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GRAFT OF THE BOUNDARY INTEGRAL METHOD ONTO THE IMAGE-SOURCE METHOD FOR VEHICLE ACOUSTICS

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Abstract

The solution obtained with the acoustical image sources method was shown to be the contribution of terms issued from a series development of the integral method (believed to be the exact solution). Missing terms in the geometrical method represent, among others, the contribution of diffraction. Taking all of them into account would indeed lead back to the full integral method. Alternatively, would it be possible to consider only a subset of these terms, therefore adding missing information to the geometrical method? This would bring the latter to its much-awaited role in the medium-frequency range.

1. INTRODUCTION

In a domain made up of an open angular sector in a 2D-space, it has been shown that the acoustic field due to the image sources corresponds to the contributions of the first terms of a series arising from the exact solution obtained by the boundary integral method [1]. It has been found that such an idea was mentioned earlier [2]. Thanks to this observation, the diffracted field in the open sector is the difference between two fields, the specular and the exact one.

One could mentally extrapolate that it would be possible to deploy in a series the acoustic field in a cavity, obtained exactly from the integral representation. The first terms of this series would correspond to the image sources contributions, the other terms revealing the diffracted field. By the help of the image sources method alone, the diffracted field is inaccessible. Were we able to find it (by other means that the integral method), adding the total diffracted field to the image-source solution would be absurd – in terms of applications – since the integral solution already leads to the solution.

However, taking into account the observation mentioned above, the diffracted field in an open sector is straightforward and could easily enrich the acoustic field by means of the very first image sources. Could an enrichment of the results obtained from the ray method in a cavity be achieved in a similar manner ?

The presentation shows the rational process which enables the integral method to be grafted onto that of image sources, in an attempt to improve the latter. The motivation arises from today's hope that the image source method will make it possible to describe the acoustic fields in the audible medium-frequency range in vehicles.

2. BRIEF RECALL OF THE DESCRIPTION OF ACOUSTIC FIELDS IN AN OPEN SECTOR

In the two-dimensional open domain, the Helmholtz equation with the right-hand term $-\delta(\mathbf{x}_S - \mathbf{x}_R)$, written as $-\delta(S-R)$, is solved by the Green function $g_\infty(S,R) = -\frac{i}{4}H_0^-(kr)$, with k the wave number and r the distance between the two points S and R. When considering the domain bound by two walls (open sector), the same excitation on the right-hand term is applied. It has been shown [1] that the elementary solution is then

$$g_\theta(S,R) = \sum_{i=0}^{\infty} T_i(S,R) = \underbrace{g_\infty(S,R) + \sum_{i=1}^{N_\theta} g_\infty(S_i,R)}_{\tilde{g}_\theta(S,R)} + \underbrace{\sum_{i=N_\theta+1}^{\infty} T_i(S,R)}_{\delta g_\theta(S,R)} \tag{1}$$

where N_θ is the number of image sources active in this case and $\delta g_\theta(S,R) = \sum\limits_{i=N_\theta+1}^{\infty} T_i(S,R)$ is the non-specular part of the acoustic field, which will be called diffracted field. The form of the terms $T_i(S,R)$ is relatively simple for $R \in \Gamma_\theta$. Since the pressure on the boundaries drives the pressure inside the domain, this study will be limited to a boundary problem for the time being.

