Plasticity in soil-structure interaction applied to cut-and-cover tunnels

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ABSTRACT: Cut-and-cover tunnels behavior at ultimate limit state depends strongly on the interactions between the foundation soil, the backfill and the reinforced concrete structure. Characterization of the potential failure modes of these types of structure necessitates taking into account every major mechanical property of each components. The influence of the structure plastic behavior on the ultimate limit state of the soil-structure system is discussed through a basic case study of soil mechanics. The ideal shape of embedded arches is also discussed.

Keywords: cut-and cover tunnels, soil-structure interaction, concrete behavior, ultimate limit state, structural design

1 INTRODUCTION

There are some uncertainties about the behavior of cut-and-cover tunnels at the ultimate limit state. This is mainly due to the complexity of cut-and-cover tunnels composed of soil, reinforced concrete and backfill material (see Fig. 1). The presence of many interfaces and interactions between these components make the modeling and analysis processes of the overall system difficult.

Most currently used calculation methods are based on the assumption that the structure behaves in a linear elastic manner. This assumption, acceptable at serviceability limit state, is no longer valid at ultimate limit state because it is characterized by important cracking and yielding of parts of the reinforced concrete structure. These phenomena may be accompanied by large stress redistribution in the surrounding soil and in the structure and thus have significant effects on the global behavior and bearing capacity of the structure.

1.1 *Research Project*

A research program currently under way at the Structural Concrete Laboratory at the Swiss Federal Institute of Technology in Lausanne aims at developing a rational analysis and design method taking into account the actual behavior of the materials and allowing a better understanding of the tunnel behavior at ultimate limit state. One of the main objectives of the research is to identify the controlling design criteria which govern the structural design of cut-and-cover tunnels.

Identification of the actual failure mechanisms will allow a better estimation of the loads acting on the structure at ultimate limit state and thus lead to a rational and consistent design procedure. A better evaluation of the level of safety will allow an optimization of the geometry of the cross section and more efficient design.

The paper presents the first steps of the research. The effects of the structure plastic behavior on ultimate limit state of a soil-structure system are first discussed through a basic case of soil mechanics: the bearing capacity of a perfectly rough strip footing on weightless and purely cohesive soil. On a more general level, the ideal shape of a three pinned embedded arch is also investigated and the main parameters identified.

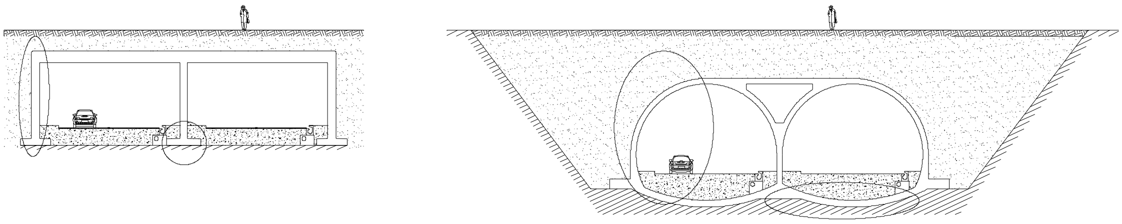

Figure 1. Soil-structure interactions in cut-and-cover tunnels

2 BEARING CAPACITY OF A STRIP FOOTING ON A WEIGHTLESS AND PURELY COHESIVE SOIL SUBJECTED TO A CENTERED LOAD

2.1 *Footing of infinite strength*

In order to illustrate the influence of the plastic behavior of the structure on the ultimate limit state of a soil-structure system, the bearing capacity of a perfectly rough strip footing on a weightless and purely cohesive soil is discussed. The footing is subjected to a concentrated load acting at the center of the foundation.

We first consider that the footing strength is infinite. The soil is assumed to be elastic perfectly plastic and to be governed by the Mohr-Coulomb yield criterion. The associated flow rule is assumed to apply.

This problem has been intensively investigated in the past (Terzaghi, Prandtl, Hill, …). The failure mechanism given by Prandtl mechanism is shown in Figure 2a. It is possible to determine the ultimate load of the footing under this mechanism using the upper bound method of limit analysis [1,2]. The corresponding ultimate load is given by Equation 1. It is important to observe that under this mechanism energy dissipation occurs only in soil.

$$Q_R = 5.14 \cdot b \cdot c \tag{1}$$

where Q_R = ultimate load, b = width of the footing and c = cohesion of soil.

