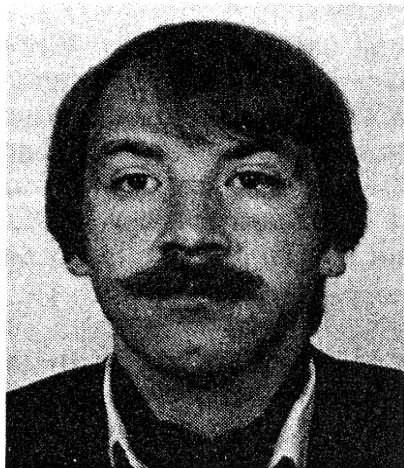# Behaviour of Beams and Punching in Slabs without Shear Reinforcement

## Comportement des poutres et poinçonnement des dalles sans armatures d'effort tranchant

## Tragverhalten von Balken und Durchstanzen von Platten ohne Schubbewehrung

**Aurelio MUTTONI**
Dr. sc. techn.
Grignoli, Martinola, Muttoni
Lugano, Switzerland

Aurelio Muttoni graduated in Civil Engineering at the Swiss Federal Institute of Technology in 1982 and subsequently undertook research on the applicability of the theory of plasticity to the design of reinforced concrete structures, obtaining his Ph. D. degree in 1989.

**Joseph SCHWARTZ**
Dr. sc. techn.
Frey, Consult. Eng.
Zug, Switzerland
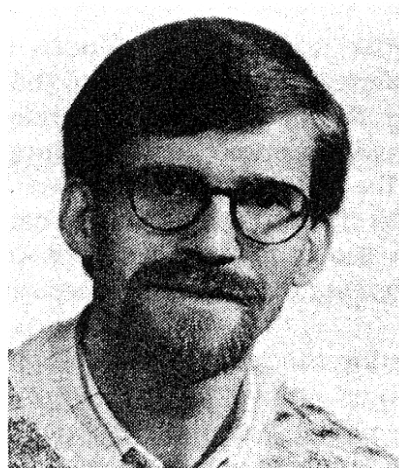
Joseph Schwartz graduated in Civil Engineering at the Swiss Federal Institute of Technology in 1981 and subsequently undertook research on the stability of masonry and reinforced concrete members, obtaining his Ph. D. degree in 1989.

## SUMMARY

The cause of shear failure is investigated in beams and slabs without shear reinforcement. The main parameters influencing the structural behaviour are discussed. A theoretical model for the determination of the punching strength as a function of different parameters including size effect is proposed.

## RÉSUMÉ

On analyse ici la cause de la rupture par cisaillement des poutres et des dalles sans armature d'effort tranchant. Un modèle théorique visant la détermination de la résistance au poinçonnement est proposé en traction de différents paramètres dont l'effet de taille.

## ZUSAMMENFASSUNG

Die Ursache des Schubbruches von Balken und Platten ohne Schubbewehrung wird untersucht. Die Hauptparameter, welche das Tragverhalten beeinflussen, werden diskutiert. Ein theoretisches Modell zur Berechnung des Durchstanzwiderstandes in Funktion verschiedener Parameter einschliesslich «size effect» wird vorgeschlagen.