3. DESCRIPTION OF THE FIELD INSIDE A CAVITY

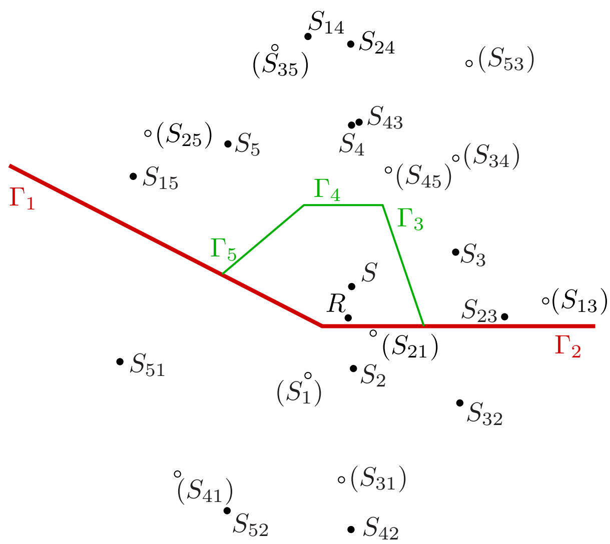

Figure 1. Shape of the bidimensional cavity studied, with visible and (in brackets) invisible image sources. Note that the cavity represents an extension of an open sector situation (walls Γ_1 and Γ_2, in red).

A bidimensional domain made up of 5 rigid walls is studied here (cf. Fig. 1). This cavity was the subject of an earlier study [3]. Using the image sources method, the pressure at a point R is

$$p^{\mathrm{sp}}(R) = g_\infty(S,R) + \sum_{n=1}^{5} g_\infty(S_n,R) + \sum_{m=1}^{5}\sum_{\substack{n=1\\ n\neq m}}^{5} g_\infty(S_{nm},R) + \sum_{l=1}^{5}\sum_{\substack{m=1\\ m\neq l}}^{5}\sum_{\substack{n=1\\ n\neq m}}^{5} g_\infty(S_{nml},R) + \dots \tag{2}$$

In fact, some image sources from this series vanish because not suitable for emitting a reflected ray reaching the receptor point R (either by being inside the cavity, so-called invalid sources, or by being "invisible" from R). For the present case, posing $\Gamma_\theta \equiv \Gamma_1 \cup \Gamma_2$ and $\Gamma_a \equiv \Gamma_3 \cup \Gamma_4 \cup \Gamma_5$, the sources are re-organised as

$$
\begin{aligned}
p_\theta^{\mathrm{sp}}(R) = {} & \left.\begin{aligned} & g_\infty(S,R) + \underbrace{\sum_{\text{images }/\Gamma_\theta} g_\infty(S_{\text{images}},R)}_{\cancel{g_\infty(S_1,R)}+g_\infty(S_2,R)+\cancel{g_\infty(S_{12},R)}+\cancel{g_\infty(S_{21},R)}} \end{aligned}\right\}\tilde{g}_\theta(S,R)\\
& \left.\begin{aligned} & + \underbrace{\sum_{\text{images }/\Gamma_a} g_\infty(S_{\text{images}},R)}_{\substack{g_\infty(S_3,R)+g_\infty(S_4,R)+g_\infty(S_5,R)+\cancel{g_\infty(S_{34},R)}+\cancel{g_\infty(S_{35},R)}\\ +g_\infty(S_{43},R)+\cancel{g_\infty(S_{45},R)}+\cancel{g_\infty(S_{53},R)}+\cancel{g_\infty(S_{54},R)}+\dots}}\\
& + \underbrace{\sum_{\text{images }/\Gamma_a}\ \sum_{\text{images }/\Gamma_\theta} g_\infty(S_{\text{images}},R)}_{\cancel{g_\infty(S_{13},R)}+g_\infty(S_{14},R)+g_\infty(S_{15},R)+g_\infty(S_{23},R)+g_\infty(S_{24},R)+\cancel{g_\infty(S_{25},R)}+\dots} \end{aligned}\right\}\tilde{B}(S,R)\\
& \left.\begin{aligned} & + \underbrace{\sum_{\text{images }/\Gamma_\theta}\ \sum_{\text{images }/\Gamma_a} g_\infty(S_{\text{images}},R)}_{\cancel{g_\infty(S_{31},R)}+g_\infty(S_{32},R)+\cancel{g_\infty(S_{41},R)}+g_\infty(S_{42},R)+g_\infty(S_{51},R)+g_\infty(S_{52},R)+\dots}\\
& + \underbrace{\sum_{\text{images }/\Gamma_\theta}\ \sum_{\text{images }/\Gamma_a}\ \sum_{\text{images }/\Gamma_\theta} g_\infty(S_{\text{images}},R) + \dots}_{\dots} \end{aligned}\right\}\tilde{C}(S,R)
\end{aligned} \tag{3}
$$

where sources that are either invalid or invisible from point R are canceled out. Sources are only represented up to the second order. Taking the order of source apparition into account, eq. (3) is written as

$$p_\theta^{\mathrm{sp}}(R) = \tilde{g}_\theta(S,R) + \tilde{B}(S,R) + \tilde{C}(S,R) \tag{4}$$

