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COMPUTER-AIDED DEVELOPMENT OF STRESS FIELDS FOR THE ANALYSIS OF STRUCTURAL CONCRETE

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Abstract: *The stress field method is a technique based on the lower-bound theorem of plasticity that can be used for dimensioning, detailing and checking of structural concrete elements. Its use has been traditionally associated with the assumption of simplified (rigid-plastic) stress-strain relationships for the materials. This assumption greatly helps in solving such models, but requires a certain level of experience to decide the most suitable load-carrying mechanism for a given case. This paper presents the possibilities of using a nonlinear finite element analysis program implementing the main hypotheses of the stress field method to develop suitable stress fields. These stress fields can be used to investigate the strength and behaviour of structural concrete members. Detailed applications are presented.*

1. INTRODUCTION

The design and check of structural concrete needs to rely on simple, accurate and safe methods. It is also advisable that these methods allow the engineer to understand the various load-carrying mechanisms of a structure. Both the strut and tie method[1] and the stress field method[2] comply with these requirements.

1.1 The strut and tie method

The strut and tie method was proposed by Schlaich et al.[1] after enhancing several theoretical aspects of the truss analogy (based on the lower bound theorem of the theory of plasticity) first proposed by Ritter[3] and later extended by Mörsch[4]. In general, various strut and tie models (STM) can be developed for a given member, as shown in Fig. 1.

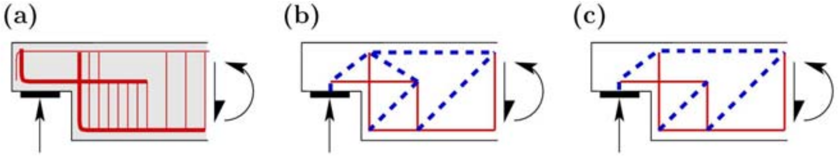

Figure 1: Various strut and tie models for a structural member with a given reinforcement layout: (a) dapped-end beam geometry, loads and reinforcement layout; and (b,c) possible strut and tie models

According to Schlaich[1] (see also MC-90[5]), one possible approach for the development of suitable strut and tie models is to identify the struts and the ties of a member from the compression and tension zones of its elastic uncracked stress field, see Figs 2a,b,c. This approach allows developing strut and tie models without knowing *a priori* the reinforcement layout, which is very advantageous for the design of new structures.

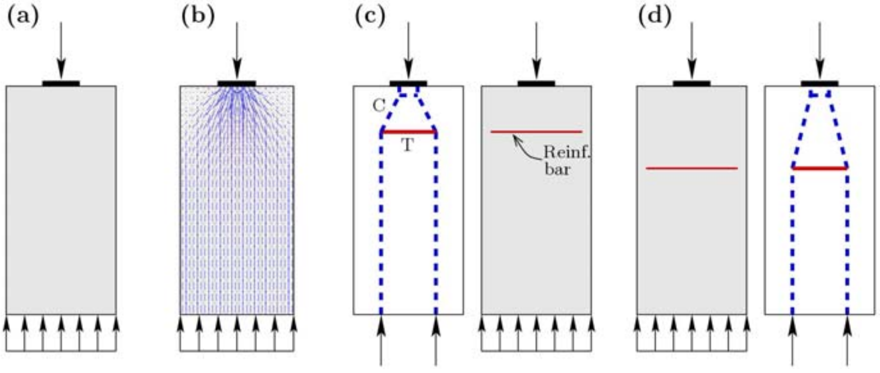

Figure 2: Local force introduction: (a) geometry and loads; (b) linear elastic (uncracked) solution; (c) possible strut and tie model inspired in the elastic stress field and corresponding reinforcement layout (C = Compression; T = Tension); and (d) actual reinforcement layout and corresponding strut and tie model

Furthermore, dimensioning according to the elastic uncracked stress field of a member ensures in most cases a suitable serviceability behaviour[1]. However, strut and tie models can also be developed for reinforcement layouts different from that inspired by the elastic stress field (see Figure 2d), which is the typical situation for existing structures. For these cases, special care has to be given to serviceability conditions[6] and to strength requirements, since wide-open cracks lead to a decrease in the compressive strength of the struts.

1.2 The stress field method

The stress field method[2] is an alternative approach to the strut and tie method. Because of the rigid-plastic constitutive laws classically adopted for steel and concrete in the stress field method, several stress fields can be developed for a structural member (as in the strut and tie method). Consequently, the development of stress fields has thus far been mainly based on experience and intuition. Recently, a rational approach for the automatic development of suitable stress fields has been proposed[7] (Figure 3) with applications to the design of structural members. One important aspect of this approach is that it allows considering the influence of cracking on the compressive strength of concrete.

