Rate-Constrained Beamforming for Collaborating Hearing Aids

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Outline

1. Motivations
2. Problem Statement
3. Information-Theoretic Background
4. Gain-Rate Analysis
5. Conclusions
Motivations (1/2)

- Hearing Aids Generalities
  - Battery-operated devices
  - Equipped with up to 3 (omni)directional microphones
  - Different types

- Main goals
  - Spectral shaping
  - Beamforming (noise reduction, enhanced speech intelligibility)
Most state-of-the-art systems involve two devices working independently of one another
  - Microphone arrays with small spatial extent (< 2 cm)
  - Limited beamforming capability
  - Poor rejection of interfering signals

Idea: combine signals from microphones of both hearing aids

Means: collaboration with a wireless communication link
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Fundamental gain-rate tradeoff
Problem Statement (1/3)

- Signal observed at hearing aid $k$ ($k = 1, 2$)

$$X_k(t) = S_k(t) + I_k(t) + N_k(t)$$

$$= h_k(t) * S(t) + g_k(t) * I(t) + N_k(t)$$

- $S$, $I$ and $N_k$ are assumed to be independent jointly Gaussian stationary random processes
- Collaboration using a wireless link of rate $R_k$.

- Fidelity criterion: mean-squared error $\mathbb{E}[\|S_k - \hat{S}_k\|^2]$
Symmetric problem: we take the point-of-view of hearing aid 1 (left ear)

The setup reduces to a remote source coding problem with side information at the decoder

We define the gain-rate function

$$G(R) = \frac{D(0)}{D(R)}$$
The problem from two different viewpoints: approximation and compression

- **Approximation**: optimal dimensionality reduction
- **Compression**: the remote Wyner-Ziv problem for jointly Gaussian vector/stationary sources
**Approximation**: the optimal transform architecture separates into

- MMSE estimation of the remote source $S_1$ using $X_2$ as if $X_1$ were available at the encoder
- Karhunen-Loève transform of this MMSE estimate

Retaining only $k_2$ out of $n_2$ coefficients results in a distortion

$$E[\|S_1 - \hat{S}_1\|^2] = \text{tr}(R_{S_1|X_1,X_2}) + \sum_{i=1}^{n_2-k_2} \lambda_i$$

where $\lambda_i$ denote the $n_2$ largest eigenvalues of $R_{S_1|X_1} - R_{S_1|X_1,X_2}$ arranged in increasing order.
Compression: the optimal rate-distortion tradeoff for vector sources is given by

\[ R(\theta) = \sum_{i=1}^{n^2} \max \left\{ 0, \frac{1}{2} \log_2 \frac{\lambda_i}{\theta} \right\} \quad \text{[bits/source vector]} \]

\[ D(\theta) = \text{tr}(R_{S_1|X_1,X_2}) + \sum_{i=1}^{n^2} \min\{\theta, \lambda_i\} \quad \text{[MSE/source vector]} \]

where \( \lambda_i \) are the \( n^2 \) largest eigenvalues of \( R_{S_1|X_1} - R_{S_1|X_1,X_2} \) and \( \theta \in (0, \max_i \lambda_i] \).
**Compression**: the optimal rate-distortion tradeoff for stationary sources is given by

\[
R(\theta) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \max \left\{ 0, \log_2 \frac{\bar{\Phi}(\Omega)}{\theta} \right\} \, d\Omega \quad \text{[bits/second]}
\]

\[
D(\theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{S_1|X_1,X_2}(\Omega) \, d\Omega \\
+ \frac{1}{2\pi} \int_{-\infty}^{\infty} \min \{ \theta, \bar{\Phi}(\Omega) \} \, d\Omega \quad \text{[MSE/second]}
\]

where \( \bar{\Phi} = \Phi_{S_1|X_1} - \Phi_{S_1|X_1,X_2} \) and \( \theta \in (0, \text{ess sup}_\Omega \bar{\Phi}(\Omega)] \)

This allows to compute the optimal gain-rate tradeoff
Gain-rate analysis (1/3)

- Assumptions: far-field case and flat power-spectrums over the frequency band \([-\Omega_0, \Omega_0]\)
- Source and ambient noise
  - We obtain as a function of the signal-to-noise ratio \(\gamma\)
    \[
    G(R, \gamma) = \frac{2\gamma + 1}{\gamma + 1} \left( \frac{\gamma}{\gamma + 1} 2^{-2\pi R/\Omega_0} + 1 \right)^{-1}
    \]
  - For \(f_0 = \Omega_0/2\pi = 4000 \text{ [Hz]}\)

![Graph of G(R, γ) in dB vs. R in kb/s and γ in dB]
Source at $\alpha_S = 0$ [deg], interferer at $\alpha_I = 10$ [deg] and ambient noise

- The spatial extent naturally offered by the head allows a better rejection of interfering signals
- For $l = 2$ [cm] (dashed) and $l = 20$ [cm] (solid)
Source at $\alpha_S = 0$ [deg], interferer at $\alpha_I = 40$ [deg] and ambient noise

The optimal rate allocation is not uniform even if the sources are white
Conclusions

- We have identified the problem of collaborating hearing aids from an information-theoretic perspective.
- The remote source coding problem with side information for sources with memory has been solved from both an approximation and a compression viewpoint.
- The optimal tradeoff between the communication bit-rate and the gain provided by collaboration has been given.
Thanks for your attention