Based on the upper bound theorem of limit analysis, this solution is an upper bound of the bearing capacity of the footing. It can be shown that this ultimate load is actually the exact solution [1,2]. This means that an equilibrium stress distribution can be found in the overall body which balances the applied load on the boundaries and does not violate the yield criterion of the materials anywhere in the body (lower bound theorem of limit analysis [1]).

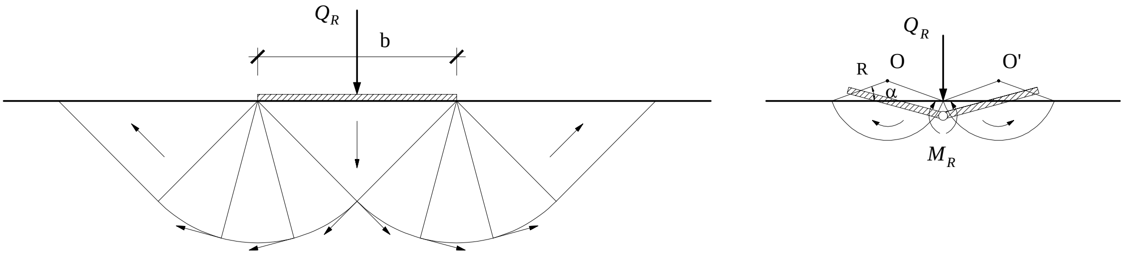

Figure 2. a) Prandtl failure mechanism. b) Mechanism with development of a plastic hinge.

2.2 *Footing of finite strength*

We consider now a foundation of finite strength assumed to be elastic perfectly plastic.

Assuming a uniform contact stress distribution under the footing at ultimate load, the maximal bending moment in the foundation M_{max} under the load defined by 1 is given by Equation 2 below.

$$M_{\max} = \frac{5.14 \cdot c \cdot b^2}{8} \tag{2}$$

If the foundation is not able to resist this bending moment, i.e. that the foundation bending strength M_R is smaller than M_{max}, the solution given by Prandtl mechanism is no longer the exact solution. Indeed, the yield criterion is violated in the foundation itself.

Figure 2b shows a kinematically admissible mechanism that assumes the development of a plastic hinge in the foundation. The plastic hinge imposes a rotation to the foundation and corresponding displacements to the soil. It is here assumed that two failure surfaces are created in the soil. The circular areas delimited by these lines of discontinuity are supposed to rotate in a rigid block motion around points O and O'. Slip occurs at the soil-foundation interface. In this case, energy dissipation not only occurs in the soil and at the interface, but also in the plastic hinge of the footing.

Using the equation of virtual works, it is possible to determine the ultimate load corresponding to the selected mechanism in function of the two variable parameters R and α (see Fig.2). After minimization in function of R and α, the ultimate load is found to be of the form of Equation 3 below.

$$Q_R = K \cdot \sqrt{\mu} \cdot b \cdot c \tag{3}$$

where $\mu = M_R / (b^2 c)$, K is a parameter depending on the interface properties. For a perfectly rough footing, $K = 6.99$.

The development of the plastic hinge induces the rising of a part of the foundation and thus a decrease of the contact surface between the structure and the soil. Consequently, the contact stresses are redistributed to a reduced area under the applied load.

2.3 *Comparison and consequences for design*

The comparison of the ultimate load of the footing predicted by both mechanisms is shown in Figure 3.