# 1. SHEAR ACTION IN BEAMS WITHOUT SHEAR REINFORCEMENT

The investigation begins with the consideration of the simple beam shown in Fig. 1. The reinforcement consists of longitudinal bars, which are completely anchored behind the supports. Not taking into account the tensile strength of the concrete and without shear reinforcement (stirrups) the only possible structural model is the so-called direct support, i.e. between the load and the reaction force a compression zone resembling a concrete strut is formed. The stress field proposed by Drucker [1] to describe this load bearing action is shown in Fig. 2. It is the solution according to plasticity theory.
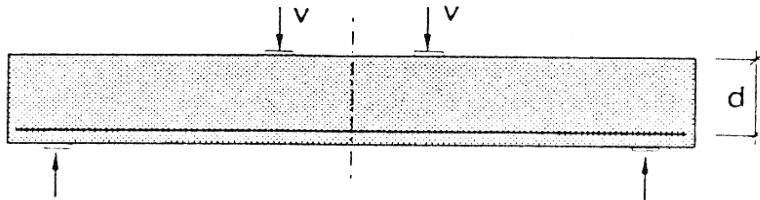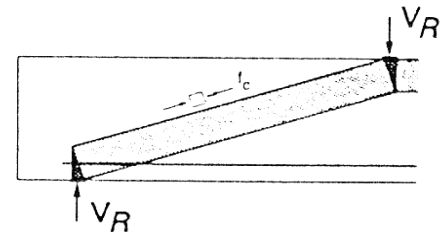
Fig. 1 Beam and loading                                   Fig. 2 Stress field

This stress field can under some circumstances deviate greatly from the actual structural action. While increasing the load V monotonically from zero to the level of the failure load $V_R$ several phases of load bearing action may be recognized. After cracks due to bending develop, the load bearing action begins to deviate from that of the elastic model which presupposes homogeneous behaviour. If one assumes that there is no load transfer over the cracks then the new internal load bearing action can be described by the stress resultants shown in Fig. 3b. This load bearing model was already recognized by Kani [2] and described as follows: *"Due to the transverse cracks the tension zone is divided up into seperate concrete elements, which can be visualized as cantilever beams fixed in the upper compression zone"*.

Further possible load bearing models are sketched together with the stress resultants in Figs. 3c and 3d. In the case of the structural model shown in Fig. 3c concrete compressive forces are assumed to be transferred across the cracks. This is possible as the roughness of the cracked surface causes an interlocking behaviour. Fig. 3d shows a structural model in which the longitudinal reinforcement also transmits forces in the transverse direction (so-called dowelling action). The effective stress field results from the combination of the three load bearing models mentioned, as also Hamadi and Regan [3], Reineck [4] and others have shown.

(a)          (b) "cantilever action" (c) "interlocking action" (d) "dowelling action"
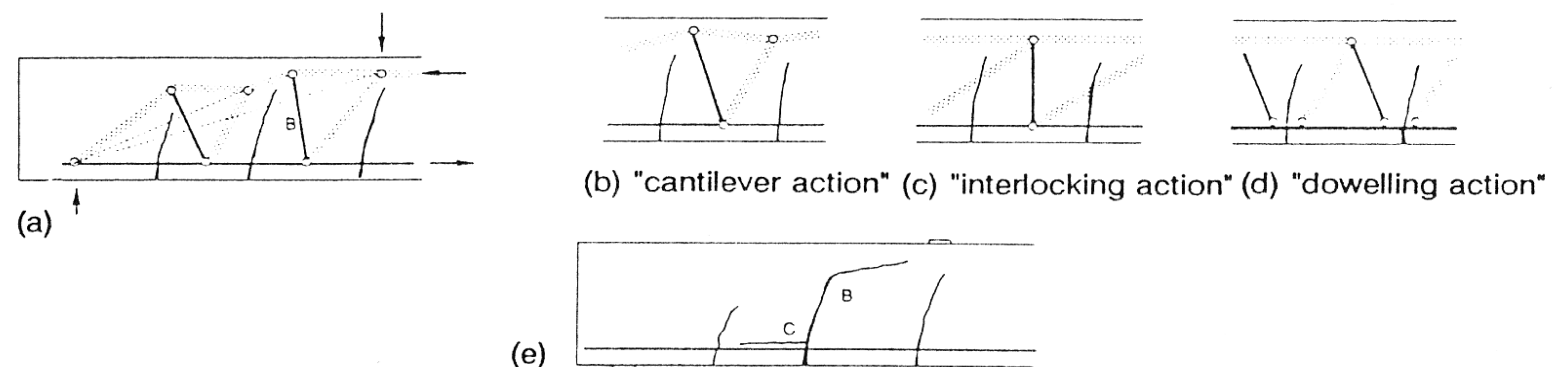
(e)

Fig. 3 Structural models after bending cracks develop with subsequent crack propagation

If the load is increased further the tensile strength is reached in region B (Fig. 3a). The original bending crack running almost vertical then propagates in a more or less horizontal direction. A typical crack pattern is shown in Fig. 3e. The propagation of this crack is accompanied by a collapse of the three abovementioned structural models. The cantilever and interlocking models lose their load bearing capacity because the concrete ties at B are broken. The dowelling action also becomes ineffective because a crack forms at C (Fig. 3e).

