Some challenges in route choice modeling

Michel Bierlaire and Emma Frejinger

transp-or.epfl.ch

Transport and Mobility Laboratory, EPFL
Route choice modeling

Given a transportation network composed of nodes, links, origin and destinations.
For a given transportation mode and origin-destination pair, which is the chosen route?
Applications

- Intelligent transportation systems
- GPS navigation
- Transportation planning
Challenges

- Alternatives are often highly correlated due to overlapping paths
- Data collection
- Large size of the choice set
http://library.epfl.ch/theses/?nr=4009
Dealing with correlation

Existing Approaches

- Few models explicitly capturing correlation have been used on large-scale route choice problems
  - C-Logit (Cascetta et al., 1996)
  - Path Size Logit (Ben-Akiva and Bierlaire, 1999)
  - Link-Nested Logit (Vovsha and Bekhor, 1998)
  - Logit Kernel model adapted to route choice situation (Bekhor et al., 2002)
- Probit model (Daganzo, 1977) permits an arbitrary covariance structure specification but cannot be applied in a large-scale route choice context
Existing Approaches

• Link based path-multilevel logit model (Marzano and Papola, 2005)
  • Illustrated on simple examples and not estimated on real data
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?
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- Which are the behaviorally important decisions?
Subnetworks

How can we explicitly capture the most important correlation structure without considerably increasing the model complexity?

- Which are the behaviorally important decisions?
- Our hypothesis: choice of specific parts of the network (e.g. main roads, city center)
- Concept: subnetwork
Subnetworks

- Subnetwork approach designed to be behaviorally realistic and convenient for the analyst
- Subnetwork component is a set of links corresponding to a part of the network which can be easily labeled
- Paths sharing a subnetwork component are assumed to be correlated even if they are not physically overlapping
Subnetworks - Example
Subnetworks - Methodology

- Factor analytic specification of an error component model (based on model presented in Bekhor et al., 2002)

\[ U_n = \beta^T X_n + F_n T \zeta_n + \nu_n \]

- \( F_n (J \times Q) \): factor loadings matrix
- \( (f_n)_{iq} = \sqrt{l_{niq}} \)
- \( T_{(Q \times Q)} = \text{diag} (\sigma_1, \sigma_2, \ldots, \sigma_Q) \)
- \( \zeta_n (Q \times 1) \): vector of i.i.d. N(0,1) variates
- \( \nu(J \times 1) \): vector of i.i.d. Extreme Value distributed variates
Subnetworks - Example

\[ U_1 = \beta^T X_1 + \sqrt{l_{1a} \sigma_a \zeta_a} + \sqrt{l_{1b} \sigma_b \zeta_b} + \nu_1 \]
\[ U_2 = \beta^T X_2 + \sqrt{l_{2a} \sigma_a \zeta_a} + \nu_2 \]
\[ U_3 = \beta^T X_3 + \sqrt{l_{3b} \sigma_b \zeta_b} + \nu_3 \]

\[
\mathbf{FTT}^T \mathbf{F}^T =
\begin{bmatrix}
l_{1a} \sigma_a^2 + l_{1b} \sigma_b^2 & \sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & \sqrt{l_{1b}} \sqrt{l_{3b}} \sigma_b^2 \\
\sqrt{l_{1a}} \sqrt{l_{2a}} \sigma_a^2 & l_{2a} \sigma_a^2 & 0 \\
\sqrt{l_{3b}} \sqrt{l_{1b}} \sigma_b^2 & 0 & l_{3b} \sigma_b^2
\end{bmatrix}
\]
Empirical Results

• The approach has been tested on three datasets: Boston (Ramming, 2001), Switzerland, and Borlänge

• Deterministic choice set generation
  Link elimination

• GPS data from 24 individuals
  2978 observations, 2179 origin-destination pairs

• Borlänge network
  3077 nodes and 7459 links

• BIOGEME (biogeme.epfl.ch, Bierlaire, 2007) has been used for all model estimations
Borlänge Road Network
Model Specifications

