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Shear Strength of Members without Transverse Reinforcement as Function of Critical Shear Crack Width

by Aurelio Muttoni and Miguel Fernández Ruiz

This paper investigates the shear strength of beams and one-way slabs without stirrups based on the opening of a critical shear crack. The shear-carrying mechanisms after the development of this crack are investigated. On this basis, a rational model is developed to estimate the shear strength of members without shear reinforcement. The proposed model is based on an estimate of the crack width in the critical shear region, taking also into account the roughness of the crack and the compressive strength of concrete. The proposed model is shown to properly describe a large set of available test data. A simplified method adopted by the Swiss code for structural concrete (SIA 262) is also introduced. Comparisons with other codes of practice are finally presented, with a highlight on the main differences between them.

Keywords: aggregate size; concrete compressive strength; crack width; shear strength.

INTRODUCTION

Traditionally, shear dimensioning and checking of structural concrete elements is performed differently on members with or without shear reinforcement.

Several well-established theories based on equilibrium considerations (strut-and-tie models[1] and stress fields[2,3]) can be applied when shear reinforcement is provided, leading to safe design solutions. Theories also considering compatibility conditions and the tensile strength of concrete (compression field-based theories[4,5] and fixed-angle softened-truss model[6]) have also been developed allowing accurate predictions of the shear response of transversely reinforced members.

The situation is, on the other hand, rather different concerning shear in members without stirrups. These members are instrumental in structural concrete, as the safety of many structural systems relies on them (refer to Fig. 1). Their shear strength has traditionally been estimated by means of purely empirical or semi-empirical expressions.[7]

Some general theories, such as the modified compression field theory[4] (MCFT), have been successfully applied to members without shear reinforcement, also leading to code-based implementations.[8] The use of such theories in practice remains complicated, however, typically requiring the help of computer programs or spreadsheets.[9] Recently, some simplified expressions based on the MCFT results have been derived[10] and proposed for the Canadian code for structural concrete.[11]

Although an important effort has been made, currently there is no generally accepted theory or physical model explaining the response of members without stirrups.

This paper presents the basis and recent improvements of the critical shear crack theory[12,13] as well as its application to one-way slabs or beams with rectangular cross sections. The method is based on an estimate of the crack width in the shear critical region and provides a rational basis for the evaluation of the shear and punching shear strength of members without stirrups. The principles of this theory were first introduced in 1991.[12] The results of the theory were later introduced in the Swiss code for structural concrete (SIA 162) in 1993. Further improvements of the theory for shear of one-way slabs and punching shear[13] were recently taken into account in the new version of the Swiss code,[14] which can be considered to be fully based on this theory for the shear design of members without stirrups.

RESEARCH SIGNIFICANCE

Currently, there is no general agreement on a theory describing the response of reinforced concrete members without shear reinforcement. Many structural systems, however, rely on such members. Their design is usually performed using empirical or semi-empirical expressions provided by codes of practice that do not consider the influence of many governing parameters (reinforcement ratio, shear span, aggregate size, and load configuration). This paper introduces a theory that provides a rational basis to estimate shear strength. The proposed theory is based on a physical model and, with a number of reasonable simplifications, has been introduced in 2003 into the Swiss code for structural concrete.[14]

INFLUENCE OF CRACKING ON SHEAR STRENGTH OF MEMBERS WITHOUT STIRRUPS

The application of the theory of plasticity to members subjected to shear was initially investigated by Drucker[15] who proposed several stress fields in which the load is carried directly by inclined struts or arches (refer to Fig. 2). According to this model, the strength of a beam is governed by the yielding of the flexural reinforcement. The stress fields proposed by Drucker were not found suitable for reinforced concrete beams, however, leading to unsafe designs.

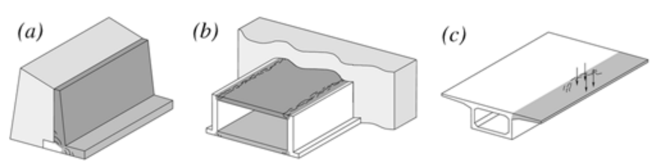

Fig. 1—Some structural elements without shear reinforcement working predominantly as one-way slabs (only shear cracks drawn): (a) wall and foundation of retaining wall; (b) upper and lower slabs of cut-and-cover tunnel; and (c) deck slab of bridge.

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The reason why the theory of plasticity is not applicable, and thus why the flexural strength cannot be reached in members without stirrups, can be understood with the help of Fig. 3. In Fig. 3(a) and (b), two geometrically identical beams tested by Leonhardt and Walther[16] are presented (shear span to effective depth ratio [a/d] equal to 2.77). In Beam EA1, deformed bars were used, whereas in Beam EB1, smooth bars were used. The failure load of Beam EA1 reached 50% of its strength according to the theory of plasticity, whereas Beam EB1 reached 86%. This difference is due to the inclined crack in Specimen EA1, which develops through the theoretical compression strut, thus reducing its strength. In Specimen EB1, thanks to the reduced bond strength, only a limited part of the inclined crack developed through the theoretical inclined strut, which significantly increased the strength of the member.

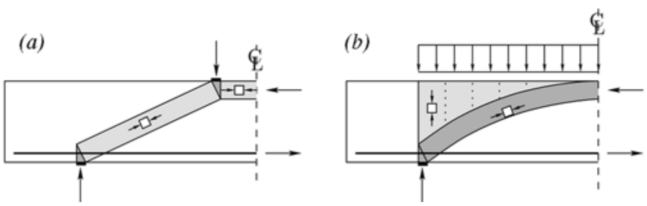

Fig. 2—Stress fields according to theory of plasticity for one-way slabs in case of: (a) concentrated loads; and (b) distributed loads.

The same phenomenon was observed in Beams BP0 and BP2 by Muttoni and Thürlimann[17] (refer to Fig. 3(d) and (e)). In this case, two beams with geometrically identical shear spans were tested to failure (a/d equal to 2.44). Beam BP0 had only flexural reinforcement, whereas Beam BP2 additionally contained a minimal reinforcement for crack control (spiral ∅6 mm at 60 mm [No. 2 at 2.36 in.]) in the region of the theoretical strut where shear failure usually develops due to diagonal tension. This reinforcement could not be used to carry shear forces because it was not enclosing the main longitudinal reinforcement, but was very effective in controlling the width of the critical shear crack. The first beam (BP0) reached 50% of the strength according to the theory of plasticity. Beam BP2, however, attained its full flexural strength because the opening of the cracks within the inclined strut remained limited thanks to the spiral reinforcement, and the strength of the strut was not decreased.