The new structural model, which could now become effective, is the plasticity solution already mentioned, i.e. the direct support model. Fig. 4a shows the crack pattern just prior to failure in a beam tested at the Institute of Structural Engineering at the Swiss Federal Institute of Technology, Zurich [5]. In this figure the stress field describing the direct support model has been introduced. The dimension of the strut has been determined

from considerations of equilibrium and full exploitation of the concrete strength $f_c$. It is clear that the main crack passes through the theoretical strut. The possibility of force transfer across the crack depends on the relative displacements of the crack boundaries and the roughness of the crack surface. The relative displacement vectors measured during the test are shown in Fig. 4b for two load steps $V = 0.87 \cdot V_R$ and $V = 0.95 \cdot V_R$. These displacements may be compared with the relationships between crack opening u and movement along crack v given in Fig. 4c. The curves represent the displacement vectors at which contact between the two crack boundaries is just no longer possible and thus force transfer ceases. They were measured on specimens, which were broken into two parts after being cut out of the test beam. A comparison of the two figures shows that practically no more force had been transferred over the crack in the test beam.
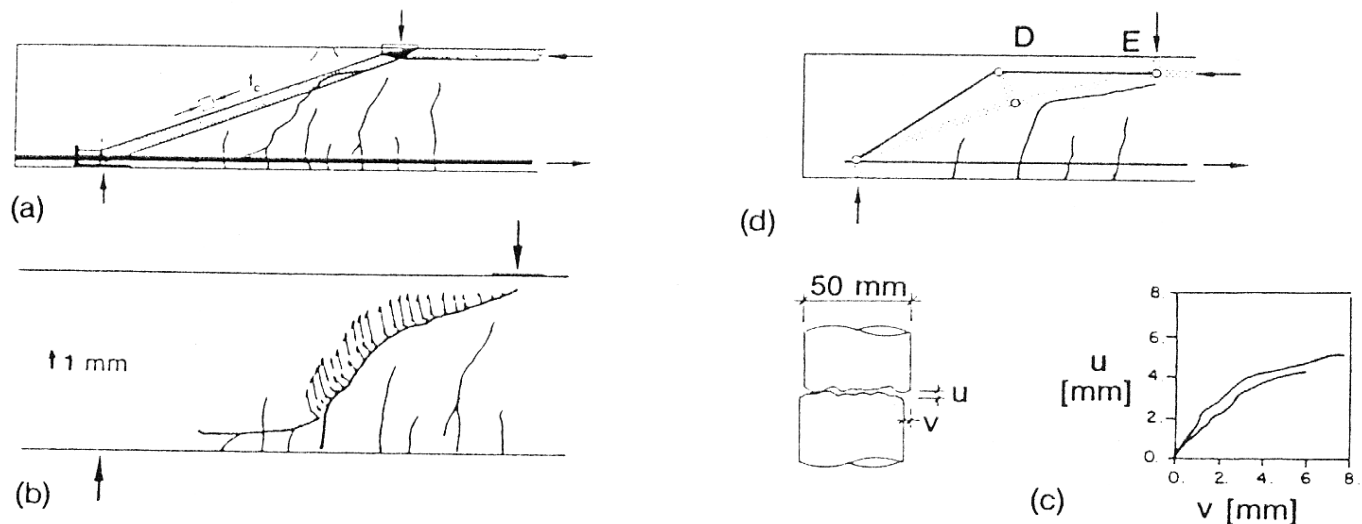
Fig. 4 Structural model; relative displacement of the critical crack

Therefore a stress field with a concrete strut running directly between the load and the support reaction is not possible. Such a strut is only admissible if it is deviated in the central region (Fig. 4d). The concrete strut acts together with a concrete tie, that also exhibits a deviation in the central part of the beam, so that the deviation forces can balance each other out. Failure occurs because the tensile strength in the region D is reached or because the strength in zone E, which is subjected to both tension and compression, is exceeded. Overstressing in this zone can be caused by the crack which propagates to the zone where the load is applied.

