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ADJUSTED COMPLEX POSITION OF EXTRA SOURCES IN THE IMAGE SOURCES METHOD

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Abstract

Using a series development of the integral solution, a formal backing of the presence of image sources in geometrical acoustics methods has been shown. Furthermore, the existence of "invisible" sources is suggested, especially in the vicinity of obtuse angles. These extra sources bring useful information but need to be adjusted in order to avoid divergences.
By more closely comparing the terms of the series development and the corresponding image source, the position of each extra source can be adjusted so as to minimize the distance between term and source contribution. In other words, a correction factor must be applied to the pure geometrical position of the "invisible" sources. In order to obtain the best results, this position may need to be defined as a complex number. This approach could have a companion idea in electromagnetism, where a function is identified with the objective of optimizing the position of the image sources.

INTRODUCTION

Widely used in room acoustics, the geometrical acoustics methods suffer from major limitations making them improper for use in cavities with dimensions nearing the considered wavelengths. The image source method (also called mirror-source or MS method) is known to lead to exact results only in very specific situations, and is not used in complex cases because of the prohibitive number of sources to be computed. Ray-tracing [1], alone or in combination with image sources [2], reduces the number of sources but suffers from additional precision problems. Recent efforts to reduce the number of needed sources provide for a fully usable image sources method even in complex situations, as exposed among other subjects in [3]. However, limitations of two very different origins remain:

1. the assumption of specular reflection on the walls doesn't take the non-locality of wave reflection into account

2. image sources issued from classical algorithms fail to convey the diffraction on the walls

Concentrating on the second shortcoming, we have shown that in nontrivial geometries (such as the vicinity of an obtuse angle), one or more missing image sources cause a loss of information [4]. Moreover, a formal backing of the image sources has been proposed by linking the series of image sources in a given situation to a series development of the exact solution in the same situation. The correspondence between the image sources and the series development terms is validated numerically and the importance of missing sources is highlighted. However, the information added by "restoring" an invisible source must be refined to avoid divergence problems.

Therefore, the position of these extra sources must be adjusted so that their contribution adds the correct missing diffraction term. Before finding a formal correlation between a given situation and the coordinates of the extra sources, a first step is to numerically adjust an extra source to show its impact on the global solution inside a cavity. This paper documents this initial approach, restricted for now in an open sector in 2D with perfectly reflecting walls. Not taking absorption into account reduces the influence of the problems caused by diffusion and so highlights the error caused by missing diffraction that we aim to correct here. The problem is here presented as an open sector for which an exact analytical solution is available. The goal is to ultimately apply the improvements brought by the extra source to a closed cavity problem, but this step will not be discussed in this paper. Furthermore, attention is focused on the pressure values on or close to the walls, since these positions are both of special interest for example for applications where outside radiation of the walls is desired and are geometrically more prone to reveal missing sources problems.

The process of placing image sources at convenient complex coordinates is also used in electromagnetics to convey the effects of a finitely conducting earth near a dipole, see for example [5] and [6].

PROPOSED METHOD

If considering the open sector situation depicted in Figure 1, the pressure in a point Q_1 tending towards wall Γ_1 can be expressed by a discretized form of the integral representation

$$p(Q_1) = 2G_\infty(Q_1, S) + 2\int_{\Gamma_2} p(M)\partial_{n_M} G(M, Q_1)dM \quad (1)$$

with $G_\infty(R, S,)$ noting the 2D Green Function in an infinite plane. Replacing a single point Q_1 by a vector of points on wall Γ_1 and by a series development, the following matrix expression is gained:

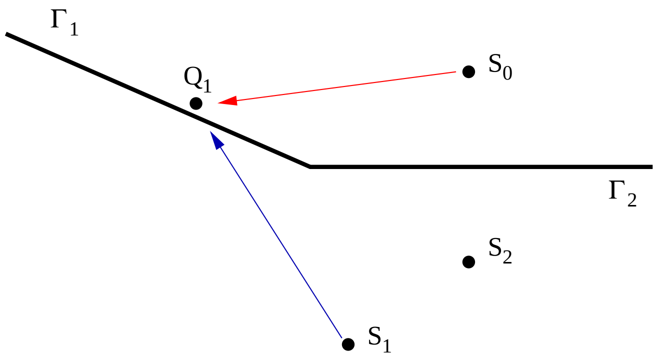


Figure 1: Considered situation in a 2D sector formed by two walls at an obtuse angle. The effect of wall Γ_2 is not visible in the classical image sources method.