The development of cracks through the inclined compression strut of a beam and its influence in the member's shear strength shows a strong dependency on the a/d, a phenomenon known as Kani's valley.[18] Figure 4 presents several tests performed by Leonhardt and Walther[16] where a/d was varied from 1.5 to 8.0. For small values of a/d (Test B2), the cracks practically do not develop through the inclined strut and thus the flexural strength can be reached. For larger values of a/d, cracks develop through the inclined struts, consequently decreasing the shear strength of the member (Tests B4 and B6). This phenomenon is less significant for very large values of a/d (Beam B10/1) where the flexural strength is again reached before the critical crack can develop.

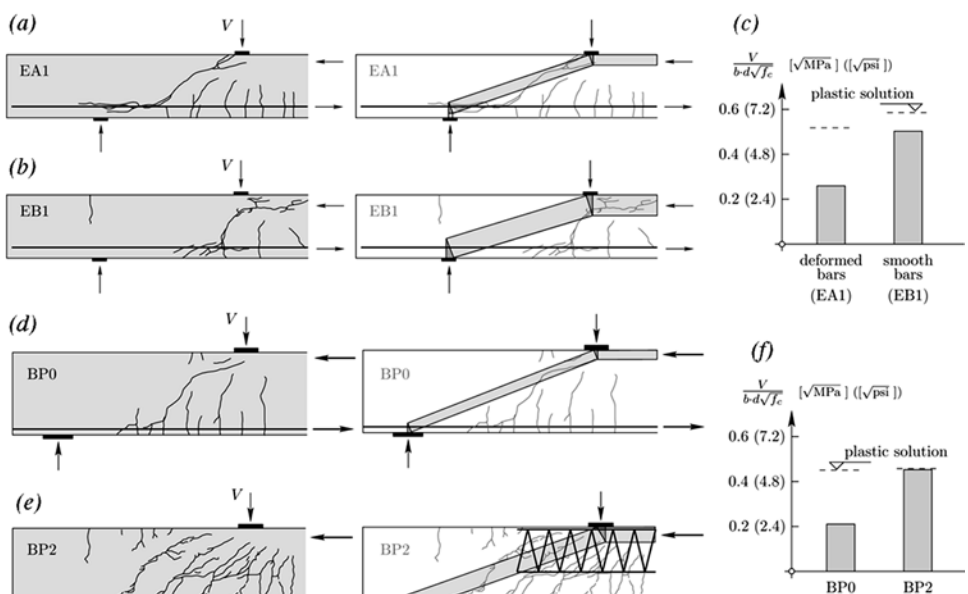

Fig. 3—Influence of crack pattern in shear strength of beams: (a) and (b) Tests EA1 (deformed bars) and EB2 (smooth bars) by Leonhardt and Walther,[16] cracking pattern and theoretical strut position; (c) failure loads compared with strength according to theory of plasticity for Beams EA1 and EB1; (d) and (e) Tests BP0 and BP2 by Muttoni and Thürlimann,[17] cracking pattern and theoretical strut position; and (f) failure loads compared with strengths according to theory of plasticity for Beams BP0 and BP2.

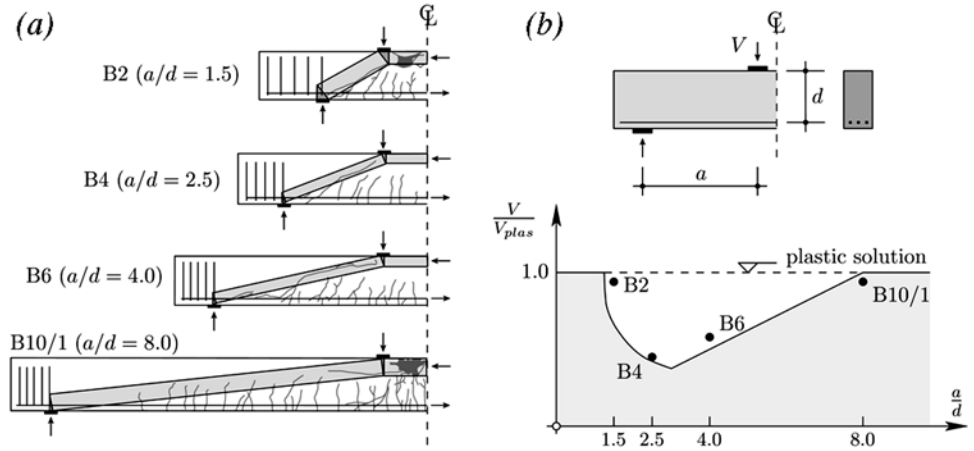

Fig. 4—Influence of a/d *on shear strength: (a) Tests B2, B4, B6, and BP10/1 by Leonhardt and Walther,*[16] *cracking pattern and theoretical strut position; and (b) Kani's valley, comparing actual strength with failure load according to theory of plasticity.*

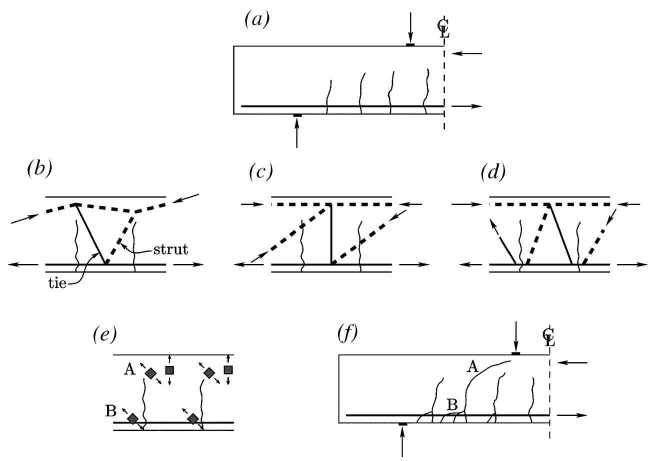

Fig. 5—Development of actual cracking pattern: (a) initial flexural cracks; (b) cantilever action; (c) aggregate-interlock action; (d) dowel action; (e) tensile stresses due to (b-d); and (f) final crack pattern.

DEVELOPMENT OF CRITICAL SHEAR CRACK

The development of the critical shear crack, whose role was discussed in the previous section, can be explained with the help of Fig. 5. Considering the flexural cracking pattern (Fig. 5(a)), various shear-carrying mechanisms may be developed by a beam,[7,12] namely, cantilever action (Fig. 5(b)), aggregate-interlock (Fig. 5(c)), and dowel action (Fig. 5(d)). These shear-carrying mechanisms induce tensile stresses in concrete (Fig. 5(e)) near the crack tip (Zone A) and at the level of the reinforcement (Zone B). Once the tensile strength of the concrete in Zones A and B is reached, the existing flexural cracks progress in a diagonal direction (Zone A) or new ones are created (Zone B). As a consequence, the capacity of the previous shear-carrying mechanisms is reduced or even cancelled.