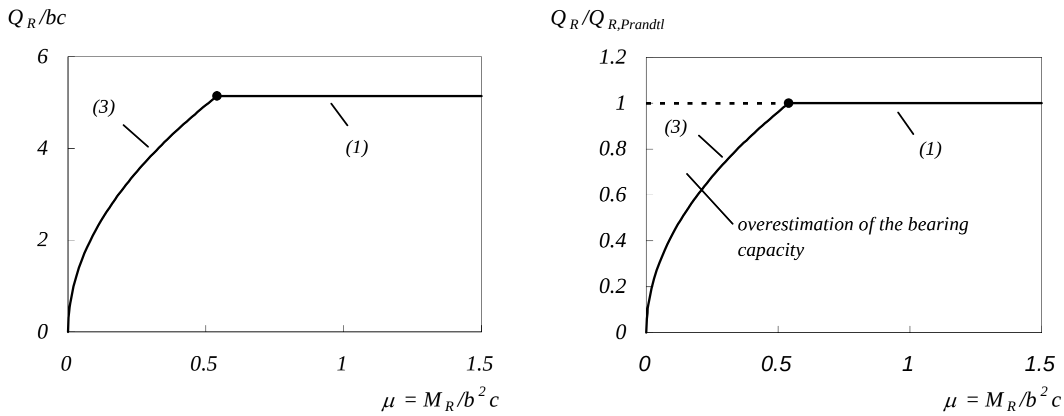

Figure 3. Bearing capacity of the footing in function of the bending strength of the footing. a) Comparison between relationships 1 and 3. b) Loss of bearing capacity in comparison to Prandtl ultimate load.

The bearing capacity of the foundation is influenced by the plastic behavior of the footing if its bending strength is lower than a threshold value. In that particular case, the development of a plastic hinge in the foundation modifies the kinematics of the failure mechanism. The failure mechanism given in Figure 2b is more likely to occur than Prandtl mechanism (Fig. 2a). Figure 3b shows the overestimation of the bearing capacity of the footing made when assuming an infinite strength of the footing. It is seen that this overestimation can become significant for low bending strengths.

The point of intersection of the two curves on Figure 3a corresponds to an optimal design for the ultimate limit state. The soil and the structure strength capacities are both fully exploited.

This basic example illustrates the potential importance of the foundation behavior on the ultimate limit state of a system composed of soil and structural elements. Plastic behavior of both reinforced concrete and soil should thus be considered in analysis, and not separately as it is generally done today. A common verification should be performed when coupling effects cannot be excluded. In the example discussed here, this is true when the foundation bending strength is below a threshold value.

3 IDEAL SHAPE OF AN EMBEDDED ARCH

3.1 *Shape of the cross section*

Besides the actual behavior of the different materials, the shape of the tunnel cross section may have significant effects on the general behavior of the structure. In practice, various cross sectional shapes are used: circular or rectangular sections, mono tube, twin or multi-tubes tunnels. Although the choice of the cross sectional shape is mainly governed by practical and economical aspects, the designer has the opportunity to adjust the shape of the structure according to static considerations.

3.2 *Ideal shape of a three pinned arch*

The ideal shape of cut-and-cover tunnel will be first discussed through the case of a three pinned arch submitted to earth load only. The ideal shape corresponds to the funicular polygon of the applied loads. The search of this shape is of particular interest because it leads to a structure submitted only to axial compression forces. This shape used to be a major concern at the time of masonry construction to ensure the required stability of the structures [3,4]. Although this aspect is of lesser importance for reinforced concrete structures, this shape usually leads to more efficient designs.

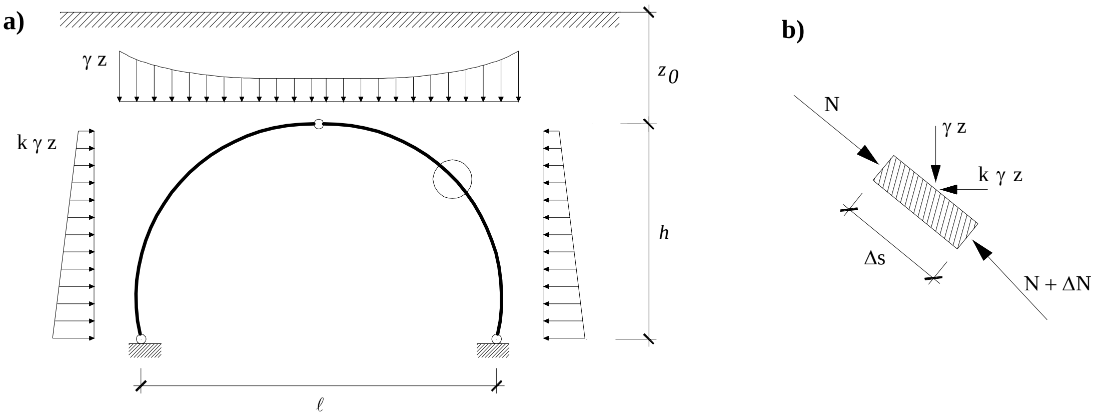

Figure 4. a) Loads acting on the embedded arch. b) Forces acting in an infinitesimal arch element.