$$\mathbf{p_1} = \underbrace{[\mathbf{g_0}(\Gamma_1) + \mathbf{g_1}(\Gamma_1)]}_{\text{terms of } 1^{\text{st}} \text{ order}} + \underbrace{[\mathbf{g_2}(\Gamma_1) + \mathbf{g_{21}}(\Gamma_1)]}_{\text{terms of } 2^{\text{nd}} \text{ order}} + \underbrace{[\mathbf{g_{12}}(\Gamma_1) + \mathbf{g_{121}}(\Gamma_1)]}_{\text{etc...}} \tag{2}$$

with the obtained term vectors $\mathbf{g_{ik\ldots}}(\Gamma_\ell)$ being interpreted as representing the pressure originating from an image source S_ℓ transferred to Γ_i, itself transferred to Γ_k and so on, or in other words as the contributions of a series of image sources S_1, S_2, S_{21} etc. It is to be noted that the term order in Equation 2 refers to the order of the series and does not correspond to the image source rank. The detail of this development as well as numerical experiments showing the concordance of the series development terms and image sources contributions has been presented in [4] and in an upcoming article.

Order of the series development terms

It can be seen from the development presented in [4] that there is no formal link between a certain source contribution and the corresponding series development term, but an intuitive link exists through Huygens' Principle. Numerical experiments (not shown here for the sake of brevity) have shown that there is factual backing to the interpretation used here, at least concerning the first four to five terms of the series. This brings confidence that the term subsequently called T_2 (the third term of the series, numbering starts with 0 to be consequent with the source numbering scheme) is related to the image source S_2 (the image of S_o by the wall Γ_2).

Effects of an extra source

In Figure 1, it is evident that the image source S_2, expressing the effects of wall Γ_2, is invisible in most points on wall Γ_1. By artificially making this source visible, the effects of the presence of Γ_2 are taken into account, but imperfectly (see Figure 4). On the other hand, by adding the contribution of the corresponding series development term (the third term in that case), the obtained result is similar to the exact solution. So in order for the extra source to bring the desired improvements, its contribution must be as close as possible to the corresponding term's. This can be achieved by adjusting the coordinates of the extra source while minimizing the error between the source and corresponding term contributions.

Definition of the error function

The contribution of each source S_i at a receptor point R is computed with the 2D Green function in infinite space

$$G_\infty(S_i, R) = G_\infty(r_i) = -\frac{I}{4}(J_n(kr_i) - IY_n(kr_i)) \tag{3}$$

with the Bessel functions of the first and second kind J_n and Y_n and $r_i = r(S_i, R)$ the Euclidean distance between S_i and R. One can define an error function between such a source contribution and a corresponding term T_i of the series development by using the distance r as a parameter:

$$\epsilon^2(r) = (G_\infty(r) - T_i) \cdot \overline{(G_\infty(r) - T_i)} \tag{4}$$

with $\overline{z}$ noting the complex conjugate of z. The work on the order on terms shows that the next important term in both the image source and terms development series is the third term T_2. Therefore all subsequent optimization procedures have been done with the third term of the series, corresponding to the first invisible source S_2.

In order for the extra source to correctly mimic the corresponding term (together with the extra phase information it contains), the distance r is chosen as a possibly complex number. The concept of a "complex position" may have no physical meaning but can easily be used in a computing algorithm.

First optimization attempt: act upon the distance source-receiver

Optimization of an image source is done by minimizing $\epsilon^2(r)$ over $r \in \mathbb{C}$ starting at r_i. In that manner, an optimized source position is obtained by positioning the extra source on S_{opt} so that $||S_{opt}R|| = r_{opt}$ with r_{opt} being the value of r at a local minimal of $\epsilon^2(r)$ and $S_{opt}R$ is in the prolongation of S_iR. Optimization is done for $i = 2$, for each receptor point R in which we are interested, in that case collocation points on wall Γ_1 since we are interested in wall pressure. Results show that the thus optimized added source adds relevant information in terms of shape (correct phase information), with an error of about 4% in amplitude.

However, doing an optimization on each and every receptor point is not realistic, the goal being an optimized source position for a range of receptor points of a sufficient number to obtain the pressure level on a whole cavity wall. An attempt was then done to find an optimized r_{opt} over several receptor points. But finding a source position so that the source is at a distance r_{opt} from several receptor points is geometrically impossible in most cases. To solve this problem, the optimization had to be done directly on the source coordinates.

