*)</td>
<td>21545(*)</td>
<td>21745(*)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>run time [s]</td>
<td>&lt; 0.1</td>
<td>&lt; 0.1</td>
<td>0.7</td>
<td>0.7</td>
<td>1.0</td>
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<table>
<thead>
<tr>
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<th>3D_10AC</th>
<th>4D_10AC</th>
<th>5D_5AC</th>
<th>5D_10AC</th>
<th>7D_16AC</th>
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<tbody>
<tr>
<td># planes</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td># flights</td>
<td>113</td>
<td>147</td>
<td>93</td>
<td>184</td>
<td>242</td>
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<td># delayed planes</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># cancelled flights</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td># delayed flights</td>
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<td>0</td>
<td>0</td>
<td>11</td>
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<td>45</td>
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<td>cost</td>
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<td>1470(*)</td>
<td>930(*)</td>
<td>1840(*)</td>
<td>5600</td>
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<td>1</td>
<td>5</td>
<td>2033</td>
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<td>3.0</td>
<td>6.5</td>
<td>1.0</td>
<td>29.1</td>
<td>3603</td>
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### Solved Instances (3): Behavior against disruptions

<table>
<thead>
<tr>
<th>Instance</th>
<th>Den2del</th>
<th>Den2grd</th>
<th>Den4del</th>
<th>Den4grd</th>
<th>Den2del2grd</th>
<th>Den6del</th>
<th>Den6grd</th>
</tr>
</thead>
<tbody>
<tr>
<td># delayed planes</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td># grounded planes</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td># affected flights</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td># cancelled flights</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>total delay</td>
<td>10</td>
<td>920</td>
<td>230</td>
<td>380</td>
<td>490</td>
<td>640</td>
<td>380</td>
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<td>max delayed flight cost</td>
<td>10</td>
<td>275</td>
<td>85</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>200</td>
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<tr>
<td>tree size</td>
<td>36100(*)</td>
<td>83200(*)</td>
<td>38300(*)</td>
<td>163800(*)</td>
<td>84900(*)</td>
<td>42400(*)</td>
<td>251800(*)</td>
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<tr>
<td>run time</td>
<td>0.7</td>
<td>0.5</td>
<td>0.6</td>
<td>0.3</td>
<td>0.5</td>
<td>1.6</td>
<td>0.2</td>
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### Instance

<table>
<thead>
<tr>
<th>Instance</th>
<th>Den3del3grd</th>
<th>Den_3x100</th>
<th>Den_1x300</th>
<th>Den_Storm1</th>
<th>Den_Storm2</th>
</tr>
</thead>
<tbody>
<tr>
<td># delayed planes</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># grounded planes</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td># affected flights</td>
<td>9</td>
<td>11</td>
<td>7</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td># cancelled flights</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>total delay</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>max delayed flight</td>
<td>950</td>
<td>675</td>
<td>2560</td>
<td>350</td>
<td>1550</td>
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<tr>
<td>cost</td>
<td>200</td>
<td>90</td>
<td>385</td>
<td>140</td>
<td>340</td>
</tr>
<tr>
<td>tree size</td>
<td>127500(*)</td>
<td>42750(*)</td>
<td>125600(*)</td>
<td>39500(*)</td>
<td>51500(*)</td>
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<tr>
<td>run time</td>
<td>0.4</td>
<td>0.3</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Solved Instances (3): Added value of maintenances

Average results of 10 randomly generated instances

<table>
<thead>
<tr>
<th>Instance</th>
<th>No maintenance</th>
<th>Dummy maintenance</th>
<th>Maintenance optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled fits</td>
<td>63.3</td>
<td>5.4</td>
<td>4.8</td>
</tr>
<tr>
<td># delayed fits</td>
<td>4.3</td>
<td>3.1</td>
<td>1.1</td>
</tr>
<tr>
<td># uncovered final states</td>
<td>2.2</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>total delay [min]</td>
<td>508</td>
<td>103.3</td>
<td>36.6</td>
</tr>
<tr>
<td>max delay [min]</td>
<td>222.2</td>
<td>35.7</td>
<td>31.6</td>
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<tr>
<td>cost</td>
<td>397214.5</td>
<td>36581.5</td>
<td>33074</td>
</tr>
<tr>
<td>optimality gap [%]</td>
<td>0.35</td>
<td>0.28</td>
<td>1.01</td>
</tr>
<tr>
<td>tree size</td>
<td>29.2</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>run time [s]</td>
<td>20.3</td>
<td>57.9</td>
<td>41.8</td>
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</table>

Considering maintenances is crucial!!!
Solved Instances (3): Added value of maintenances

Example of instance

<table>
<thead>
<tr>
<th>Instance</th>
<th>No maintenance</th>
<th>Dummy maintenance</th>
<th>Maintenance optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td># cancelled flts</td>
<td>57</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td># delayed flts</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>total delay [min]</td>
<td>546</td>
<td>61</td>
<td>79</td>
</tr>
<tr>
<td>max delay [min]</td>
<td>191</td>
<td>34</td>
<td>50</td>
</tr>
<tr>
<td>cost</td>
<td>339195</td>
<td>13310(*)</td>
<td>5760(*)</td>
</tr>
<tr>
<td>tree size</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>run time [s]</td>
<td>8.8</td>
<td>30.5</td>
<td>47.0</td>
</tr>
</tbody>
</table>
Future Work

• Benchmark solutions against practitioners

• Allow repositioning flights and early departures

• Extend Pricing Solver for acceleration

• Include in APM solutions
Conclusions

• Developed a flexible and fast algorithm

• Solutions are very promising

• Maintenance planning is an added value
THANKS for your attention!

Any Questions?