- Six different models: MNL, PSL, EC_1, EC'_1, EC_2 and EC'_2
- EC_1 and EC'_1 have a simplified correlation structure
- EC'_1 and EC'_2 do not include a Path Size attribute
- Deterministic part of the utility

\[ V_i = \beta_{PS} \ln(PS_i) + \beta_{EstimatedTime} EstimatedTime_i + \beta_{NbSpeedBumps} NbSpeedBumps_i + \beta_{NbLeftTurns} NbLeftTurns_i + \beta_{AvgLinkLength} AvgLinkLength_i \]
Estimation Results

- Parameter estimates for explanatory variables are stable across the different models.
- Path size parameter estimates:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>EC₁</th>
<th>EC₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Size</td>
<td>-0.28</td>
<td>-0.49</td>
<td>-0.53</td>
</tr>
<tr>
<td>Scaled estimate</td>
<td>-0.33</td>
<td>-0.53</td>
<td>-0.56</td>
</tr>
<tr>
<td>Rob. T-test 0</td>
<td>-4.05</td>
<td>-5.61</td>
<td>-5.91</td>
</tr>
</tbody>
</table>

- All covariance parameters estimates in the different models are significant except the one associated with R.50 S.
## Estimation Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Nb. Estimates</th>
<th>Nb. Estimated Parameters</th>
<th>Final L-L</th>
<th>Adjusted Rho-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNL</td>
<td>-</td>
<td>12</td>
<td>-4186.07</td>
<td>0.152</td>
</tr>
<tr>
<td>PSL</td>
<td>-</td>
<td>13</td>
<td>-4174.72</td>
<td>0.154</td>
</tr>
<tr>
<td>EC₁ (with PS)</td>
<td>1</td>
<td>14</td>
<td>-4142.40</td>
<td>0.161</td>
</tr>
<tr>
<td>EC'</td>
<td>1</td>
<td>13</td>
<td>-4165.59</td>
<td>0.156</td>
</tr>
<tr>
<td>EC₂ (with PS)</td>
<td>5</td>
<td>18</td>
<td>-4136.92</td>
<td>0.161</td>
</tr>
<tr>
<td>EC₂'</td>
<td>5</td>
<td>17</td>
<td>-4162.74</td>
<td>0.156</td>
</tr>
</tbody>
</table>

1000 pseudo-random draws for Maximum Simulated Likelihood estimation
2978 observations
Null log likelihood: -4951.11
BIOGEME (biogeme.epfl.ch) has been used for all model estimations.
Forecasting Results

- Comparison of the different models in terms of their performance of predicting choice probabilities
- Five subsamples of the dataset
  - Observations corresponding to 80% of the origin destination pairs (randomly chosen) are used for estimating the models
  - The models are applied on the observations corresponding to the other 20% of the origin destination pairs
- Comparison of final log-likelihood values
Forecasting Results

- Same specification of deterministic utility function for all models
- Same interpretation of these models as for those estimated on the complete dataset
- Coefficient and covariance parameter values are stable across models
Forecasting Results

Some challenges in route choice modeling – p.21/63
Conclusion - Subnetworks

- Models based on subnetworks are designed for route choice modeling of realistic size
- Correlation on subnetwork is explicitly captured within a factor analytic specification of an Error Component model
- Estimation and prediction results clearly shows the superiority of the Error Component models compared to PSL and MNL
- The subnetwork approach is flexible and the model complexity can be controlled by the analyst
Network-free data

Data collection and processing

- Link-by-link descriptions of chosen routes necessary for route choice modeling but never directly available.
- Data processing in order to obtain network compliant paths:
  - Map matching of GPS points
  - Reconstruction of reported paths
- Difficult to verify and may introduce bias and errors.
Modeling with network-free data

- An observation $i$ is a sequence of individual pieces of data related to an itinerary. Examples: sequence of GPS points or reported locations.
- For each piece of data we define a Domain of Data Relevance (DDR) that is the physical area where it is relevant.
- The DDRs bridge the gap between the network-free data and the network model.
Example - GPS data
Example - Reported trip

Intersection
Main St and Cross St

Home

City center
Mall

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Domain of Data Relevance

- For each piece of data $d$ we generate a list of relevant network elements $e$ (links and nodes).