The $\tilde{\cdot}$ sign specifies that these terms are purely specular contributions. Equation (4) will make it possible to observe the diffraction effects on the image sources method that are due only to the open sector, as well as the diffraction due to the non-specular terms on the first consideration of walls Γ_a.

4. ENRICHMENT OF THE TERM $\tilde{g}_\theta(S,R)+\tilde{B}(S,R)$

4.1. Formulation

As a preliminary, recall that the exact (integral) solution with rigid walls and expressed with the kernel $g_\theta(S,R)$ is

$$p^{\text{ex}}(R)=g_\theta(S,R)-\int_{\Gamma_a\equiv\Gamma_3\cup\Gamma_4\cup\Gamma_5} p^{\text{ex}}(R')\partial_{n(R')}g_\theta(R',R)dR' \tag{5}$$

Discretization of Γ_a into facets Γ_{aj} and approximation by collocation lead to

$$\begin{aligned}p^{\text{ex}}(R)&=g_\theta(S,R)+\left\langle \ldots-\int_{\Gamma_{aj}}\partial_{n(R')}g_\theta(R',R)dR'\ldots\right\rangle\cdot\left\{\begin{array}{c}\vdots\\ p_j^{\text{ex}}(R'\in\Gamma_{aj})\\ \vdots\end{array}\right\}\\ &=g_\theta(S,R)+\mathbf{b}^T(R)\cdot\mathbf{p}^{\text{ex}}(\Gamma_a)\end{aligned} \tag{6}$$

with the line $\mathbf{b}^T=\langle\ldots\rangle$ and column $\mathbf{p}=\{\vdots\}$ vectors. The notation p^{ex} is maintained in the discretized form even though only a numerical approximation of it is gained in the process. When R is chosen at the collocation points, one gets

$$\mathbf{p}^{\text{ex}}(\Gamma_a)=(\mathbf{I}-\mathbf{A})^{-1}\cdot\mathbf{g}_\theta(S,\Gamma_a)=(\mathbf{I}+\mathbf{A}-\mathbf{A}^2+\mathbf{A}^3-\ldots)\cdot\mathbf{g}_\theta(S,\Gamma_a) \tag{7}$$

where the matrix $\mathbf{A}$, made up of the lines $\mathbf{b}^T$, contains the main value of the integrand $\partial_{n(R')}g_\theta(R',R)dR'$ on its diagonal when R and R' are merged on Γ_a. In the process of enriching p^{sp} in $p^{\text{sp++}}$ via $\delta g_\theta(S,R)$ and $\delta B(S,R)$, this main value will not intervene. Since $\mathbf{p}^{\text{ex}}(\Gamma_a)$ is available through (7), eq. (6) leads to

$$p^{\text{ex}}(R\in\Gamma_\theta)=g_\theta(S,R)+\mathbf{b}^T(R)\cdot\mathbf{I}\cdot\mathbf{g}_\theta(S,\Gamma_a)+\mathbf{b}^T(R)\cdot(\mathbf{A}-\mathbf{A}^2+\mathbf{A}^3-\ldots)\cdot\mathbf{g}_\theta(S,\Gamma_a) \tag{8}$$

where

$$\begin{aligned}\mathbf{b}^T(R)\cdot\mathbf{I}\cdot\mathbf{g}_\theta(S,\Gamma_a)&=\left\langle\ldots-\int_{\Gamma_{aj}}\partial_{n(R')}(\tilde{g}_\theta+\delta g_\theta)(R',R)dR'\ldots\right\rangle\cdot\left\{\begin{array}{c}\vdots\\ (\tilde{g}_\theta+\delta g_\theta)(S,R'\in\Gamma_{aj})\\ \vdots\end{array}\right\}\\ &=(\tilde{\mathbf{b}}^T(R)+\delta\mathbf{b}^T(R))\cdot(\tilde{\mathbf{g}}_\theta(S,\Gamma_a)+\delta\mathbf{g}_\theta(S,\Gamma_a))\end{aligned} \tag{9}$$

is the discretized form of the integral

$$B(S,R)=-\int_{\Gamma_a}g_\theta(S,R')\partial_{n(R')}g_\theta(R',R)dR' \tag{10}$$