The development of the critical shear crack, however, does not necessarily imply the collapse of the member. A new shear-carrying mechanism, the arching action, may be developed by the beam. Figure 6 shows two possibilities for developing the arching action. The first one is the development of an elbow-shaped strut[12] that deviates the compression strut to avoid the cracks. The development of an elbow-shaped strut strongly depends on the actual crack pattern and is limited by the tensile strength of the member (cracks may appear close to the point of introduction of the load as shown in Fig. 6(a)).

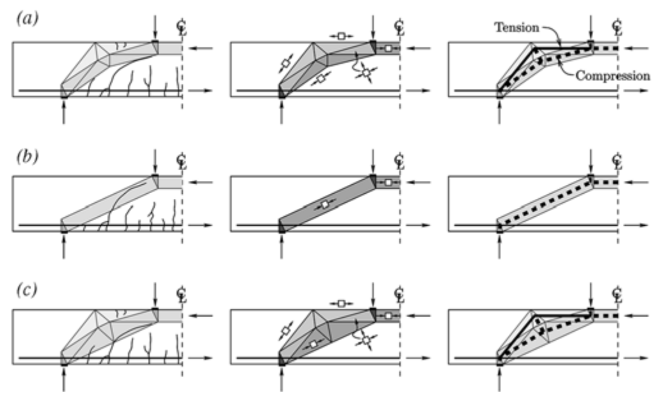

Fig. 6—Load-carrying mechanisms after development of critical shear crack: (a) elbow-shaped strut; (b) straight strut (enabled by aggregate interlock); and (c) combined response.

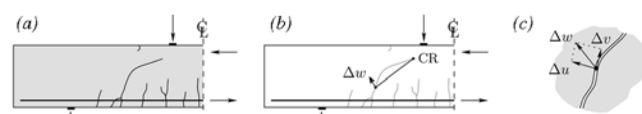

Fig. 7—Aggregate interlock after development of critical shear crack: (a) actual crack pattern; (b) center of rotation (CR) assuming two rigid bodies; and (c) normal (Δu*) and tangential (*Δv*) relative displacements between lips of crack.*

The second physical mechanism that allows the development of the arching action is the direct strut that develops thanks to the aggregate interlock in the critical shear crack (Fig. 6(b)). If the center of rotation is located at the tip of the crack (which is a rather reasonable assumption as confirmed by some experimental measurements[12,19]), an opening of the crack induces a transverse sliding between its lips (refer to Fig. 7). Thus, aggregate interlock is activated and a strut (with a limited strength) can develop through the critical crack. The aggregate interlock depends on the crack geometry,

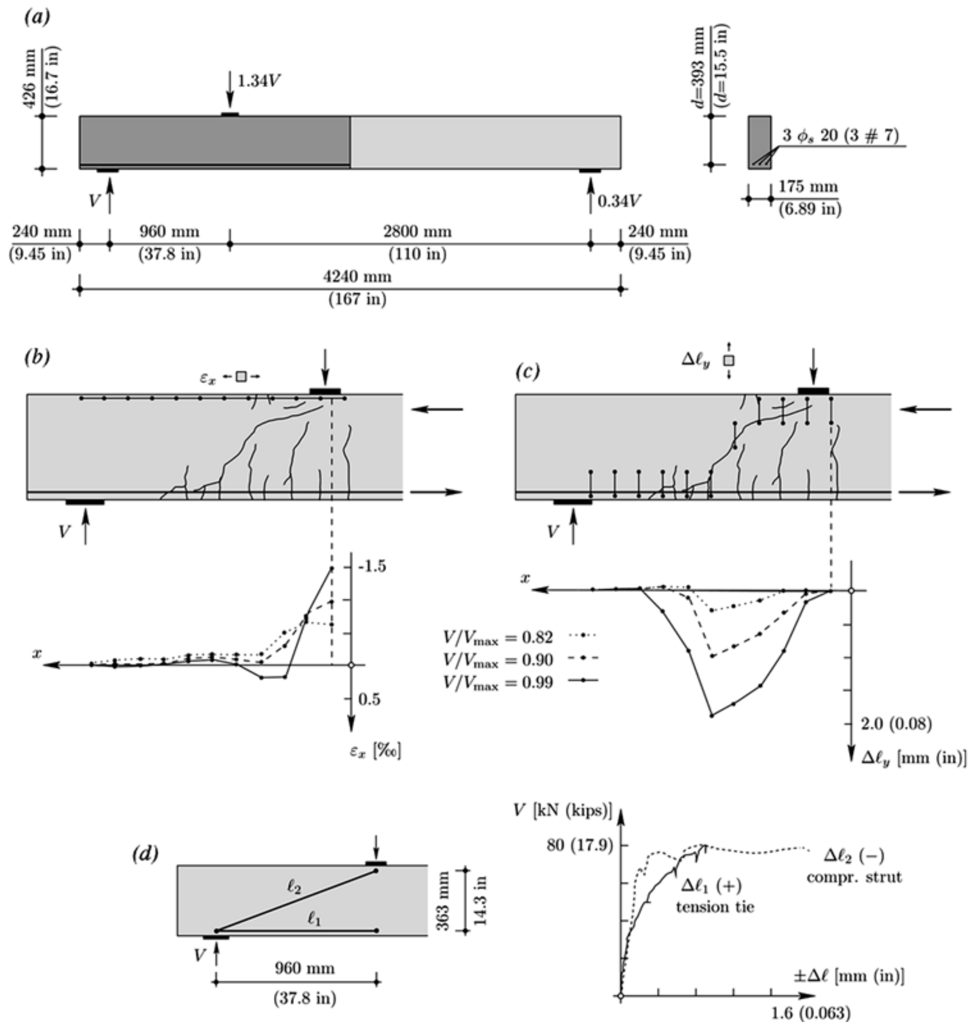

Fig. 8—Test BP0 by Muttoni and Thürlimann[17] (f_c = 30.5 MPa [4.42 ksi]; d_g = 16 mm [5/8 in.]): (a) test setup; (b) crack pattern and horizontal strains along top fiber (points of measurement indicated); (c) crack pattern and vertical elongation along diagonal crack (points of measurement indicated); and (d) measured elongations in directions of compression strut and of tension tie.

crack opening, and roughness of the lips (which in turn is a function of the aggregate size). Actually, both aggregate interlock and the elbow-shaped strut are active at failure. The response of a member consists of a combination of these two mechanisms, as proposed in Fig. 6(c).

For certain cases, typically with large *a*/*d*, failure of the member takes place during the propagation of the critical shear crack. In these cases, dowel action may still have a certain capacity to transmit shear forces at failure. However, this contribution is neglected in this paper.