It may be easily seen that the load capacity of the stress field (with a bent strut) shown in Fig. 4d is smaller than that given by the plasticity theory (with a straight strut). Therefore the failure load is very dependent upon the crack pattern. Thus there results, especially for tests on small beams, a big scatter in the experimentally determined failure loads. Since the crack pattern depends on the initial state of stress and the load history, the failure load is also influenced by these factors. For this reason a theoretical estimate of the failure load is always encumbered by great difficulties and uncertainties. The position of the critical crack and thus the load capacity depends greatly on a number of parameters, for example slenderness of beam, geometry and axial force. These influences are described in [6].

The correlation between crack pattern and failure load indicates the possibility of improving the structural performance and increasing the failure load by favourably influencing the crack pattern by means of constructional measures. The zone where the strut corresponding to direct support should be formed can for example be kept free of cracks by prestressing the beam such that the load is supported directly from the beginning. This explains why prestressed beams exhibit a greater shear resistance than non-prestressed reinforced beams. A similar improvement in performance is evident when the reinforcment in the region subjected to shear action is not bonded to the concrete. Fig. 5 shows the crack patterns and the stress fields for two tests which differ only in the quality of the bonding. For beams having bond-free reinforcement or with a poor quality of bonding cracks form in the central part. As shown in Fig 5b the inclined strut is hardly affected by these cracks, so that the failure load can be as much as twice that of beams with good bonding quality (Fig. 5a). A further increase of the failure load can be achieved by placing reinforcement in the critical region. Fig. 6 shows the arrangement of reinforcement and the crack pattern at failure of a test reported by Muttoni and Thürlimann [6]. Wide cracks were only obtained in the unreinforced region, which allowed the direct support system to develop. As a result the load could be increased up to the point of yielding of the

longitudinal bars. The failure load was about double that of a similar beam which did not have reinforcement in the region of the concrete strut (Fig. 4a).
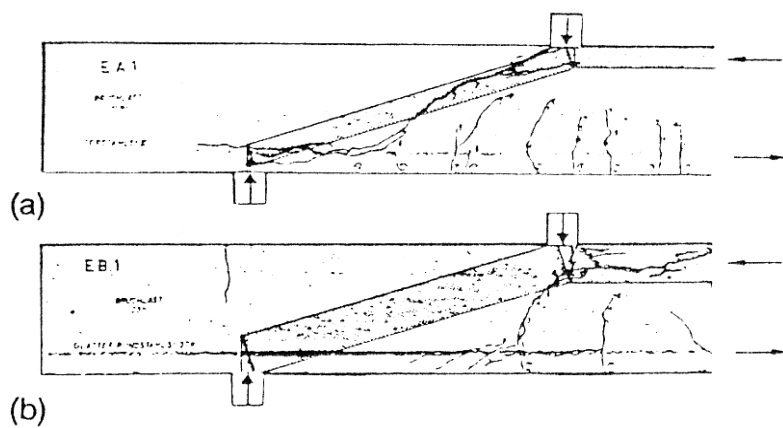


(a)

(b)