We define an indicator function

$$ \delta(d, e) = \begin{cases} 
1 & \text{if } e \text{ is related to the DDR of } d \\
0 & \text{otherwise}
\end{cases} $$
Model estimation

- We aim at estimating the parameters $\beta$ of route choice model $P(p|C_n(s); \beta)$
- We have a set $S_i$ of relevant od pairs
- The probability of reproducing observation $i$ of traveler $n$, given $S_i$ is defined as

$$P_n(i|S_i) = \sum_{s \in S_i} P_n(s|S_i) \sum_{p \in C_n(s)} P_n(i|p) P_n(p|C_n(s); \beta)$$
Model estimation

- Measurement equation \( P_n(i|p) \)

- Reported trips

\[
P_n(i|p) = \begin{cases} 
1 & \text{if } i \text{ corresponds to } p \\
0 & \text{otherwise}
\end{cases}
\]

- GPS data

\( P_n(i|p) = 0 \) if \( i \) does not correspond to \( p \)

If \( i \) corresponds to \( p \) then \( P_n(i|p) \) is a function of the distance between \( i \) and \( p \)
Model estimation

- Measurement equation $P_n(i|p)$ for GPS data
- Distance between $i$ and a the closest point on a link $\ell$ is $D(d, p) = \min_{\ell \in A_{pd}} \Delta(d, \ell)$

![Diagram showing distance calculation]

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Model estimation

\[ P_n(i|S_i) = \sum_{s \in S_i} P_n(s|S_i) \sum_{p \in C_n(s)} P_n(i|p)P_n(p|C_n(s); \beta) \]

\[ P(i|s) = P(i|p_1)P(p_1|C(s); \beta) + P(i|p_2)P(p_2|C(s); \beta) \]
Empirical Results

- Simplified Swiss network (39411 links and 14841 nodes)
- RP data collection through telephone interviews
- Long distance car travel
- The chosen routes are described with the origin and destination cities as well as 1 to 3 cities or locations that the route pass by
- 940 observations available after data cleaning and verification
Empirical Results
Empirical Results

- No information available on the exact origin destination pairs

\[ P(s|i) = \frac{1}{|S_i|} \forall s \in S_i \]

- \( P(r|i) \) is modeled with a binary variable

\[ \delta_{ri} = \begin{cases} 
1 & \text{if } r \text{ corresponds to } i \\
0 & \text{otherwise} 
\end{cases} \]
Empirical Results

- Two origin-destination pairs are randomly chosen for each observation
- 46 routes per choice set are generated with a choice set generation algorithm
- After choice set generation 780 observations are available
  - 160 observations were removed because either all or none of the generated routes crossed the observed zones
Empirical Results

- Probability of an aggregate observation \( i \)

\[
P(i) = \sum_{s \in S_i} \frac{1}{|S_i|} \sum_{r \in C_s} \delta_{ri} P(r|C_s)
\]

- We estimate Path Size Logit (Ben-Akiva and Bierlaire, 1999) and Subnetwork (Frejinger and Bierlaire, 2007) models

- BIOGEME (biogeme.epfl.ch) used for all model estimations
Empirical Results - Subnetwork