Because the three pinned arch is statically determinate, the ideal shape of the arch can be obtained directly from equilibrium formulation. The arch is assumed to be submitted to both vertical and horizontal distributed loads which correspond to the state of stress in the surrounding soil as shown in Figure 4a. The vertical pressure is given by the soil unit weight times the depth. The horizontal pressure is related to the vertical stress by Equation 4.

$$\sigma_h = k \cdot \sigma_v = k \cdot \gamma \cdot z \tag{4}$$

where k = earth pressure coefficient, γ = soil unit weight, z = depth considered

The equilibrium formulation of an infinitesimal arch element (Fig. 4b) leads to the governing differential equation of the arch submitted to earth load given by Equation 5 below.

$$z''\cdot\left(\frac{H_0}{\gamma}+\frac{k}{z}\cdot z_0{}^2-\frac{k}{2}\cdot z^2\right)-k\cdot z'^2\cdot z-z=0 \quad (5)$$

where H_0 = horizontal force acting in the structure at the crown of the arch, z_0 = earth cover and z = position of the arch in function of the x axis.

For given H_0 and z_0, the solution of Equation 5 has been computed by numerical methods. The ideal shape of the arch depends on its dimensions (h and ℓ on Fig. 4), on the earth cover and on the earth pressure coefficient. Figure 5 shows ideal curves for a typical section and for various values of k.

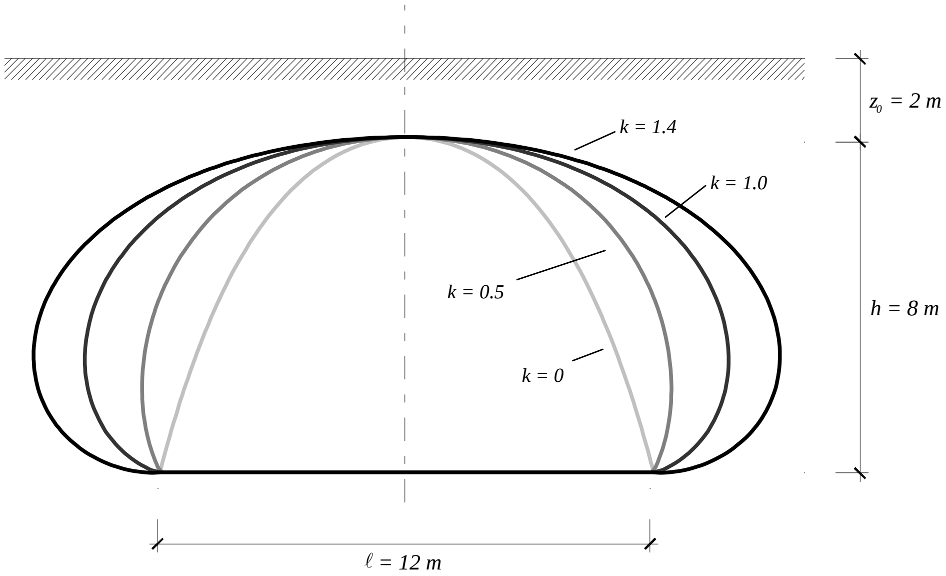

Figure 5. Ideal shape for a typical section (z_0 = 2 m, h = 8 m, ℓ = 12 m) and for various values of the earth pressure coefficient.

The choice of the shape of the structure should be chosen in accordance to the ideal shape in order to minimize bending in the structural element and consequently the amount of steel reinforcement.

3.3 *Influence of the earth pressure coefficient on the ideal shape*

The earth pressure coefficient is of particular interest for cut-and-cover tunnel. Indeed, this coefficient depends directly on the backfill material used and on the degree of compaction chosen. It can be seen on Figure 5 that this parameter plays an important role in the behavior of the structure. For a given shape, an error in the estimation of the earth pressure coefficient leads to an important displacement of the funicular polygon from its assumed position and thus to important bending of the structure.

This influence is not constant and uniform however, but depends on the earth cover and on the dimensions of the tunnel. For example, Figure 6 shows the ideal shape of a structure under both active (k_a) and at rest (k_0) conditions for a typical soil (friction angle φ = 30°) and for two different earth covers. The maximal distance between the two curves decreases when the earth cover increases. Consequently the influence of the earth pressure coefficient on the ideal shape also decreases.