Refinement of the optimization: act upon the coordinates of the extra source

$\epsilon^2(r)$ is now minimized over $x_{S_i} \in \mathbb{C}$ and $y_{S_i} \in \mathbb{C}$ with $r(x_S, y_S) = \sqrt{(x_R - x_S)^2 + (y_R - y_S)^2}$. This provides a set of complex 2D coordinates for which the position of the extra source is optimized. By defining a compound error function

$$\epsilon_k^2(r_1, r_2, ..., r_\ell) = \sum_{k=1}^{\ell} \epsilon^2(r_k) \tag{5}$$

it is possible to obtain an optimized source for several receptor points R_1 to R_ℓ. Numerical experiments show that two points on the receptor range are enough to obtain a good compromise over all wall receptor points (see Figure 2(b)). Best results are obtained when the two chosen points are equally distributed over the receptor range ($1/3$ and $2/3$ of the length).

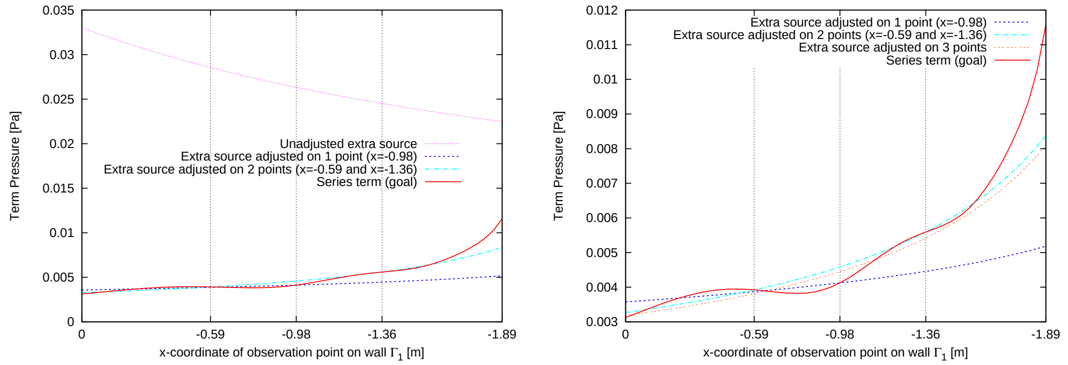
NUMERICAL EXPERIMENTS

For the experiments shown here, the configuration of an open sector in 2D, similar to Figure 1, is used. The aperture angle between the two walls is chosen as $\theta = 5\pi/8$. Coordinates of points are given in meters, relatively to the origin which is at the walls intersection. Both walls have a length of 5 m and are seen as perfectly reflective. The primary source S_0 is at coordinates $(-0.7, 3.8)$ and has an unary amplitude of 1 Pa. The "invisible" source under investigation here is S_2, making the presence of wall Γ_2 having no effect in the conventional image source method. The reference solution used here is an integral solution of this problem, exact except for the finiteness of the walls and discretization. The series development of this solution gives the series of terms which are used to optimize the placement of $S_{2_{opt}}$.

After it had been made clear that the optimization had to be done on the coordinates of the extra source, a first test was needed to know for how many points in the receptor range the optimization has to be done. Figure 2 shows the term contribution to be approached on wall Γ_1 for a fixed frequency. Figure 2(a) shows the difference between the desired contribution and that of the extra source S_2 before optimization, that is at the position obtained by reflecting S_0 on Γ_2. Figure 2(b) is a close-up showing that the differences are small when optimizing on more than two points.

Figure 3 shows the total obtained pressure on wall Γ_1 for a fixed frequency of 500Hz. It can be seen that the non-optimized extra source provided missing information (the oscillation), albeit imperfectly. Mostly, an important phase-shift is observed. The optimized source, on the other hand, brings a correct phase information.

Figure 4 shows the pressure spectrum on a particular point on Γ_1. Again, the correction on the coordinates of the extra source removes the phase-shift. The position for which the spectrum is shown is between the two points for which the optimization is done. Other positions on the wall give similar results, with the exception of the walls extremities, for which the exact solution is of discutable accuracy since it forsees walls of infinite length.



(a) *With extra source at unadjusted coordinates* (b) *Zoomed-in graphic*

Figure 2: Adjustment of the coordinates of the extra source. Values on the receptor points (wall Γ_1) for a fixed frequency of 500Hz.