- Subnetwork: main motorways in Switzerland
- Correlation among routes is explicitly modeled on the subnetwork
- Combined with a Path Size attribute
- Linear-in-parameters utility specifications
Empirical Results - Subnetwork
<table>
<thead>
<tr>
<th>Parameter</th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>In(path size) based on free-flow time</td>
<td>1.04 (0.134)</td>
<td>1.10 (0.141)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Freeway free-flow time 0-30 min</td>
<td>-7.12 (0.877)</td>
<td>-7.45 (0.984)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-7.12</td>
<td>-7.04</td>
</tr>
<tr>
<td>Freeway free-flow time 30min - 1 hour</td>
<td>-1.69 (0.875)</td>
<td>-2.26 (1.03)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-1.69</td>
<td>-2.14</td>
</tr>
<tr>
<td>Freeway free-flow time 1 hour +</td>
<td>-4.98 (0.772)</td>
<td>-5.64 (1.00)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-4.98</td>
<td>-5.33</td>
</tr>
<tr>
<td>CN free-flow time 0-30 min</td>
<td>-6.03 (0.882)</td>
<td>-6.25 (0.975)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-6.03</td>
<td>-5.91</td>
</tr>
<tr>
<td>CN free-flow time 30 min +</td>
<td>-1.87 (0.331)</td>
<td>-2.16 (0.384)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-1.87</td>
<td>-2.04</td>
</tr>
<tr>
<td>Main free-flow travel time 10 min +</td>
<td>-2.03 (0.502)</td>
<td>-2.46 (0.624)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.03</td>
<td>-2.33</td>
</tr>
<tr>
<td>Small free-flow travel time</td>
<td>-2.16 (0.685)</td>
<td>-2.75 (0.804)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.16</td>
<td>-2.60</td>
</tr>
<tr>
<td>Proportion of time on freeways</td>
<td>-2.2 (0.812)</td>
<td>-2.31 (0.865)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-2.2</td>
<td>-2.18</td>
</tr>
<tr>
<td>Proportion of time on CN</td>
<td>0 fixed</td>
<td>0 fixed</td>
</tr>
<tr>
<td>Proportion of time on main</td>
<td>-4.43 (0.752)</td>
<td>-4.40 (0.800)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-4.43</td>
<td>-4.16</td>
</tr>
<tr>
<td>Proportion of time on small</td>
<td>-6.23 (0.992)</td>
<td>-6.02 (1.03)</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>-6.23</td>
<td>-5.69</td>
</tr>
<tr>
<td>Covariance parameter</td>
<td>0.217 (0.0543)</td>
<td>4.00</td>
</tr>
<tr>
<td>Scaled Estimate</td>
<td>0.217</td>
<td>0.205</td>
</tr>
</tbody>
</table>
### Empirical Results

<table>
<thead>
<tr>
<th></th>
<th>PSL</th>
<th>Subnetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance parameter</td>
<td></td>
<td>0.217</td>
</tr>
<tr>
<td>(Rob. Std. Error) Rob. T-test</td>
<td></td>
<td>(0.0543) 4.00</td>
</tr>
<tr>
<td>Number of simulation draws</td>
<td>-</td>
<td>1000</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Final log-likelihood</td>
<td>-1164.850</td>
<td>-1161.472</td>
</tr>
<tr>
<td>Adjusted rho square</td>
<td>0.145</td>
<td>0.147</td>
</tr>
</tbody>
</table>

Sample size: 780, Null log-likelihood: -1375.851
Empirical Results

- All parameters have their expected signs and are significantly different from zero
- The values and significance level are stable across the two models
- The subnetwork model is significantly better than the Path Size Logit (PSL) model
Concluding remarks

- Network-free data are more reliable
- Data processing may bias the result
- We prefer to model explicitly the relationship between the data and the model
Choice set generation

Path enumeration

- Dial’s approach avoids path enumeration
- Computationally convenient but behaviorally incorrect
- MNL inappropriate due to significant path overlap
- Generalized cost must be link-additive
- Heterogeneity in terms of behavior, equipments, etc. cannot be accounted for.
- With other DCM models, choice sets must be explicitly defined
- Path enumeration heuristics have been proposed:
  - Deterministic approaches: link elimination (Azevedo et al., 1993), labeled paths (Ben-Akiva et al., 1984)
  - Stochastic approaches: simulation (Ramming, 2001) and doubly stochastic (Bovy and Fiorenzo-Catalano, 2006)
Path enumeration