EXPERIMENTAL EVIDENCE FROM TEST BP0[17]

This section discusses the role of the various shear-carrying mechanisms previously introduced with reference to Test BP0 by Muttoni and Thürlimann[17] (Fig. 8(a)).

In this specimen, measurements of the longitudinal strain were performed along the upper face of the beam (Fig. 8(b)). These measurements show that, before failure, tensile strains developed close to the point of introduction of the load, also confirmed by the development of vertical cracks in the upper face of the element, in agreement with the elbow-shaped strut previously introduced.

Regarding the dowel action, its effect is very limited at failure because the width of the crack developing parallel to the reinforcement exceeded 2 mm (0.08 in.) (refer to Fig. 8(c)). Another interesting measurement taken in Beam BP0 is the change in the length of the strut shown in Fig. 8(d). It can be seen that at approximately 90% of the failure load, the strut exhibited a sharp strain increase for small load increments. This is due to the development of the critical shear crack, and thus to the bending of the elbow-shaped strut.

CRITICAL SHEAR CRACK THEORY

The shear strength of members without stirrups, traditionally correlated to the square root of the concrete compressive strength (after the works of Moody et al.[20]), is strongly dependent on the critical shear crack width and on its roughness. The critical shear crack theory reflects this dependency as expressed in Eq. (1)

$$\frac{V_R}{bd} = \sqrt{f_c}\, f(w, d_g) \tag{1}$$

where f_c is the concrete compressive strength, w is the critical shear crack width, and d_g is the maximum aggregate size. The following hypotheses are accepted:

1. The shear strength is checked in a section (depending on the load configuration) where the width of the critical shear crack can be adequately represented by the strain at a depth $0.6d$ from the compression face[13] (refer to Fig. 9).

2. The critical crack width w is proportional to the product of the longitudinal strain in the control depth ε times the effective depth of element d

$$w \propto \varepsilon d \tag{2}$$

A similar hypothesis has been proposed for punching shear[12] and for shear in beams.[21,22] It should be noted that Eq. (2) is valid for a rectangular cross section without skin reinforcement on the side faces, as in reinforced concrete slabs. On the contrary,[23] reinforcement on the side faces reduces the critical crack width, thus increasing the shear strength of the member.

The longitudinal strain ε is evaluated in the critical region assuming that plane sections remain plane and a linear-elastic behavior in compression for concrete (neglecting its tensile strength), as shown in Fig. 9(b).

If no axial force is applied, the strain in the control depth can be derived based on the bending moment M in the critical section

$$\varepsilon = \frac{M}{bd\rho E_s(d - c/3)} \frac{0.6d - c}{d - c} \tag{3}$$

The depth of the compression zone c is

$$c = d\rho\frac{E_s}{E_c}\left(\sqrt{1 + \frac{2E_c}{\rho E_s}} - 1\right) \tag{4}$$

where E_c can be taken as $E_c \approx 10{,}000 f_c^{1/3}$ in MPa ($= 276{,}000 f_c^{1/3}$ in psi).

Taking into consideration the effects of the critical crack width, the aggregate size and the concrete compressive strength, the following analytical expression is proposed to evaluate the shear strength[13]

$$\frac{V_R}{bd\sqrt{f_c}} = \frac{1}{6} \cdot \frac{2}{1 + 120\frac{\varepsilon d}{16 + d_g}} \quad \text{SI units: MPa, mm}$$

$$\frac{V_R}{bd\sqrt{f_c}} = 2 \cdot \frac{2}{1 + 120\frac{\varepsilon d}{\frac{5 \text{ in.}}{8} + d_g}} \quad \text{US units: psi, in.} \tag{5}$$

The shear strength can be obtained by substituting Eq. (3) into Eq. (5) and solving the resulting quadratic equation. For high-strength concrete (f_c > 60 MPa [9000 psi]) or lightweight concrete, d_g should be taken equal to zero because the cracking surface develops through the aggregates as proposed by Collins et al.[9,10]

This expression is compared in Fig. 10 with the results of 150 shear tests with concentrated loads and 16 tests with uniform loading for normal and high-strength concrete. Further details of the agreement with 285 shear tests (comprising tests with tension and compression axial forces, lightweight aggregate, and high-strength concrete) are given in Table 1.

The expression provided in ACI 318-05[24] is also plotted in Fig. 10. It can be noted that for small values of $\varepsilon d/(16 + d_g)$, the code gives rather conservative results (for ε = 0, Eq. (5) predicts two times the shear strength from ACI 318-05). For large values of $\varepsilon d/(16 + d_g)$, however, very unconservative shear strengths are obtained. This can be traced back to the fact that the ACI formula was originally proposed in 1963[36] when only tests with relatively small effective depths were available and the influence of size effect was thus not included.

All test results presented in Fig. 10 correspond to values of a/d greater than 2.9 (or 2.5 for the continuous beams detailed in Reference 23). These are usual cases in practice. Conservative results are obtained for values of a/d between 1 and 3.

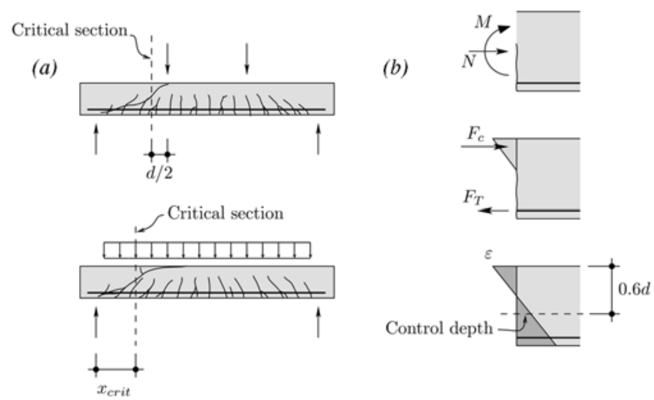

Fig. 9—Critical shear crack model: (a) critical section for point loading and distributed loading; and (b) determination of longitudinal strain in control depth using internal forces N and M.