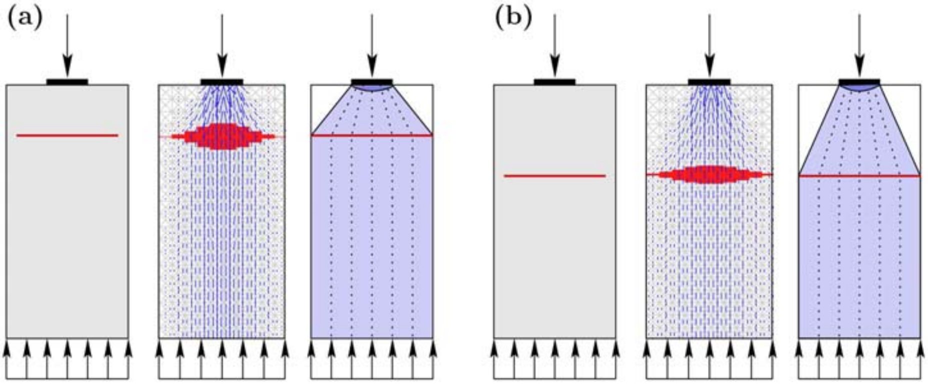

Figure 3: Computer-aided development of stress fields (a,b) geometry and reinforcement layout, nonlinear FEM-based stress fields and corresponding discontinuous stress fields

This paper explores the possibilities of the FEM-based stress fields presented in referenced publication[7] for investigating the behaviour and strength of existing structures. To that end, the FEM-based stress fields are compared to the measurements of an actual test, explaining the various results obtained. Finally an application for the checking of an existing structure is also presented.

2. APPLICATION OF FEM-BASED STRESS FIELDS TO THE INVESTIGATION OF A TEST SPECIMEN

Figure 3 shows a scheme of beam VN2 tested by Kaufmann and Marti in shear and bending[8].

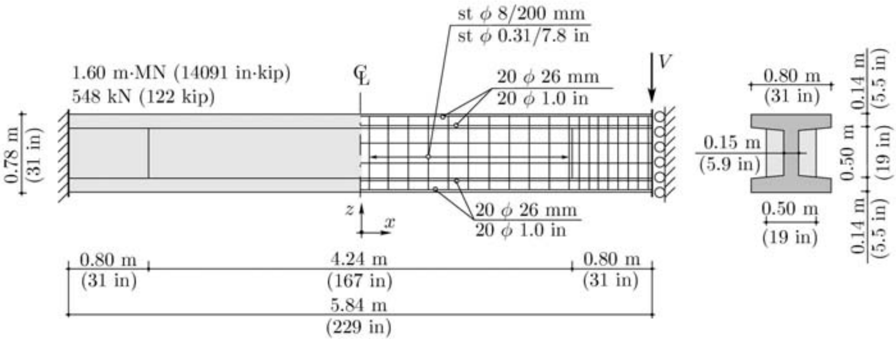

Figure 3: Geometry and reinforcement layout for beam VN2[8] (f_c = 49 MPa (7.1 ksi), E_c = 31000 MPa (4492 ksi); $f_{y,8\text{mm}}$ = 484 MPa (70.1 ksi); $f_{y,26\text{mm}}$ = 539 MPa (78.1 ksi); E_s = 210000 MPa (30434 ksi))

Since the geometry and reinforcement of the element are known, the development of a FEM-based stress field is straightforward following the approach proposed in referenced publication[7]. Some results are shown in Figure 4.

It can be noted that an inclined compression field is developing in the web (Figure 4a) with significant compressive stresses (black indicates concrete crushing). Yielding of the stirrups is also predicted by the FEM (shown in brown in Figure 4b, where red means that the steel remains elastic), in good agreement with the test measurements[8]. In the FEM analysis, the concrete compressive strength is reduced to account for transverse cracking. This reduction is introduced according to Vecchio and Collins[9] by means of a coefficient named η_ε whose minimum value in this case (see Figure 4c) is 0.49. This value is smaller than 0.60 (usually adopted to dimension the web of beams) due to the extensive yielding of the stirrups.

Figure 4d compares the numerical results at failure for the inclination of the compression field with the measurements performed by Kaufmann and Marti[8], showing a good agreement. It can be noted that a very small inclination of the compression field is obtained (approx. 17°) due to the limited reinforcement ratio in the web (ρ_w = 0.33%) which leads to extensive yielding of the stirrups. A failure load equal to 98 % of the measured one (V_{max} = 548 kN) is obtained with the FEM, with the member failing by crushing of the web as observed during the test. Further comparisons of the FEM-based stress fields to test results can be found elsewhere[7].