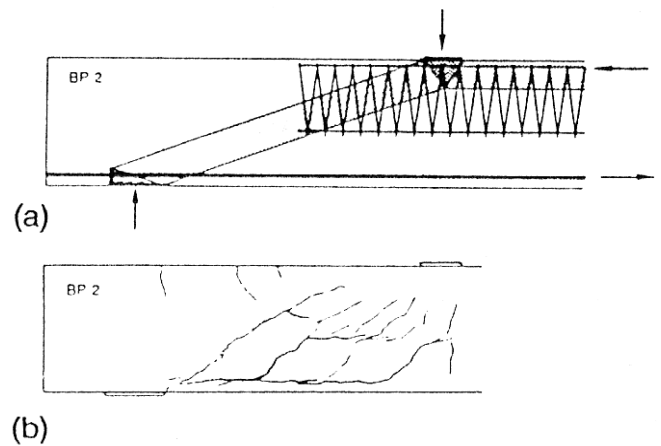Fig. 5 Influence of bonding



(a)

(b)

Fig. 6 Influence of reinforcement in the critical zone

## 2. PUNCHING SHEAR

With only small modifications the considerations concerning the internal structural behaviour of beams may be applied to the slab element shown in Fig. 7. Fig 8a shows the position of the stress resultants after tangential cracks have developed.
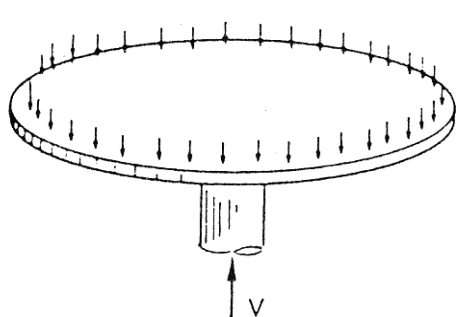


Fig. 7 Slab specimen



(a)                                    $V_R = 76.1$ kN    (b)                                    $V_R = 44.0$ kN

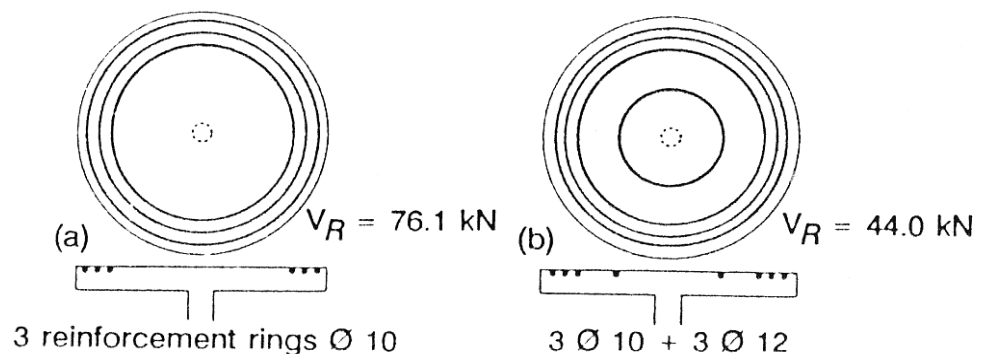3 reinforcement rings Ø 10          3 Ø 10 + 3 Ø 12

Fig. 9 Punching shear tests with ring reinforcement

As numerous punching shear tests have shown the curvatures in the radial direction are concentrated in the region of the columns, so that ring-shaped concentric bending cracks are only obtained in this region. In the other zones, however, only radial cracks are present. Since shear force is not transferred in the tangential direction, the stress state is not influenced by these cracks. In the region of tangential cracking a part of the shear force can be resisted by the interlocking action at the crack boundaries and by the dowelling action of the reinforcing bars (see Fig. 3). From Fig. 8a it is evident that the horizontal components of the compression and tension diagonals can be resisted by the reinforcement in the radial direction, by the radial compression zone and also by the deviation of the tangentially directed forces in the reinforcement and in the compression zone. On increasing the load the strength of the concrete tie is reached at A, so that the bending crack is propagated in the horizontal direction. The structural action that then results is similar to that obtained in the beam. As no force can be transferred across the crack a bent strut is formed with a tie that is also bent (Fig. 8b).