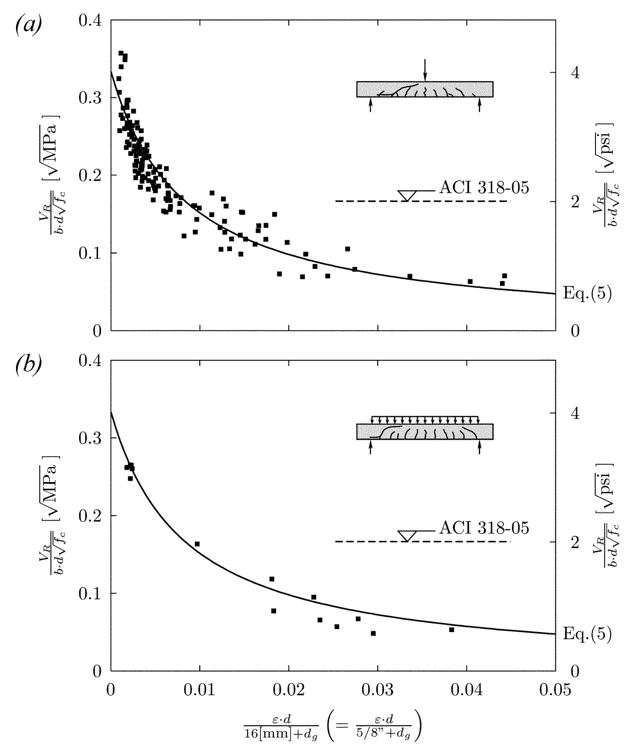

Fig. 10—Comparison of Eq. (5) and ACI 318-05[24] with test results for normal and high-strength concrete members: (a) concentrated loading; and (b) distributed loading.

SIMPLIFIED DESIGN METHOD—CODE PROPOSAL

In this section, a simplified design method based on Eq. (5) is introduced. The following hypotheses are adopted.

1. The value of ε is estimated assuming that the depth of the compression zone c is equal to $0.35d$ (which is a reasonable value accounting for various reinforcement ratios and concrete strengths), thus

$$\varepsilon = \varepsilon_s \frac{0.6d - c}{d - c} \cong 0.41\varepsilon_s \tag{6}$$

2. ε_s (the reinforcement strain) is assumed proportional to the bending moment m_{Ed}. At yielding ($m_{Ed} = m_{Rd}$), its value is $\varepsilon_s = f_{yd}/E_s$, where f_{yd} is the design strength of the reinforcement ($= f_{yk}/\gamma_s \approx \phi_s f_y$).

3. The flexural strength can be estimated (according to the theory of plasticity) as: $m_{Rd} = \rho d^2 f_{yd}(1 - \rho f_{yd}/(2f_{cd}))$, where f_{cd} is the design compressive strength of concrete ($= f_{ck}/\gamma_c \approx \phi_c f_c'$).

Table 1—Comparison between several test results and Eq. (5) and (8)

	Year	No.	Refined method Eq. (5)		Simplified method Eq. (8)	
			Mean value	Coefficient of variation	Mean value	Coefficient of variation
Concentrated loads						
Normal-strength concrete (f_c < 60 MPa [9000 psi]) without axial force	1957[25]	12	0.94	0.04	1.09	0.07
	1959[26]	3	1.01	0.14	0.98	0.14
	1962[16]	12	1.03	0.10	1.13	0.11
	1969[27]	7	0.99	0.09	1.15	0.12
	1979[18]	69	0.97	0.08	1.12	0.11
	1984[28]	3	0.99	0.02	1.03	0.11
	1986[29]	6	0.93	0.08	1.02	0.08
	1987[30]	3	1.22	0.15	1.13	0.10
	1999[23]	20	1.01	0.12	1.03	0.12
	Σ	135	0.98	0.10	1.10	0.11
High-strength concrete (f_c > 60 MPa [9000 psi]) without axial force	1986[29]	7	1.00	0.09	1.08	0.06
	1999[23]	8	0.97	0.09	0.94	0.08
	Σ	15	0.98	0.09	1.01	0.10
Lightweight concrete without axial force	1963[31]	24	0.93	0.07	1.02	0.07
	1967[32]	20	1.06	0.09	1.16	0.10
	1984[28]	6	1.00	0.06	1.07	0.08
	Σ	50	0.99	0.10	1.08	0.10
Normal-strength concrete with compression axial force	1957[25]	18	1.00	0.09	1.17	0.08
	1969[27]	5	0.93	0.02	1.15	0.07
	Σ	23	0.99	0.09	1.17	0.08
Normal-strength concrete with tension axial force	1969[27]	10	0.98	0.09	1.12	0.10
	2002[33]	36	1.05	0.09	1.08	0.10
	Σ	46	1.03	0.09	1.09	0.10
Distributed loading						
Normal-strength concrete (f_c < 60 MPa [9000 psi]) without axial force	1962[16]	7	0.99	0.03	1.12	0.03
	1985[34]	4	0.96	0.13	0.93	0.14
	1989[35]	5	0.90	0.10	0.86	0.10
	Σ	16	0.95	0.09	0.99	0.14
Total		285	0.99	0.10	1.09	0.11

Note: All beams with rectangular cross section; without shear reinforcement (or skin reinforcement along side faces); without prestressing and failing in shear.

With the previous hypotheses, introducing the concrete partial safety factor γ_c (or the concrete strength reduction factor ϕ_c) and referring the shear strength to the target 5% fractile, Eq. (5) leads to

$$\frac{V_{Rd}}{bd\sqrt{f_{ck}}} = \frac{0.3/\gamma_c}{1 + \frac{50}{16 + d_g} \cdot \frac{f_{yk}}{\gamma_s E_s} d \frac{m_{Ed}}{m_{Rd}}} \quad \text{SI units: MPa, mm}$$

$$\frac{V_{Rd}}{bd\sqrt{f_c'}} = \frac{3.6\phi_c}{1 + \frac{50}{\frac{5\text{ in.}}{8} + d_g} \cdot \frac{\phi_s f_y}{E_s} d \frac{m_{Ed}}{m_{Rd}}} \quad \text{US units: psi, in.} \tag{7}$$

Equation (7) can be simplified introducing the usual value of some of its parameters. For instance, if the following values are adopted: E_s = 205,000 MPa (29,700 ksi); f_{yd} = 435 MPa ($\phi_s f_y$ = 63 ksi, design strength of the reinforcement); d_g = 32 mm (5/4 in.) and $\gamma_c = 1.5$ ($\approx 1/\phi_c$), it becomes

$$\frac{V_{Rd}}{bd\sqrt{f_{ck}}} = \frac{0.2}{1 + 0.0022d\frac{m_{Ed}}{m_{Rd}}} \quad \text{SI units: MPa, mm}$$

$$\frac{V_R}{bd\sqrt{f_c'}} = \frac{2.3}{1 + 0.056d\frac{m_{Ed}}{m_{Rd}}} \quad \text{US units: psi, in.} \tag{8}$$

This equation has been adopted by the Swiss Code for structural concrete.[14] Design based on Eq. (8) is very simple because m_{Ed}/m_{Rd} can be set to 1.0 as an initial (and conservative) value. If the shear strength of the member is found to be insufficient, a more refined estimate of m_{Ed}/m_{Rd} can be introduced later.