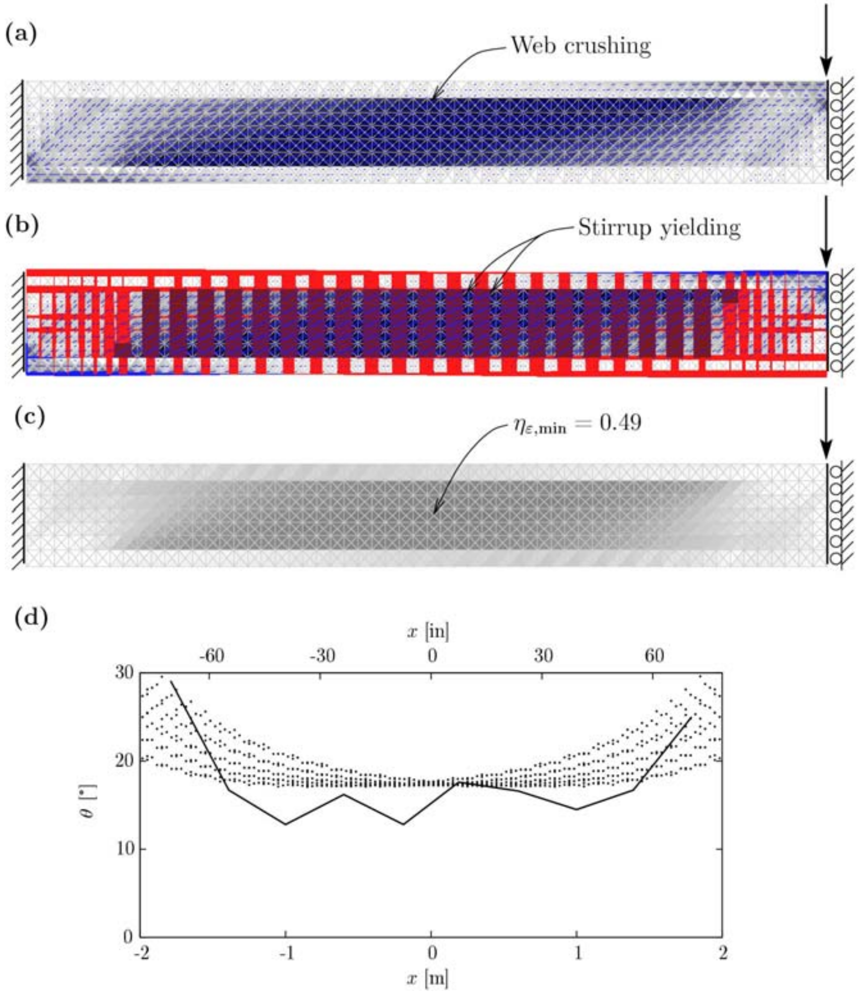

Figure 4: FEM results for beam VN2 by Kaufmann and Marti[8]: (a) plot of concrete principal stresses directions; (b) plot of ratio σ_s/f_y for the different steels (superimposed to the concrete principal stresses directions); (c) plot of coefficient η_ε (minimum value: 0.49; maximum value: 1.00); and (d) comparison of the FEM results for the inclination of the compression field with the test measurements at failure

3. APPLICATION OF FEM-BASED STRESS FIELDS TO THE CHECKING OF AN EXISTING STRUCTURE

The use of the FEM-based stress fields is very attractive when unusual stress fields have to be developed. For instance, Figure 5 shows the support region of an actual prestressed precast beam of 18.93 m (62.3 feet) span, where the anchorage length of the prestressing wires is 1200 mm (47.2 in). The influence of the position (*s*) of the support plate in the shear strength of the beam was investigated following an improper placement of the beams.

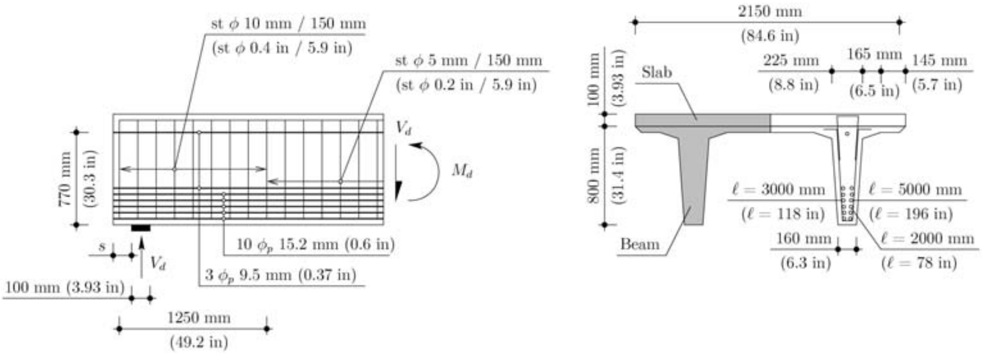

Figure 5: Geometry and prestressing wires for the prestressed beam with open cross-section (ℓ = unbonded length; $f_{c,beam}$ = 45 MPa (6.5 ksi), $E_{c,beam}$ = 32000 MPa (4637 ksi); $f_{c,slab}$ = 38 MPa (5.5 ksi), $E_{c,slab}$ = 30500 MPa (4420 ksi); f_{p0} = 1239 MPa (179 ksi); f_{pk} = 1770 MPa (256 ksi); E_p = 195000 MPa (28260 ksi))