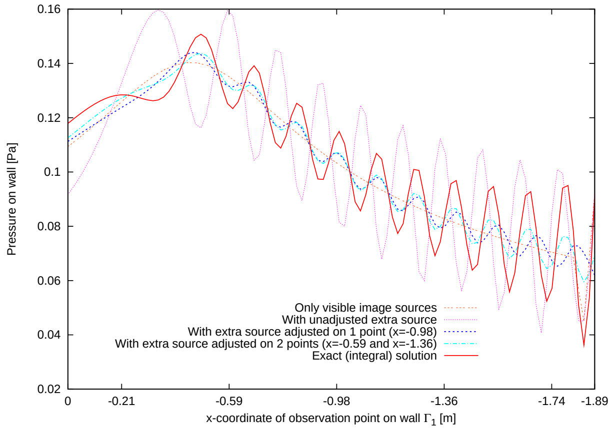


Figure 3: Pressure at 500Hz on every point of Γ_1

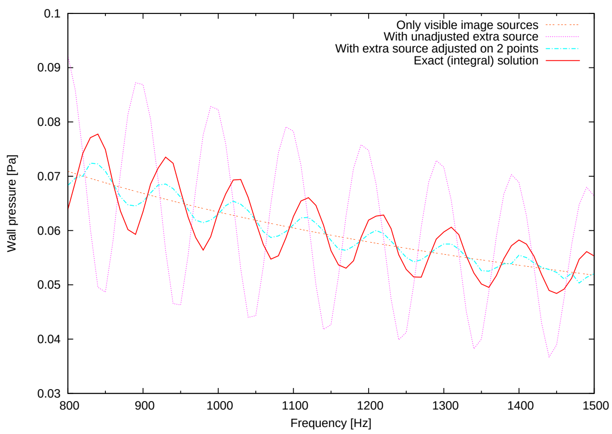
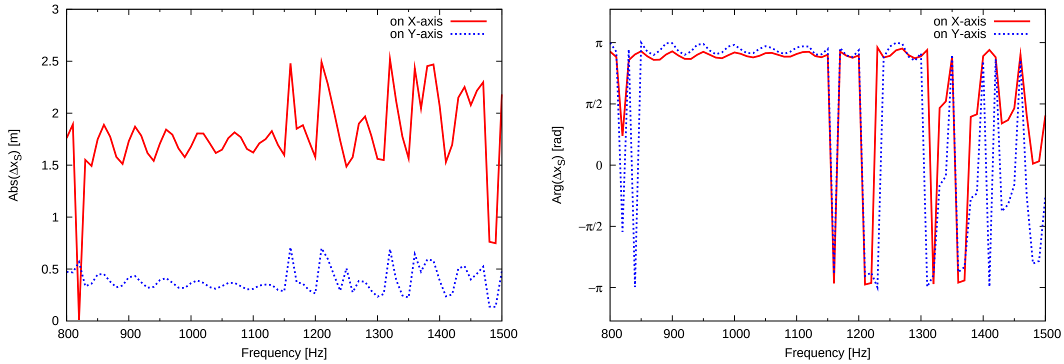


Figure 4: Spectrum on a point $(-1.24m, 3.00m)$ on wall Γ_1

For the results shown here, the optimized source coordinates have been computed once for each frequency. Figure 5 shows the variations of the coordinates in function of the frequency. Apart from an oscillatory behavior (probably due to the fact that the optimization is done by finding a local minimum on an oscillating function), the values are relatively constant over the frequency. Computing an optimized source for a whole range of frequencies is therefore possible.



(a) *Absolute value* (b) *Argument*

Figure 5: Variation of the adjusted extra source coordinates in function of the frequency.

CONCLUSIONS AND OUTLOOK

As shown by prior work, missing image sources in the vicinity of obtuse angles result in incomplete pressure information on certain receptor points, in particular on the walls. Restoring the visibility of these missing sources provide the missing information, but with important errors (phase shifts). By situating the extra sources on complex coordinates optimized so that their contribution is close to their corresponding series development term, the missing information is provided with far less errors.

The results shown here are to be understood as the first exploratory steps in the possibilities offered by optimized extra sources in ray-tracing. It is still in the very early stage and needs to be further developed in order to be usable inside an existing image sources software. The following remarks will be an outlook into what remains to be done.

- Computation is to be done with absorbing walls, characterized by a – possibly complex – wall impedance. Formal development to take finite impedance into account is available. However, this primary work had to be done with perfectly reflecting walls because it was sought to separate the effects of diffraction (occurring, it is believed, through the missing image sources corrected here) from those of diffusion due to specular reflexion.

- Optimization is done once for each frequency, but Figure 5 hints that it is possible to compute an optimized source position for a certain range of frequencies. It is however expected that the differences may be less stable when the walls have a finite impedance.

- The most obvious drawback of this method is that it requires an *a priori* knowledge of the exact solution. This solution is only available for open sector situations, as depicted here. To be usable in a closed cavity situation, the problem should be addressed for every obtuse angle present in the cavity. Eventually, a purely geometrical relation between situation, frequency and source position is sought, so that an algorithm is able to rapidly compute the positions of the needed extra sources. A possibility to obtain this relation could be to compute extra source positions for a certain raster of situations and then interpolate between these values for the situation at hand. A similar work has been done by the present authors when addressing the problem of specular reflexion [7].

Work is currently being done to implement this method inside an existing 2D image sources software.

Acknowledgement

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