The following points should be considered:

1. The aggregate size needs to be set to zero for high-strength concrete (f_{ck} > 60MPa [f_c' > 9000 psi]) or for lightweight aggregates. This accounts for the fact that in these cases, cracks develop through the aggregates.

2. The value of m_{Ed}/m_{Rd} has to be multiplied by 1.5 in development or anchorage zones (refer to Fig. 11) to account for the localization of the cracks. The shear strength reduction

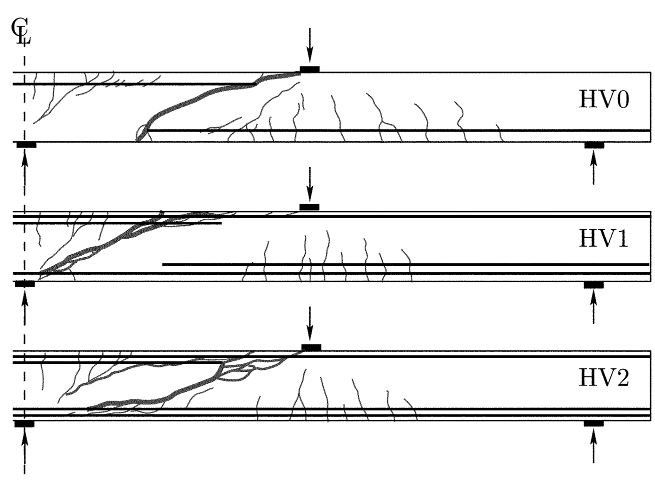

Fig. 11—Cracking pattern in Beams HV0, HV1, and HV2[37] with reinforcement cutoffs.

in cut-off zones is also taken into account in ACI 318-05[24] where the shear strength is reduced by a factor 1.5.

3. If the reinforcing bars are not parallel to the principal shear directions, m_{Ed}/m_{Rd} has to be multiplied by the following coefficient

$$\frac{1}{\sin^4\theta + \cos^4\theta} \tag{9}$$

where θ is the angle between the direction of the reinforcement and the principal shear direction. This coefficient[38] accounts for the fact that flexural cracks are wider when the reinforcement is not parallel to the principal shear direction.

4. When an axial force is applied to the member, the critical crack width may be increased or diminished. To take this phenomenon into account, m_{Ed} has to be replaced by $(m_{Ed} - m_{Dd})$ and m_{Rd} by $(m_{Rd} - m_{Dd})$, where m_{Dd} is the decompression moment (bending moment causing $\varepsilon_s = 0$), whose value can be taken as

$$\begin{aligned} n_d < 0&: m_{Dd} = -n_d\left(\frac{h}{2} - \frac{d}{3}\right) \\ n_d > 0&: m_{Dd} = -n_d\left(\frac{h}{2} - d'\right) \end{aligned} \tag{10}$$

where d' is the distance from the extreme compression fiber to the centroid of the longitudinal compression reinforcement. Also, an effective shear span a_{eff} has to be considered to account for the effect of the axial force on the arching action

$$a_{eff} = a + \frac{Nh}{2V} \tag{11}$$

5. The value of 0.0022 (0.056) in Eq. (8) is valid when the internal forces are obtained from an elastic analysis. If they result from a plastic analysis, considering internal force redistributions, this value should be taken to 0.003 (0.077). This accounts for the fact that yielding of the longitudinal reinforcement produces a notable increase in the width of the critical crack, consequently reducing the shear strength of the member.[19]

INFLUENCE OF LOAD CONFIGURATION AND MEMBER GEOMETRY ON SHEAR STRENGTH

The control section for beams subjected to concentrated loading is taken at $d/2$ from the point of load introduction (refer to Fig. 9(a)). This choice is justified because the shear force is constant along the shear span, but close to midspan, the bending moments are maximal (thus, the cracks are wider and, consequently, the shear strength is minimal).

For beams subjected to distributed loading, however, the choice of the critical section is not so straightforward. Close to the midspan, the bending moments are maximal but the shear force is minimal. On the contrary, near the supports, the shear force increases but so also does the shear strength (because the bending moments are reduced). The critical section is located at the point where the shear strength equals the value of the shear force. When investigating this location, the presence of lap splices, reinforcement cutoffs and changes in the effective depth of the beam need to be considered because they can lead to a localization of the critical crack, thus reducing the shear strength of the member.

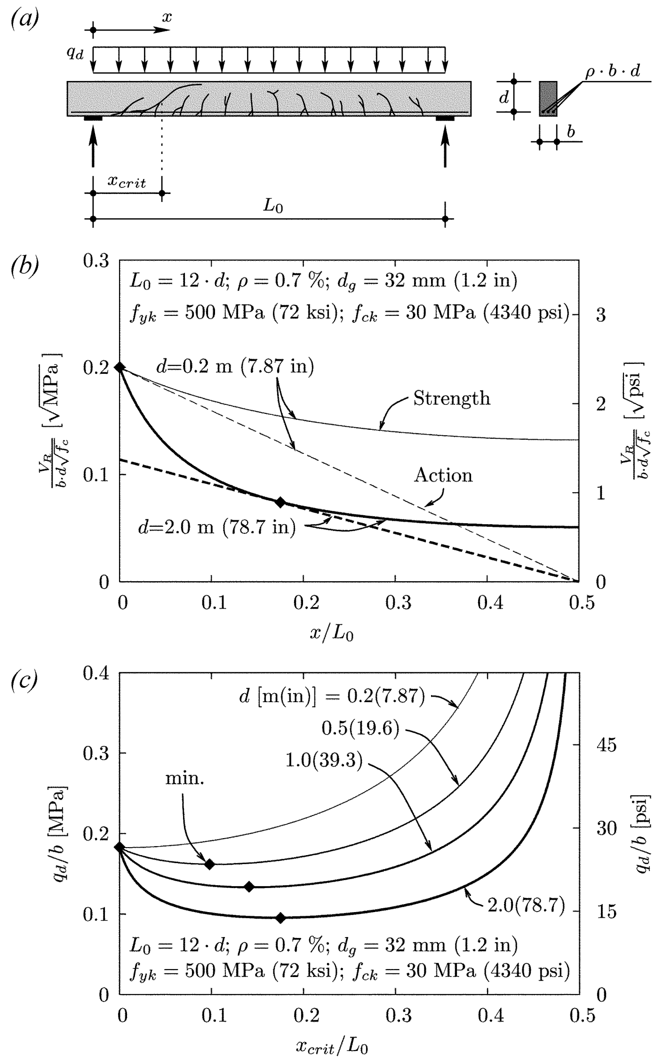

Fig. 12—Shear critical section for distributed loading: (a) geometry and mechanical parameters investigated; (b) location of critical sections (tangent between strength and action lines) for two effective depths; and (c) required load to develop shear failure at each section of beam and minimum (indicating shear critical section) for various effective depths.