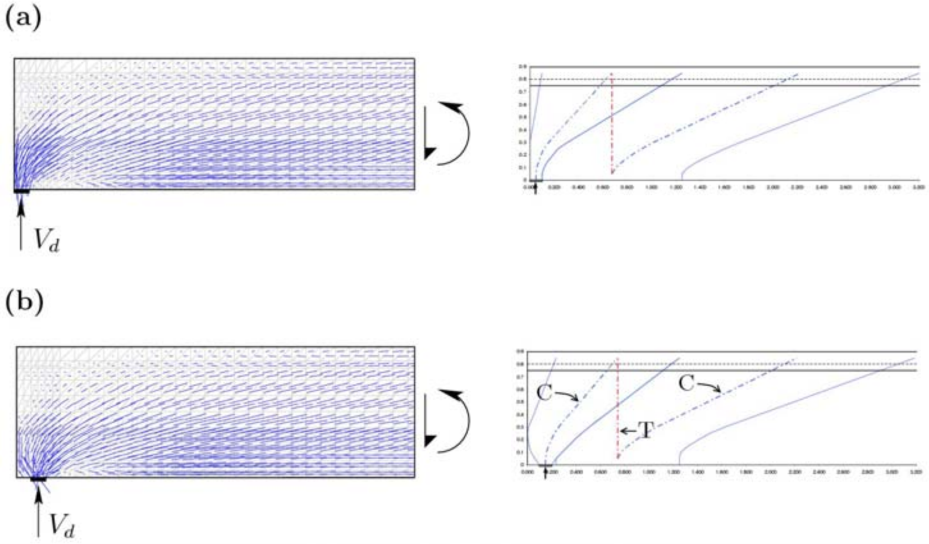

Figure 6: FEM results for the prestressed beam. Plots of FEM concrete stress fields and adopted (analytical) stress fields for: (a) bearing plate at the edge of the beam; and (b) bearing plate at 100 mm (3.9 in) of the edge of the beam

The FEM-based stress fields were used as a guide to develop suitable stress fields for the region, see Figure 6, allowing to determine the reduction in the shear strength of the beam. From the study, it was concluded that it was not safe to have support plates at distances to the edge of the beam smaller than 50 mm (1.96 in.).

4. I-CONCRETE, ON-LINE ENVIRONMENT OF STRESS FIELDS IN STRUCTURAL CONCRETE

The i-concrete project was initiated at the Ecole Polytechnique Fédérale de Lausanne to foster and to encourage the application of stress field models in the design of structural concrete. The site is meant both as a teaching tool for undergraduate and graduate students and as a reference tool for practicing engineers.

It offers a series of classical examples with their solution using stress fields, and a comparison with available test results. Users can benefit from the interactive calculation provided by Java applets that offer an FEM solution similar to those described in the paper.

The main idea is to help students develop an understanding for the behaviour of concrete structures at the ULS, by confronting their solutions to numerical simulations and, whenever possible, to available test results. Access is free at http://i-concrete.epfl.ch.

5. CONCLUSIONS

This paper investigates the possibilities of using the FEM method for developing stress fields with the aim of checking existing structures or with the aim of investigating test results. The main conclusions of the paper are:

1. FEM-based stress fields are a valuable tool to understand the behaviour of existing structural members
2. FEM-based stress fields can accurately predict the strength and failure mode of actual structural members
3. Strut and tie models or rigid-plastic stress fields can be developed from the FEM-based stress fields. This is particularly interesting when unusual details or members need to be investigated

NOTATION

f_c = compressive strength of concrete
f_y = yield strength of reinforcing steel
f_p = yield strength of prestressing steel
f_{p0} = initial prestressing
ℓ = unbonded length
s = distance of beam end to support plate
x, z = coordinate
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of reinforcing steel
V = shear force
η_ε = strength reduction factor accounting for transverse cracking
θ = inclination of the compression field
ρ_w = reinforcement ratio of the web
σ_s = steel stress
C = compression strut
T = tension tie
st = stirrup

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