Rate-Constrained Beamforming for Collaborating Hearing Aids

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Outline

Motivations (1/2)

- Hearing Aids Generalities
  - Battery-operated devices
  - Equipped with up to 3 (omni)directional microphones
  - Different types

- Main goals
  - Spectral shaping
  - Beamforming (noise reduction, enhanced speech intelligibility)

Motivations (2/2)

- Most state-of-the-art systems involve two devices working independently of one another
  - Microphone arrays with small spatial extent (< 2 cm)
  - Limited beamforming capability
  - Poor rejection of interfering signals
- Idea: combine signals from microphones of both hearing aids
- Means: collaboration with a wireless communication link

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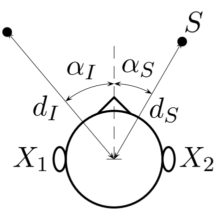
Fundamental gain-rate tradeoff

Problem Statement (1/3)

- Signal observed at hearing aid k ($k = 1, 2$)

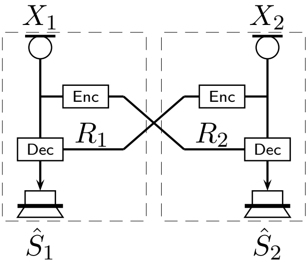
$$
\begin{aligned}
X_k(t) &= S_k(t) + I_k(t) + N_k(t) \\
&= h_k(t) * S(t) + g_k(t) * I(t) + N_k(t)
\end{aligned}
$$

- S, I and N_k are assumed to be independent jointly Gaussian stationary random processes



Problem Statement (2/3)

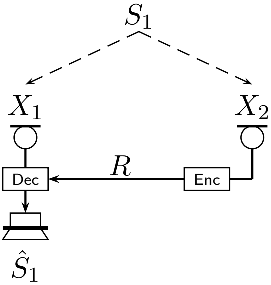
- Collaboration using a wireless link of rate R_k



- Fidelity criterion: mean-squared error $\mathrm{E}[\|S_k - \hat{S}_k\|^2]$

Problem Statement (3/3)

- Symmetric problem: we take the point-of-view of hearing aid 1 (left ear)
- The setup reduces to a remote source coding problem with side information at the decoder



- We define the gain-rate function

$$G(R) = D(0)/D(R)$$

Information-Theoretic Background (1/4)

- The problem from two different viewpoints: approximation and compression
  - Approximation: optimal dimensionality reduction
  - Compression: the remote Wyner-Ziv problem for jointly Gaussian vector/stationary sources

Information-Theoretic Background (2/4)

- Approximation: the optimal transform architecture separates into
  - MMSE estimation of the remote source S_1 using X_2 as if X_1 were available at the encoder
  - Karhunen-Loève transform of this MMSE estimate

  Retaining only k_2 out of n_2 coefficients results in a distortion

$$\mathrm{E}[\|S_1 - \hat{S}_1\|^2] = \mathrm{tr}(R_{S_1|X_1,X_2}) + \sum_{i=1}^{n_2-k_2} \lambda_i$$

where λ_i denote the n_2 largest eigenvalues of $R_{S_1|X_1} - R_{S_1|X_1,X_2}$ arranged in increasing order.

Information-Theoretic Background (3/4)

- Compression: the optimal rate-distortion tradeoff for vector sources is given by

$$
\begin{aligned}
R(\theta) &= \sum_{i=1}^{n_2} \max\left\{0, \frac{1}{2}\log_2 \frac{\lambda_i}{\theta}\right\} \quad \text{[bits/source vector]} \\
D(\theta) &= \operatorname{tr}(R_{S_1|X_1,X_2}) + \sum_{i=1}^{n_2} \min\{\theta, \lambda_i\} \quad \text{[MSE/source vector]}
\end{aligned}
$$

where λ_i are the n_2 largest eigenvalues of $R_{S_1|X_1} - R_{S_1|X_1,X_2}$ and $\theta \in (0, \max_i \lambda_i]$.

Information-Theoretic Background (4/4)

- Compression: the optimal rate-distortion tradeoff for stationary sources is given by

$$R(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left\{0, \log_2 \frac{\bar{\Phi}(\Omega)}{\theta}\right\} d\Omega \quad \text{[bits/second]}$$

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_1|X_1,X_2}(\Omega)\, d\Omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} \min\left\{\theta, \bar{\Phi}(\Omega)\right\} d\Omega \quad \text{[MSE/second]}$$

where $\bar{\Phi} = \Phi_{S_1|X_1} - \Phi_{S_1|X_1,X_2}$ and $\theta \in (0, \operatorname{ess\,sup}_\Omega \bar{\Phi}(\Omega)]$

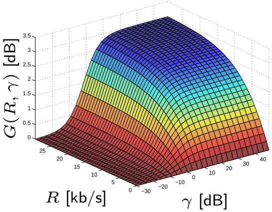
- This allows to compute the optimal gain-rate tradeoff

Gain-rate analysis (1/3)

- Assumptions: far-field case and flat power-spectrums over the frequency band $[-\Omega_0, \Omega_0]$
- Source and ambient noise
  - We obtain as a function of the signal-to-noise ratio γ

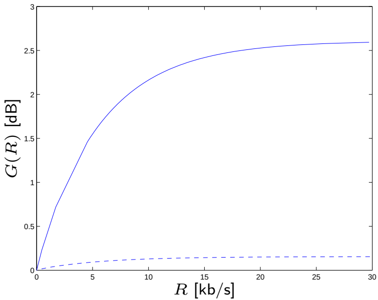
$$G(R,\gamma) = \frac{2\gamma+1}{\gamma+1}\left(\frac{\gamma}{\gamma+1}\,2^{-2\pi R/\Omega_0}+1\right)^{-1}$$

  - For $f_0 = \Omega_0/2\pi = 4000$ [Hz]



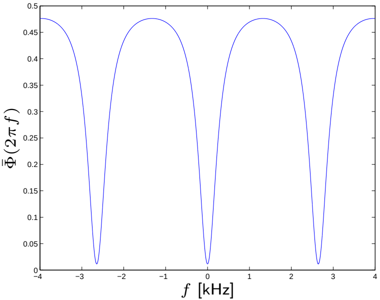
Gain-rate analysis (2/3)

- Source at $\alpha_S = 0$ [deg], interferer at $\alpha_I = 10$ [deg] and ambient noise
  - The spatial extent naturally offered by the head allows a better rejection of interfering signals
  - For $l = 2$ [cm] (dashed) and $l = 20$ [cm] (solid)



Gain-rate analysis (3/3)

- Source at $\alpha_S = 0$ [deg], interferer at $\alpha_I = 40$ [deg] and ambient noise
  - The optimal rate allocation is not uniform even if the sources are white



Conclusions

- We have identified the problem of collaborating hearing aids from an infomation-theoretic perspective
- The remote source coding problem with side information for sources with memory has been solved from both an approximation and a compression viewpoint
- The optimal tradeoff between the communication bit-rate and the gain provided by collaboration has been given

Thanks for your attention