The discrete form of $\tilde{B}(S,R) = -\int_{\Gamma_a} \tilde{g}_\theta(S,R')\partial_{n(R')}\tilde{g}_\theta(R',R)dR'$ is then

$$\tilde{\mathbf{b}}^T(R)\cdot\tilde{\mathbf{g}}_\theta(S,\Gamma_a) = \left\langle \ldots - \int_{\Gamma_{aj}} \partial_{n(R')}\tilde{g}_\theta(R',R)dR' \ldots \right\rangle \cdot \left\{ \begin{array}{c} \vdots \\ \tilde{g}_\theta(S,R'\in\Gamma_{aj}) \\ \vdots \end{array} \right\} \tag{11}$$

Accepting the interpretation of the Huyghens' Principle made in [1] that proved to be so efficient, (11) would be the incident pressure (regardless of sign) on Γ_a issued by the true source S and its images S_θ through Γ_θ, reflected upon Γ_a towards point R. In other words, (11) represents the contribution of all images of these sources through Γ_a, i.e. part $\tilde{B}(S,R)$ in (4). Following (8), one obtains

$$p^{\text{ex}}(R\in\Gamma_\theta) = \tilde{g}_\theta(S,R)+\delta g_\theta(S,R)+\tilde{B}(S,R)+\delta B(S,R)+\underbrace{\mathbf{b}^T(R)\cdot(\mathbf{A}-\mathbf{A}^2+\ldots)\cdot\mathbf{g}_\theta(S,\Gamma_a)}_{\tilde{C}(S,R)+\delta C(S,R)} \tag{12}$$

In this equation, $g_\theta = \tilde{g}_\theta + \delta g_\theta$ is easily available analytically, as well as $B = \tilde{B} + \delta B$ through the more delicate operation (10). The part $C = \tilde{C} + \delta C$ is here said to be "non-manageable". Henceforth, an enhanced expression of the specular pressure at R would be

$$p^{\text{sp++}}(R) = g_\theta(S,R) - \int_{\Gamma_a} \partial_{n(R')}g_\theta(R',R)g_\theta(S,R')dR' + \tilde{C}(S,R) \tag{13}$$

Since the terms $\tilde{g}_\theta$ and $\tilde{B}$ (i.e. the contribution of the true source S, its images S_θ through Γ_θ and the images of these images through Γ_a) are included in p^{sp}, one obtains

$$p^{\text{sp++}}(R) = p^{\text{sp}}(R) + \delta g_\theta(S,R) + \delta B(S,R) \tag{14}$$

4.2. Access to parts of $B(S,R)$

The numerical approximation of $B(S,R)$, leading to $\delta B(S,R)$ through substraction of $\tilde{B}(S,R)$ (i.e. the contributions of S, S_θ and their images through Γ_a), gives access to the enhancement stated in (14). However, certain parts of B can be reached easier by understanding what their physical meaning could be. Indeed, B can be developed into

$$B(S,R) = \underbrace{\overbrace{\underbrace{-\int_{\Gamma_a}\tilde{g}_\theta\partial_n\tilde{g}_\theta dR'}_{\substack{\text{specular reflection of } \tilde{g}_\theta \\ \text{i.e. of } S \text{ and its images } S_\theta}}}^{\tilde{B}(S,R)} \overbrace{-\int_{\Gamma_a}\delta g_\theta\partial_n\tilde{g}_\theta dR'}}_{\substack{\text{specular reflection of } g_\theta \text{ i.e. of } S,\, S_\theta, \\ \text{and the effect of } \Gamma_a \text{ on diffracted terms.}}} \overbrace{\underbrace{- \int_{\Gamma_a}(\tilde{g}_\theta+\delta g_\theta)\partial_n\delta g_\theta dR'}_{\text{"backscattering" of } (\tilde{g}_\theta+\delta g_\theta)}}^{\delta B(S,R)} \tag{15}$$