- Underlying assumption in existing approaches: the actual choice set is generated
- Empirical results suggest that this is not always true
- Our approach:
  - Choice set contains all paths
  - Too large for computation
  - Solution: sampling of alternatives
Sampling of Alternatives

- Multinomial Logit model (e.g. Ben-Akiva and Lerman, 1985):

\[
P(i|C_n) = \frac{q(C_n|i)P(i)}{\sum_{j \in C_n} q(C_n|j)P(j)} = \frac{e^{V_{in} + \ln q(C_n|i)}}{\sum_{j \in C_n} e^{V_{jn} + \ln q(C_n|j)}}
\]

- If purely random sampling, \( q(C_n|i) = q(C_n|j) \) and

\[
P(i|C_n) = \frac{e^{V_{in} + \ln q(C_n|i)}}{\sum_{j \in C_n} e^{V_{jn} + \ln q(C_n|j)}} = \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}}
\]

\( C_n \): set of sampled alternatives
\( q(C_n|j) \): probability of sampling \( C_n \) given that \( j \) is the chosen alternative
Importance Sampling of Alternatives

• Attractive paths have higher probability of being sampled than unattractive paths

• In this case, $q(C_n|i) \neq q(C_n|j)$

\[
P(i|C_n) = \frac{e^{V_{in} + \ln q(C_n|i)}}{\sum_{j \in C_n} e^{V_{jn} + \ln q(C_n|j)}} \neq \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}}
\]

• Path utilities must be corrected in order to obtain unbiased estimation results
Stochastic Path Enumeration

- Key feature: we must be able to compute $q(C_n|i)$
- One possible idea: a biased random walk between $s_o$ and $s_d$ which selects the next link at each node $v$.
- Initialize: $v = s_o$
- Step 1: associate a weight with each outgoing link $\ell = (v, w)$:

$$\omega(\ell|b_1) = 1 - (1 - x_\ell b_1)$$

where

$$x_\ell = \frac{SP(v, s_d)}{C(\ell) + SP(w, s_d)},$$

is 1 if $\ell$ is on the shortest path, and decreases when $\ell$ is far from the shortest path
Stochastic Path Enumeration

\[ \omega(\ell | b_1) \]

\[ b_1 = 1, 2, 5, 10, 30 \]
Stochastic Path Enumeration

- Step 2: normalize the weights to obtain a probability distribution

\[ q(\ell | E_v, b_1) = \frac{\omega(\ell | b_1, b_2)}{\sum_{m \in E_v} \omega(m | b_1)} \]

- Random draw a link \((v, w^*)\) based on this distribution and add it to the current path

- If \(w^* = s_d\), stop. Else, set \(v = w^*\) and go to step 1.

Probability of generating a path \(j\):

\[ q(j) = \prod_{\ell \in \Gamma_j} q(\ell | E_v, b_1). \]
Sampling of Alternatives

• Following Ben-Akiva (1993)
• Sampling protocol
  1. A set $\tilde{C}_n$ is generated by drawing $R$ paths with replacement from the universal set of paths $\mathcal{U}$
  2. Add chosen path to $\tilde{C}_n$
• Outcome of sampling: $(\tilde{k}_1, \tilde{k}_2, \ldots, \tilde{k}_J)$ and $\sum_{j=1}^{J} \tilde{k}_j = R$

$$P(\tilde{k}_1, \tilde{k}_2, \ldots, \tilde{k}_J) = \frac{R!}{\prod_{j \in \mathcal{U}} \tilde{k}_j!} \prod_{j \in \mathcal{U}} q(j)^{\tilde{k}_j}$$

• Alternative $j$ appears $k_j = \tilde{k}_j + \delta_{cj}$ in $\tilde{C}_n$
Sampling of Alternatives