The location of the control section for usual cases (beams with constant reinforcement and effective depth subjected to uniform loading) is investigated in Fig. 12. Two representative cases are presented in Fig. 12(b), corresponding to small and large effective depths. The control section (located at the point where the shear strength line and the shear force line are tangent) is located close to the supports for small depths, shifting to $0.17L_0$ for large effective depths. Figure 12(c) plots the value of the uniform load causing the shear failure at each section of the beam for various effective depths (where the minimum of each curve is located at the control section). The curves are very flat around the minimum. Consequently, variations on the location of the shear control section lead to small differences in the failure load of the beam. Therefore, it is proposed to check the shear strength of such members at $d/2$ and at $L_0/6$ from the support (the effective failure load being the minimum of these two values).

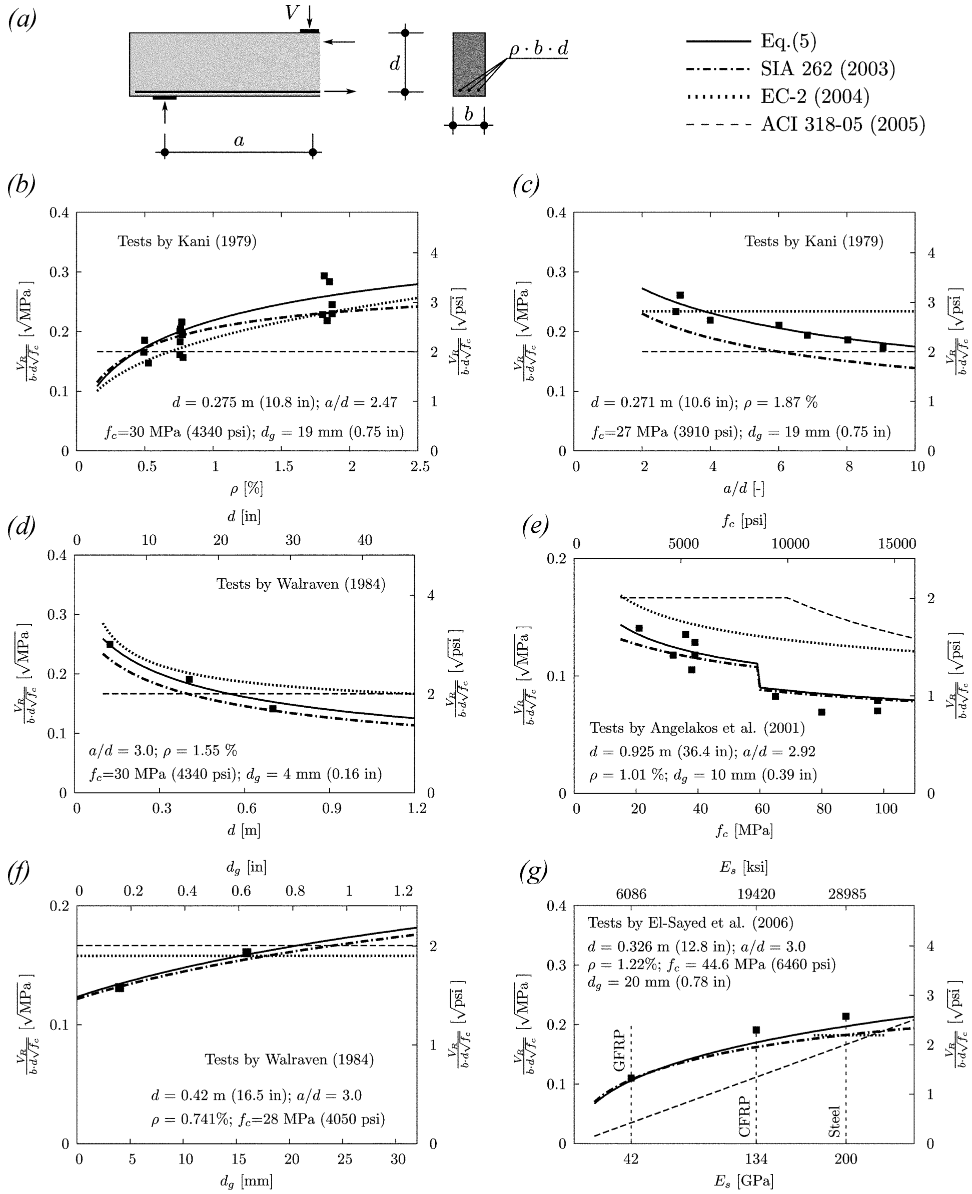

Fig. 13—Comparison of Eq. (5), SIA 262,[14] EC-2,[39] and ACI 318-05[24] (corrected by ACI 440.1R-01[40] in plot (g)) for various cases: (a) geometrical and mechanical parameters; (b) and (c) tests by Kani;[18] (d) and (f) tests by Walraven;[28] (e) tests by Angelakos et al.;[9] and (g) tests by El-Sayed et al.[41] with steel, carbon fiber; and glass fiber reinforcement.

COMPARISON WITH TEST RESULTS AND VARIOUS CODES OF PRACTICE

The results of Eq. (5) and (8), on which the current Swiss code[14] is based, are compared in this section with some experimental data as well as the provisions detailed in other codes of practice: EC-2[39] and ACI 318-05[24] in Fig. 13 and CSA A23.3-04[11] and AASHTO LRFD[8] in Fig. 14 ($\gamma_c = \phi_c = 1.0$).

The influence of the following parameters has been investigated:

1. Parameters influencing the width of the shear critical crack: effective depth of the beam d, reinforcement ratio ρ, modulus of elasticity of the reinforcement E_s, and a/d;

2. The aggregate size d_g, which governs aggregate interlock: its value is set to zero when the compressive strength of concrete is larger than 60 MPa (9000 psi) or for lightweight concrete, as previously stated; and

3. The compressive strength of concrete f_c: this parameter appears in Eq. (1) and proposes a linear increase of the shear strength with the square root of f_c. When the concrete compressive strength is increased, however, larger strains are developed by the reinforcement (resulting in larger widths of the critical shear crack) due to the increase in the shear failure load. Thus, a less than proportional relationship between the square root of f_c and the shear strength results.

The comparison shows that formulations based on physically sound models (AASHTO LRFD and CSA based on the MCFT, and Eq. (5) and SIA 262 based on the critical shear crack theory) give the best agreement when compared with test data, leading to similar trends for the various parameters. Empirical formulations[24,39] present larger scatters and may neglect the role of some governing parameters.