where the interpretations are made regardless of sign. The first element $\tilde{B}(S,R) = \tilde{B}_3(S,R) + \tilde{B}_4(S,R)+\tilde{B}_5(S,R)$ (representing the effect of walls Γ_3, Γ_4 and Γ_5 respectively) can be obtained by a geometrical construction with receptor points R_3, R_4 and R_5 as images of R through the walls Γ_a. In fact let

$$\tilde{B}_j(S,R) = -\int\limits_{\Gamma_j} \tilde{g}_\theta(S,R')\partial_{n(R')}\tilde{g}_\theta(R',R)dR' \tag{16}$$

with $j = 3,4,5$. Following Huyghens' Principle as in [1] and for walls of infinite length, $\tilde{B}_j(S,R)$ can represent the pressure issued from S and S_θ emitted on R_j, which is the image of R through Γ_j. In this case, the term $-\int\limits_{\Gamma_j}(\tilde{g}_\theta + \delta g_\theta)\partial_n\tilde{g}_\theta dR'$ $(j = 3,4,5)$ could be understood as the specular reflection on Γ_j of the incident pressure on the same wall coming from the open sector Γ_θ, including the diffraction part. It would also be equal to $g_\theta(S,R_j)$. After a numerical approximation of $B_j(S,R) = -\int\limits_{\Gamma_j} g_\theta\partial_n g_\theta dR$, the contribution $-\int\limits_{\Gamma_j}(\tilde{g}_\theta + \delta g_\theta)\partial_n(\delta g_\theta)dR'$ could be deduced. A straightforward physical meaning of the latest can not be given, but it could correspond to the reflection of the incident pressure on Γ_j radiated by the open sector Γ_θ. For want of a better name, this quantity is referred to as the "backscattering" of g_θ upon Γ_j.

Figure 2 shows the comparison of $B_j(S,R)$ with $-g_\theta(S,R_j)$. The differences observed reveal not only the backscattering, absent from $g_\theta(S,R_j)$, but also that in the latter expression the walls are considered one after the other, thus not taking the interaction between these walls Γ_a into account.