- Let \( C_n = \{ j \in U \mid k_j > 0 \} \)

\[
q(C_n|i) = q(\tilde{C}_n|i) = \frac{R!}{(k_i - 1)!} \prod_{j \in C_n \atop j \neq i} k_j^{-1} q(i)^{k_i - 1} \prod_{j \in C_n \atop j \neq i} q(j)^{k_j} = K_{C_n} \frac{k_i}{q(i)}
\]

\[
K_{C_n} = \frac{R!}{\prod_{j \in C_n} k_j!} \prod_{j \in C_n} q(j)^{k_j}
\]

\[
P(i|C_n) = \frac{e^{V_{in} + \ln\left(\frac{k_i}{q(i)}\right)}}{\sum_{j \in C_n} e^{V_{jn} + \ln\left(\frac{k_j}{q(j)}\right)}}
\]
Numerical Results

- Estimation of models based on synthetic data generated with a postulated model
- Evaluation of
  - Sampling correction
  - Path Size attribute
  - Biased random walk algorithm parameters
Numerical Results
Numerical Results

- True model: Path Size Logit
  \[ U_j = \beta_{PS} \ln PS^U_j + \beta_L \text{Length}_j + \beta_{SB} \text{SpeedBumps}_j + \varepsilon_j \]

  \[ \beta_{PS} = 1, \quad \beta_L = -0.3, \quad \beta_{SB} = -0.1 \]

  \[ \varepsilon_j \text{ distributed Extreme Value with scale 1 and location 0} \]

- \[ PS^U_j = \sum_{\ell \in \Gamma_j} \frac{L_{\ell} L_j}{L_{\ell}} \frac{1}{\sum_{p \in U} \delta_{\ell p}} \]

- 3000 observations
Numerical Results

- Four model specifications

<table>
<thead>
<tr>
<th>Path Size</th>
<th>Sampling Without</th>
<th>Correction Without</th>
<th>Sampling With</th>
<th>Correction With</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$M_{PS(C)}^{NoCorr}$</td>
<td>$M_{PS(C)}^{Corr}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>$M_{PS(U)}^{NoCorr}$</td>
<td>$M_{PS(U)}^{Corr}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$PS^U_i = \sum_{\ell \in \Gamma_i} \frac{L_{\ell}}{L_i} \sum_{j \in U} \frac{1}{\delta_{\ell j}}$$

$$PS^C_{in} = \sum_{\ell \in \Gamma_i} \frac{L_{\ell}}{L_i} \sum_{j \in C_n} \frac{1}{\delta_{\ell j}}$$
Numerical Results

- Model $M_{PS(C)}^{NoCorr}$:
  \[ V_{in} = \mu \left( \beta_{PS} \ln PS_{in}^C - 0.3 \text{Length}_i + \beta_{SB} \text{SpeedBumps}_i \right) \]

- Model $M_{PS(C)}^{Corr}$:
  \[ V_{in} = \mu \left( \beta_{PS} \ln PS_{in}^C - 0.3 \text{Length}_i + \beta_{SB} \text{SpeedBumps}_i \right) + \ln \left( \frac{k_i}{q(i)} \right) \]

- Model $M_{PS(U)}^{NoCorr}$:
  \[ V_{in} = \mu \left( \beta_{PS} \ln PS_{in}^U - 0.3 \text{Length}_i + \beta_{SB} \text{SpeedBumps}_i \right) \]

- Model $M_{PS(U)}^{Corr}$:
  \[ V_{in} = \mu \left( \beta_{PS} \ln PS_{in}^U - 0.3 \text{Length}_i + \beta_{SB} \text{SpeedBumps}_i \right) + \ln \left( \frac{k_i}{q(i)} \right) \]
## Numerical Results