It has been shown experimentally that often the concrete compression in the radial direction in the compression zone near the column (point B) initially increases under monotonically increasing load and after reaching a certain load level begins to decrease until shortly before failure negative values may be exhibited. This observation is in agreement with the proposed model. The compression decreases once the main bending crack has propagated and its negative value (tensile strain) indicates that at B a tie has formed.  Since the structural behaviour and the failure mechanisms are analogous to the those of the beam, for slabs also the same constructional solutions may in principle be employed. The crack pattern can be favourably influenced by omitting reinforcement in the region of the columns. As in the case of beams having reinforcement without

stirrups, whereby cracks can only develop in the vicinity of load application, for such a slab tangential cracks can only develop above the edge of the column. This kind of behaviour has been observed by Bollinger [7] in tests carried out on slab elements. The test shown in Fig 9a was reinforced with concentric rings placed at the boundary of the slab only. The punching shear force for this test was consistently higher than that for similar slabs which also had reinforcement in the region of the column. For the test shown in Fig. 9b the additional reinforcing ring in the critical region (greater amount of reinforcement) even caused a substantial reduction in the punching shear force.
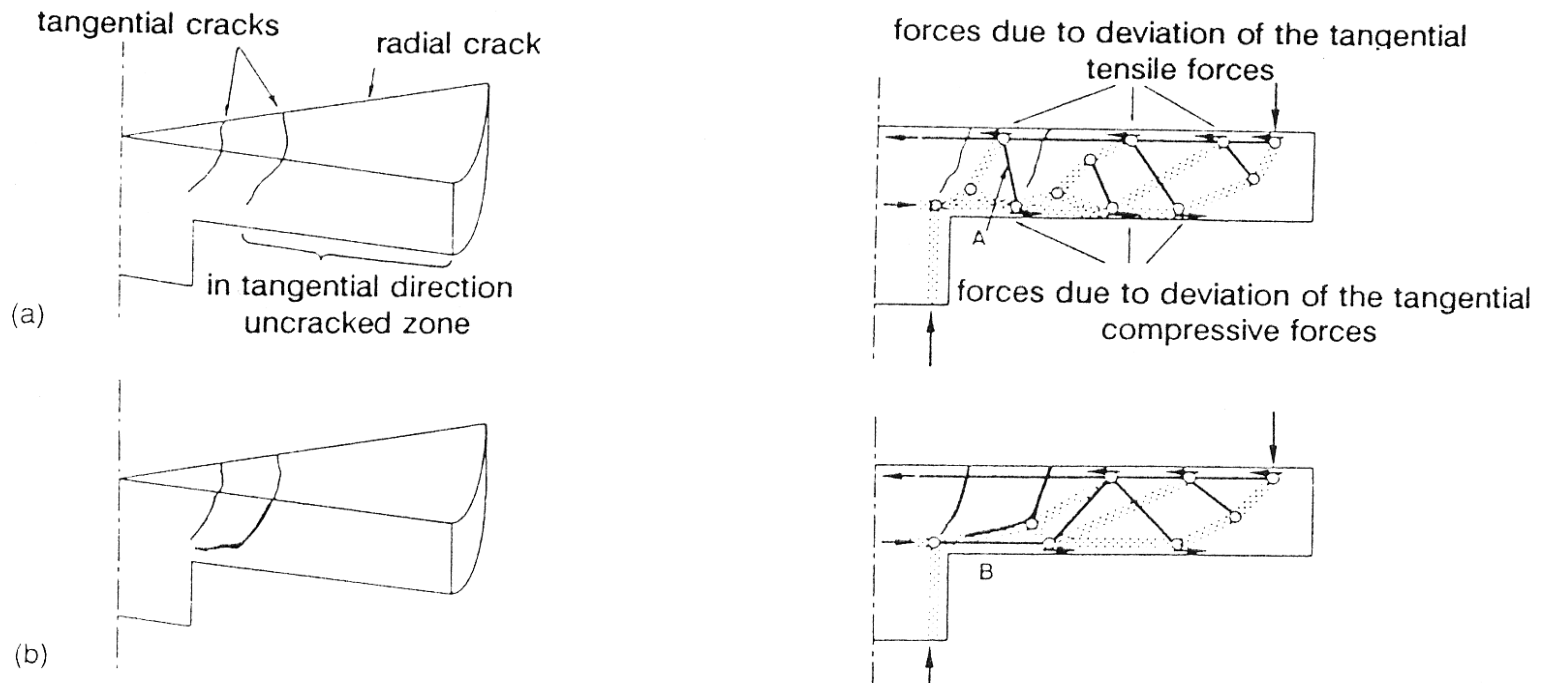