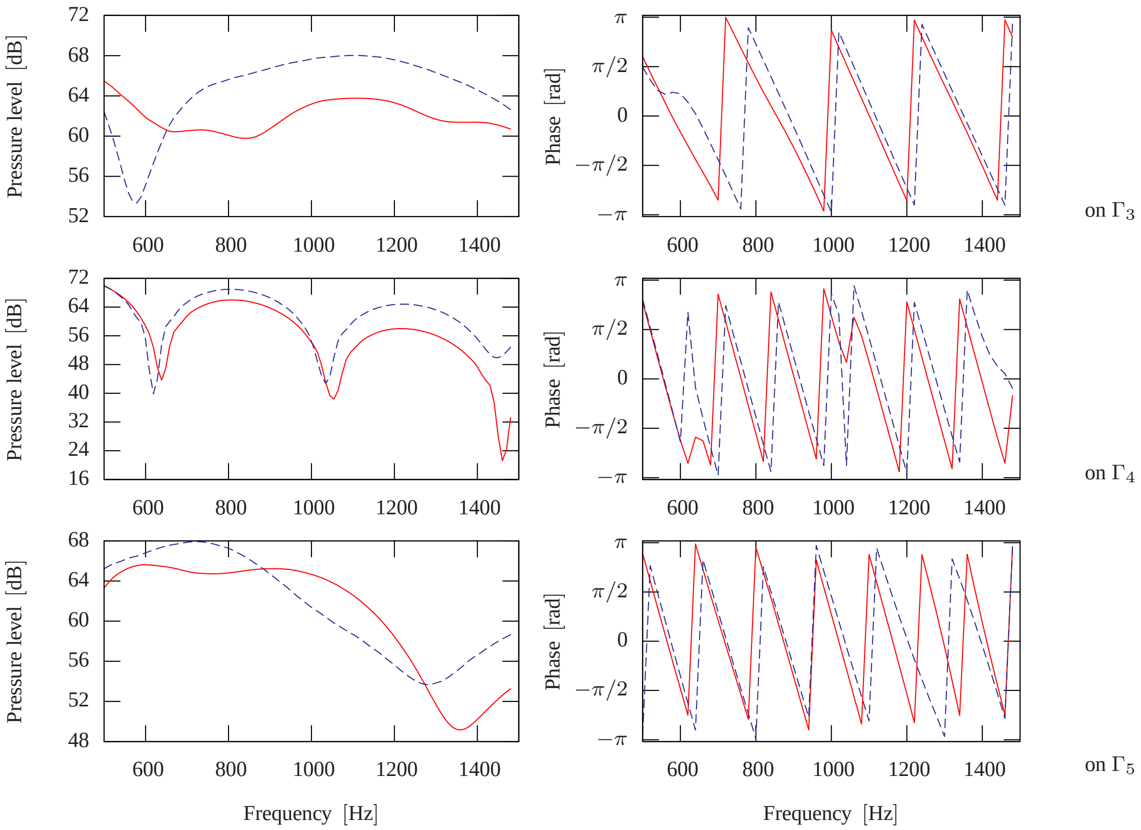

Figure 2. Comparison of the two ways to gain access to part of B: — B_j, -- $g_\theta(S,R_j)$.

It remains to be verified if at the particular point $R(0.17\text{m}, 0.05\text{m})$ observed here the contribution of $\delta g_\theta(S,R) + \delta B(S,R)$ is of sufficient importance to modify $\tilde{g}_\theta(S,R) + \tilde{B}(S,R)$.

To this end, one should compare the contribution of sources acting in the latter term to the total amount of $g_\theta(S,R) + B(S,R)$. As previously stated, it is complicated to correctly account for the interaction of the walls in $\tilde{B}$, since the number of acting sources is too large to be handled without errors. Nevertheless, a comparison can be temporarily accepted, while knowing that the interaction between the walls Γ_a is not accounted for. Within this remark, the compared terms are then

$$\tilde{g}_\theta(S,R) - \sum_{j=3,4,5} \tilde{g}_\theta(S,R_j) \text{ and } g_\theta(S,R) + B(S,R) \tag{17}$$

Figure 3 shows the difference between these two terms, which should be nearing $\delta g_\theta(S,R) + \delta B(S,R)$. It can be seen that a difference is visible in amplitude, but less so on the phase, which is consistent with the observations on Figure 2. Will this small difference in amplitude be enough to modify the global pressure level inside the cavity?

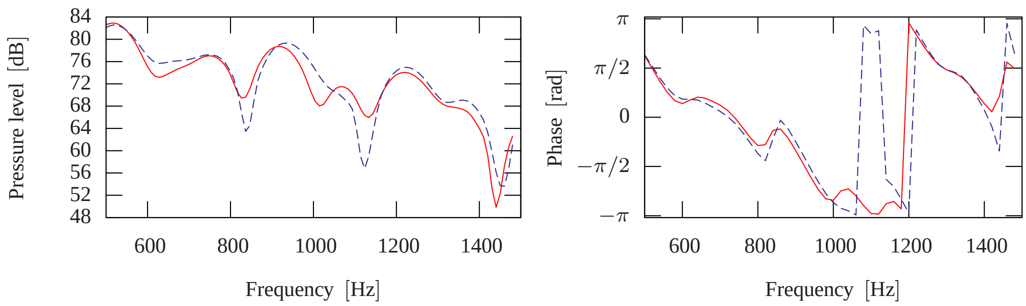

Figure 3. Approximation of $\delta g_\theta(S,R) + \delta B(S,R)$ as the difference between — $g_\theta(S,R) + B(S,R)$ and -- $\tilde{g}_\theta(S,R) - \sum_{j=3,4,5} \tilde{g}_\theta(S,R_j)$