The shape of the structure should thus be determined accordingly to the actual earth pressure coefficient and to the chosen compaction process. The ideal curves computed here are based on the assumption that the horizontal stress distribution is linear and related to the vertical stress by means of the earth pressure coefficient and that no other load is acting on the structure. In the case of cut-and-cover tunnels the horizontal stresses can however be considerably affected by the construction process. The contact stresses against the structure will depend on the interaction between the backfill material and the structure during the different construction stages.

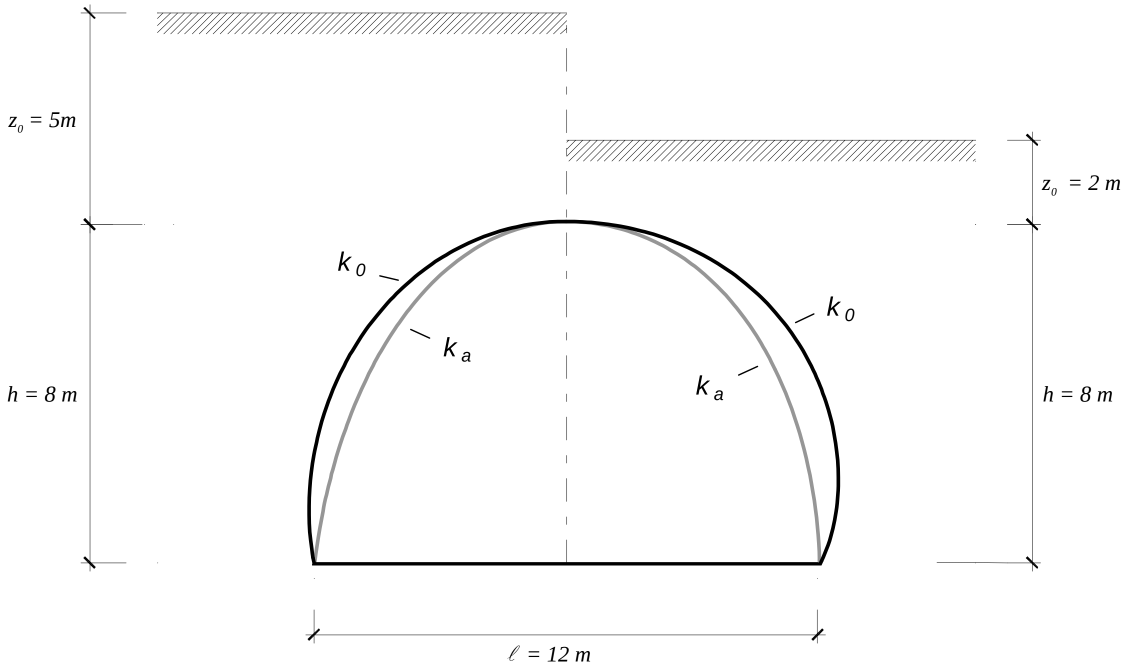

Figure 6. Ideal shape for two different earth covers (z_0 = 2 m and 5 m).

4 FUTURE WORK

The ideal shape of a three pinned arch constitutes the first step of a more general study on embedded arches. Euler' buckling strength and second order effects will be investigated for the three pinned arch and also the more realistic case of fixed arches subjected to various load cases. Special consideration will be given to the consequences of the development of plastic hinges on the stability of structures. Finally, particularities of cut-and-cover tunnels will be considered and intensively investigated, i.e. the construction stages and the interface properties.

5 CONCLUSIONS

The study of the basic case of the bearing capacity of a perfectly rough strip footing on a weightless and purely cohesive soil subjected to a centered load emphasizes the consequences of the structure plastic behavior on a soil-structure system at the ultimate limit state. It was found that the bearing capacity can decrease considerably if the footing bending strength is below a threshold value. A failure mechanism which takes into account the development of a plastic hinge in the footing has been presented. The corresponding kinematics differs considerably from the well known Prandtl mechanism.

The ideal shape of an embedded three pinned arch has also been presented. This shape depends on the dimensions of the arch, the earth cover and the earth pressure coefficient. The influence of the latest was found to be very important. Static considerations based on the lateral backfill properties and compaction process should play a major role in the selection of the shape of a cut-and-cover tunnel.

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