Fig. 8 Structural behaviour in the cracked state

Since in general the critical crack is responsible for the reduction in the failure load the punching shear force is inversely proportional to the width of crack. This is confirmed in Fig.10 in which the load is given as a function of the rotation $\varphi$ in the region of the column for four slabs with different amounts of reinforcement.
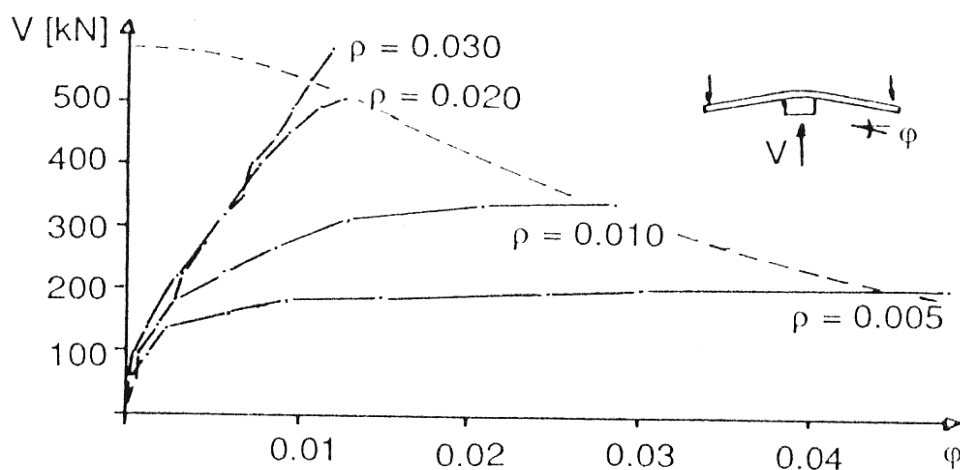


Fig. 10 Load-displacement diagrams of slabs

Since for the four slabs the thickness was kept constant the crack width is also proportional to the rotation $\varphi$. In Fig. 11a the punching shear forces and the corresponding deformations are shown for further test specimens. Since the crack width inside the slab is a direct function of the slab thickness, for this figure the product of rotation $\varphi$ times statical depth d was chosen for the abscissa. The ordinate represents normalized nominal punching shear strength. The relationship between punching shear resistance and deformation is described satisfactorily by the curve given in Fig. 10. Good agreement with test results is also obtained when the slab thickness h alone is varied (Fig. 11b).
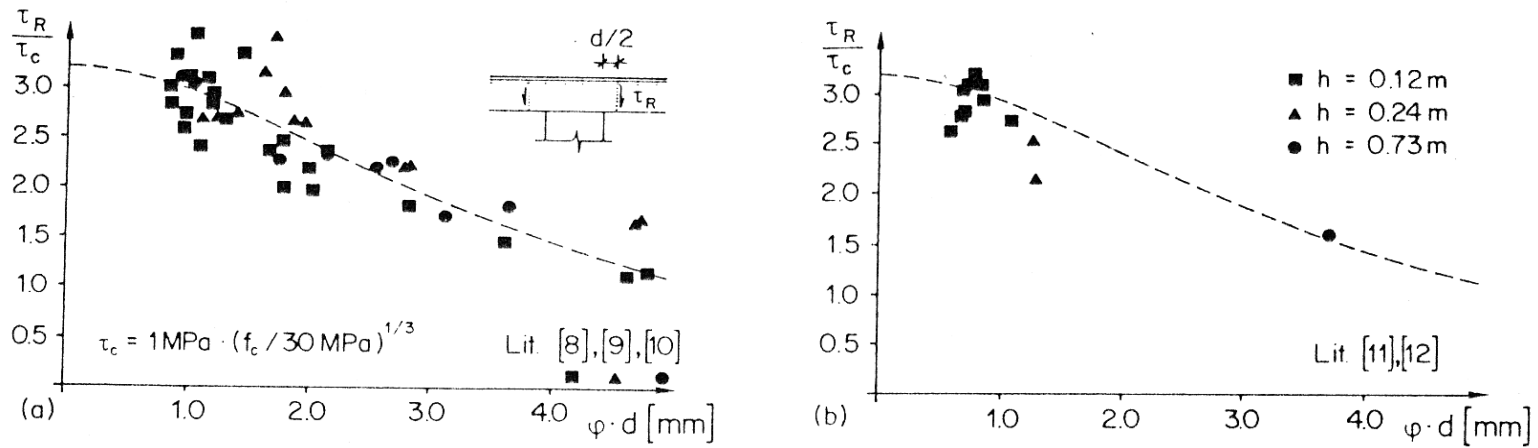
Fig. 11 Nominal punching shear strength in function of φ•d