CONCLUSIONS

This paper investigates the shear strength of members without transverse reinforcement, introducing the fundamentals of the critical shear crack theory. The main conclusions are:

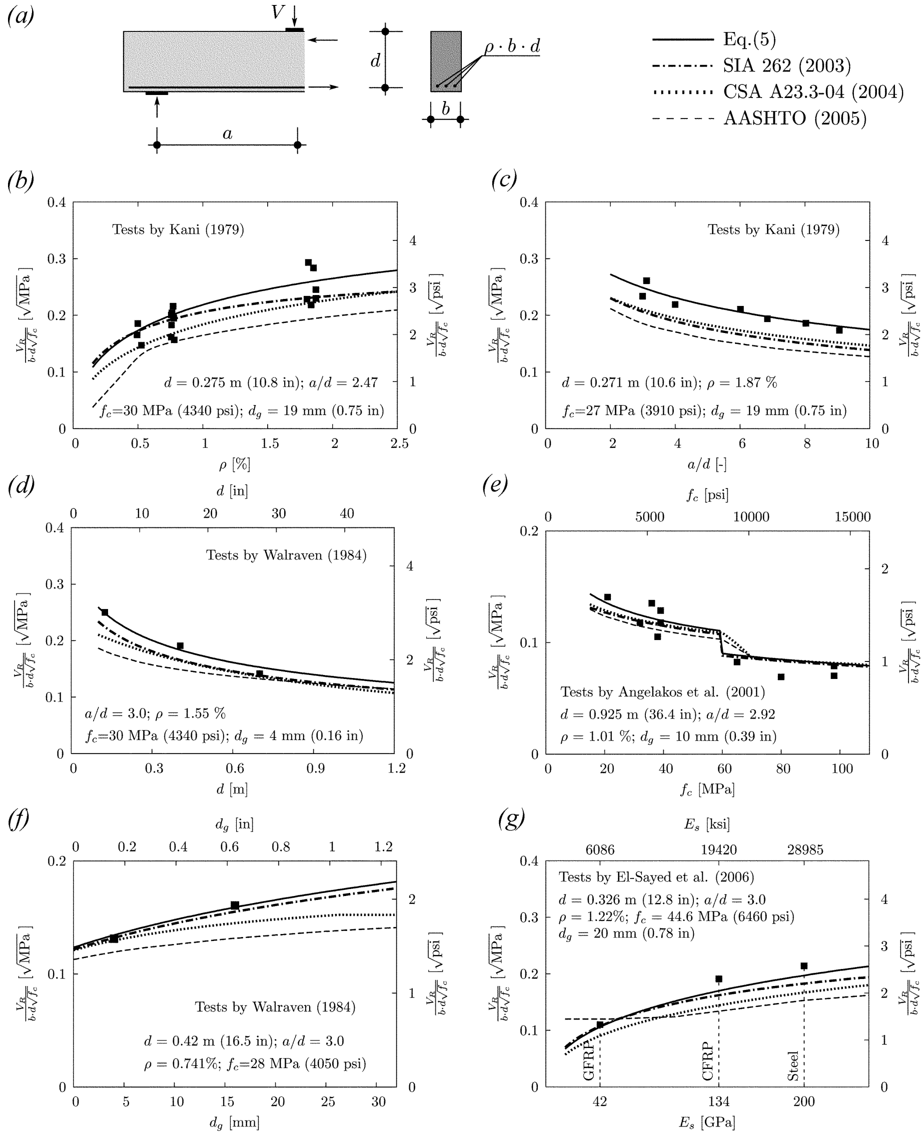

Fig. 14—Comparison of Eq. (5), SIA 262,[14] CSA A23.3-04,[11] and AASHTO LRFD[8] for various cases: (a) through (g) same geometry and tests as in Fig. 13.

1. The plasticity-based solutions with an inclined compression strut overestimate the shear strength when a critical shear crack develops inside the theoretical strut;

2. Shear is initially resisted by three shear-carrying mechanisms: cantilever action, aggregate interlock, and dowel action. These mechanisms create a state of tensile stresses in the concrete that leads to the development of the critical shear crack;

3. The development of the critical shear crack cancels the three previous shear-carrying mechanisms. A new one, the arching action, is activated;

4. The parameters governing the arching action (and thus the shear strength) are the location of the critical shear crack, its width, and the aggregate size;

5. The shear strength can be satisfactorily estimated considering the effect of the previous parameters. To that end, an analytical expression is proposed with good agreement to 285 test results;

6. A simplified design method (adopted by the Swiss code[14]) is detailed as well as its application to practical cases. The influence of parameters such as the load configuration, member geometry, presence of an axial force, and reinforcement arrangement can be easily investigated using this simplified method; and

7. Comparisons between several test results, the expressions proposed in this paper, and those of some current codes of practice are presented. Empirical models[24,39] do not satisfactorily agree with many of the test results and the influence of some governing parameters is not suitably reflected. AASHTO LRFD,[8] and CSA A23.3-04,[11] based on the MCFT, show a good agreement in the cases investigated. Also, very good results are obtained with the Swiss code[14] based on the critical shear crack theory. Both theories are physically sound and, although developed from different approaches, propose similar expressions with the same governing parameters, auguring a promising agreement in this field.

NOTATION

a = shear span
a_{eff} = effective shear span
b = thickness of member
c = depth of compression chord
d = effective depth
d' = distance from extreme compression fiber to centroid of longitudinal

compression reinforcement

d_g = aggregate size
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of reinforcement
f_c = compressive strength of concrete
f_c' = specified concrete uniaxial strength in compression (American practice)
f_{ck} = characteristic compressive cylinder strength of concrete (European practice)
f_y = Yield strength of reinforcement
f_{yd} = Design yield strength of reinforcement (European practice)
h = height of cross section
L_0 = equivalent span (distance between points of zero bending moment)
ℓ = length of element
m_d = design (factored) moment divided by width of cross section
m_{Ed} = design (factored) moment per unit length in critical section
m_{Rd} = plastic design (factored) moment per unit length in critical section
q_d = design (factored) value of distributed load
u = perpendicular relative displacement between lips of crack
V = shear force
V_R = shear strength
v = parallel relative displacement between lips of crack
v_d = design (factored) shear force divided by width of cross section
w = total relative displacement between lips of crack
x = location of given section
x_{crit} = location of critical shear section
ε = strain
ε_s = steel strain
ϕ_c = concrete strength reduction factor (American practice)
ϕ_s = steel strength reduction factor (American practice)
γ_c = concrete partial safety factor (European practice)
γ_s = steel partial safety factor (European practice)
θ = longitudinal reinforcement ratio
ρ = angle between reinforcement and shear principal direction

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