4.3. Graft

To obtain an enhanced version of the specular pressure according to Equation (14), one needs to substract from p^{sp} the sources participating in $\tilde{g}_\theta + \tilde{B}$ so as to gain access to the "non-manageable" part $\tilde{C}$. This value is then added to $g_\theta + B$ (obtained by the integral method) to finally get $p^{\text{sp}^{++}}$. Figure 4 presents the results of this operation for a particular point R. One can see that this process does indeed modify the specular results, but an enhancement is only visible in certain parts of the spectrum. Conversely, for other frequencies, the modified values tend to get further apart from the reference solution. This leads to the idea that a global modification could only be obtained by grafting yet more terms to the specular results. As previously stated, this would remove any interest in the grafting procedure, since the integral method would then be easier to apply.

5. CONCLUSION

Is it possible to enhance the image sources method so that it becomes competitive in the low- and medium-frequency range for vehicle-sized cavities? In the configuration studied here, a partial answer to this question is given: the image sources method can *not* be enhanced, given the not significant modifications obtained. Such an answer is important considering the vast amount of work invested in the implementation of geometrical methods. Further than an actual

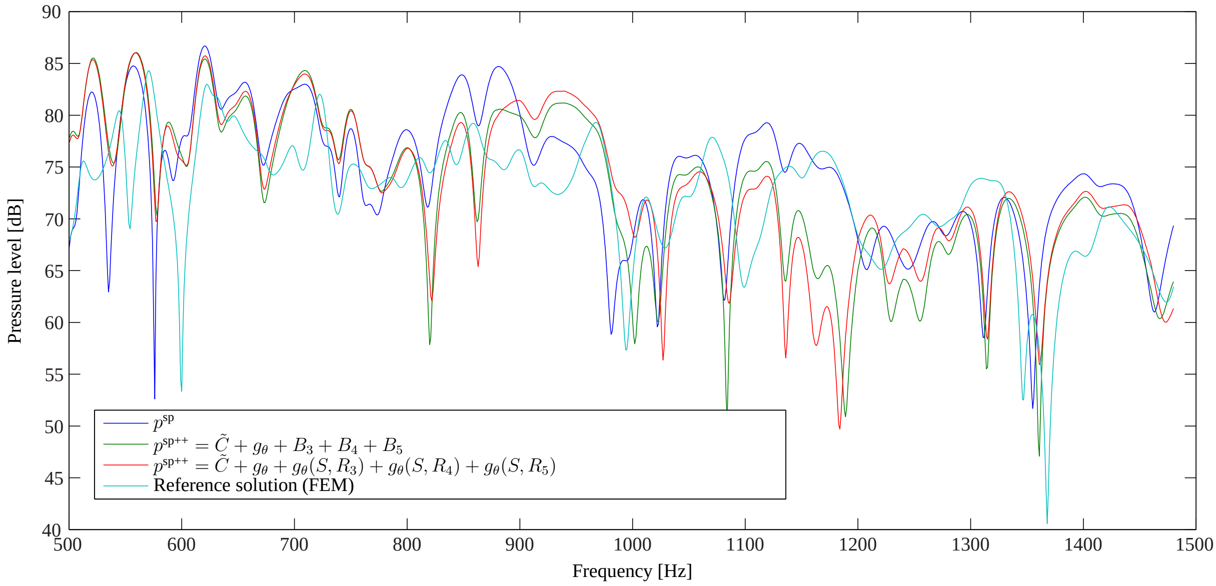

Figure 4. Spectra of $p^{\mathrm{sp}++}(R)$ and $p^{\mathrm{sp}}(R)$ as compared to a reference solution obtained by finite elements.

answer to the particular application problem tackled here, however, the present development of a grafting method is definitely of interest. The specificity of the studied situation does not allow for a general conclusion but for other configurations the answer to the previously asked question is now to be obtained by following the path that has been laid down in this paper.

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