With the empirically determined curve for describing the ductility restrictions for axisymmetric slabs the punching shear force can be found analytically.
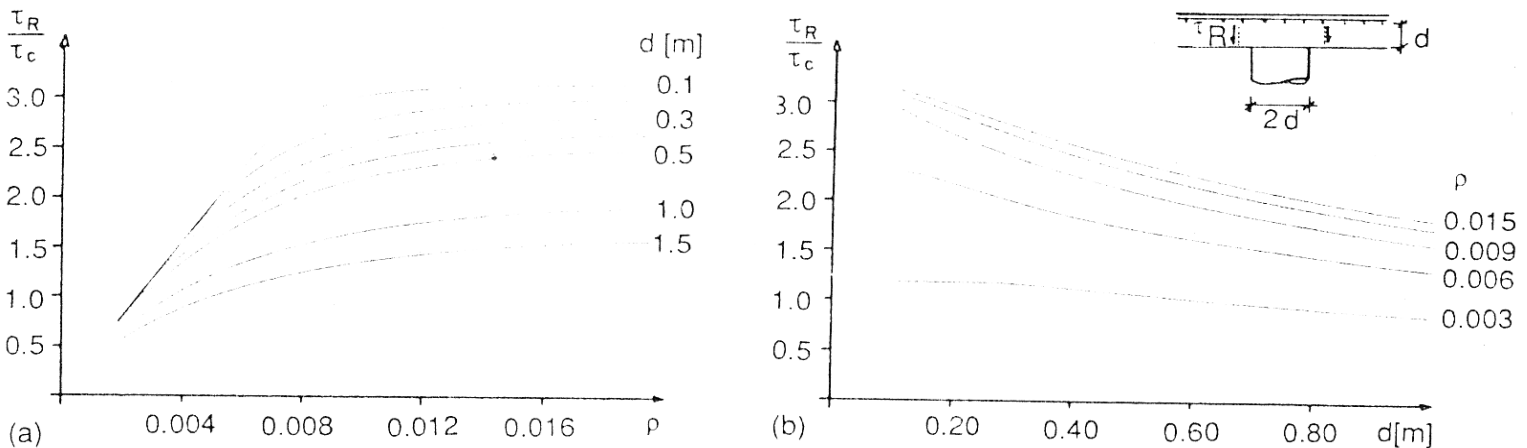


Fig. 12 Nominal punching shear strength in function of amount of reinforcement and slab thickness

From Fig. 12 it is evident that with increasing slab thickness a considerable reduction in punching shear resistance may result, i.e. the geometry or size-effect is important.

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