<table>
<thead>
<tr>
<th></th>
<th>True PSL</th>
<th>$M_{PS(C)}^{NoCorr}$ PSL</th>
<th>$M_{PS(C)}^{Corr}$ PSL</th>
<th>$M_{PS(U)}^{NoCorr}$ PSL</th>
<th>$M_{PS(U)}^{Corr}$ PSL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$\beta_L$ fixed</strong></td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td><strong>$\mu$</strong></td>
<td>1</td>
<td>0.182</td>
<td>0.923</td>
<td>0.141</td>
<td>0.977</td>
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<tr>
<td>standard error</td>
<td></td>
<td>0.0277</td>
<td>0.0246</td>
<td>0.0263</td>
<td>0.0254</td>
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<tr>
<td>$t$-test w.r.t. 1</td>
<td></td>
<td>-29.54</td>
<td>-3.13</td>
<td>-32.64</td>
<td>-0.91</td>
</tr>
<tr>
<td><strong>$\beta_{PS}$</strong></td>
<td>1</td>
<td>1.94</td>
<td>0.308</td>
<td>-1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>standard error</td>
<td></td>
<td>0.428</td>
<td>0.0736</td>
<td>0.383</td>
<td>0.0539</td>
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<tr>
<td>$t$-test w.r.t. 1</td>
<td></td>
<td>2.20</td>
<td>-9.40</td>
<td>-5.27</td>
<td>0.37</td>
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<tr>
<td><strong>$\beta_{SB}$</strong></td>
<td>-0.1</td>
<td>-1.91</td>
<td>-0.139</td>
<td>-2.82</td>
<td>-0.0951</td>
</tr>
<tr>
<td>standard error</td>
<td></td>
<td>0.25</td>
<td>0.0232</td>
<td>0.428</td>
<td>0.024</td>
</tr>
<tr>
<td>$t$-test w.r.t. -0.1</td>
<td></td>
<td>-7.24</td>
<td>-1.68</td>
<td>-6.36</td>
<td>0.20</td>
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</tbody>
</table>
# Numerical Results

<table>
<thead>
<tr>
<th></th>
<th>True PSL</th>
<th>$M_{PS(C)}^{NoCorr}$ PSL</th>
<th>$M_{PS(C)}^{Corr}$ PSL</th>
<th>$M_{PS(U)}^{NoCorr}$ PSL</th>
<th>$M_{PS(U)}^{Corr}$ PSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final log likelihood</td>
<td>-6660.45</td>
<td>-6147.79</td>
<td>-6666.82</td>
<td>-5933.62</td>
<td></td>
</tr>
<tr>
<td>Adj. rho-square</td>
<td>0.018</td>
<td>0.093</td>
<td>0.017</td>
<td>0.125</td>
<td></td>
</tr>
</tbody>
</table>

Null log likelihood: -6784.96, 3000 observations
Algorithm parameters: 10 draws, $b_1 = 5$, $b_2 = 1$, $C(\ell) = L_\ell$
Average size of sampled choice sets: 9.66
BIOGEME (Bierlaire, 2007 and Bierlaire, 2003) has been used for all model estimations
Extended Path Size

- Compute Path Size attribute based on an *extended choice set* $C_{n}^{\text{extended}}$
- Simple random draws from $\mathcal{U} \setminus C_n$ so that $|C_n| \leq |C_{n}^{\text{extended}}| \leq |\mathcal{U}|$
Extended Path Size

![Graph showing the extended path size with different scales for Path Size, Speed Bump, and Scale Parameter.](image)

- **Path Size**
- **Speed Bump**
- **Scale Parameter**

Average number of paths in $C_r^{extended}$

$t$-test w.r.t. true value
Extended Path Size

- Assume that the true choice set is the set of all paths
- Draw a subset for estimating the choice probability
- Draw a larger subset to compute the path size
- Various heuristics based on the same definition of the link weights can be used
Conclusions

- New point of view on choice set generation and route choice modeling
- Path generation is considered an importance sampling approach
- We present a path generation algorithm and derive the corresponding sampling correction
- Path Size should be computed on largest possible sets
- Numerical results are very promising