Monetary-Fiscal Interactions with a Conservative Central Bank

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Abstract

We study macroeconomic stabilization when monetary and fiscal policies interact via their effects on output and inflation and the monetary authority is more conservative than the fiscal. We find that monetary-fiscal interactions result in poor macroeconomic stabilization. With both policies discretionary, the Nash equilibrium is suboptimal with higher output and lower inflation than optimal; the Nash equilibrium may be extreme with output higher and inflation lower than either authority want. Leadership equilibria are not second best. Monetary commitment is completely negated by fiscal discretion and yields the same outcome as discretionary monetary leadership for all realizations of shocks. But fiscal commitment is not similarly negated by monetary discretion. Optimal macroeconomic stabilization requires either commitment of both monetary and fiscal policies, or identical targets for both authorities – output socially optimal and inflation appropriately conservative – or complete separation of tasks.

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1 Introduction

A large literature has focused on how monetary policy should be chosen so as to stabilize the economy against unexpected shocks. This literature has studied how monetary policy is chosen depending on the preferences of the central bank and it has found that monetary policy should be delegated to an independent and conservative central bank – see Sargent and Wallace (1981), Barro and Gordon (1983), Rogoff (1985a) and Svensson (1996) and the works that followed. Independent means that the central bank has full control over the instruments of monetary policy and therefore over the decision of how much of the fiscal deficit to monetize. By conservative this literature means that the central bank should have a lower inflation and/or output target and/or a lower weight on the output target than society does. Central bank conservatism eliminates the inflation bias stemming from the time inconsistency of monetary policy, which is commonly believed to have contributed to high inflation in the 1970s and early 1980s.

Following these seminal works, many countries have made their central banks increasingly independent and more conservative than the politicians who run fiscal policy in recent years. For example, the Maastricht Treaty that established the Economic and Monetary Union of Europe determines that the European Central Bank should primarily be concerned with price stability in the Euro area and it forbids it from financing the deficits of the member countries or bailing them out even in a fiscal crisis.

A crucial feature of this literature is that fiscal policy is either absent or assumed exogenously fixed and constant. In reality, however, macroeconomic stabilization consists of both monetary and fiscal policies; most importantly, these two policies interact via their effects on output and inflation.

The goal of this paper is to explicitly model fiscal policy to consider such interactions and how they affect economic outcomes. We develop a model whose natural rate of output is suboptimally low because firms enjoy monopolistic power over the goods they produce. Fiscal policy consists of government spending over the goods produced in the economy financed by lump-sum taxes. Government spending can bring output to its efficient level, but it may not be socially optimal to do so because public spending reduces private consumption and welfare. This leaves an output gap that creates a time-consistency problem for both monetary and fiscal policies. With some prices preset, the central bank thinks it can raise output in the short run by an unanticipated monetary expansion that boosts private demand; similarly, the fiscal authority thinks it can raise output by an unanticipated increase in public spending that boosts public demand. In equilibrium, these incentives are perfectly anticipated by rational firms and they simply bias policies away from optimality.

But time inconsistency is not the only problem here. Central bank conservativeness creates a conflict of objectives between monetary and fiscal policies that makes them interact suboptimally. The equilibria are suboptimal and possibly extreme no matter whether policies are discretionary or pre-committed.

We can summarize our results as follows:

1. If neither type of policy has commitment or leadership, the Nash equilibrium has higher

output and lower inflation than optimal; these outcomes may be extreme with respect to both authorities' goals, depending on the realization of the stochastic shocks.

2. Giving leadership (first-mover advantage) to fiscal policy typically improves welfare over Nash from an ex-ante perspective. However, even fiscal leadership is not optimal.

3. The time-consistency problem of monetary policy can be solved by commitment to a rule specifying how the actual policy choice will respond to all possible realizations of the stochastic shocks. But with discretionary fiscal policy chosen by a strategic fiscal authority, the ex post reaction function of the fiscal authority acts as a constraint on the monetary rule. We find that this entirely negates the advantage of monetary commitment – the optimal monetary rule is no different than discretionary leadership of monetary over fiscal policy for every realization of the shocks. Fiscal commitment, on the other hand, eliminates the time inconsistency of fiscal policy and, if coupled with an appropriately designed central bank, it leads to the social optimum. The asymmetry between monetary and fiscal commitment stems from the fact that fiscal policy has *direct* negative effects on social welfare while monetary policy does not.

4. Commitment achieves the second best if it can be extended to both monetary and fiscal policy.

5. If commitment to a policy rule is not an option, the second best can be achieved by appropriately assigning goals to policies so as to avoid any conflict of objectives. Two alternative assignments are possible. First, the two authorities should have identical targets, the output target being the social optimum and the inflation target being conservative. Second, the two authorities should have separate targets, with the monetary authority targeting only inflation and the fiscal authority targeting output and the welfare losses due to fiscal policy.

2 Literature Review

Our paper takes as a starting point the findings of the literature on commitment and discretion in monetary policy initiated by Kydland and Prescott (1977) and Barro and Gordon (1983). These works assume that distortions create short-run benefits from unexpected inflation and study the equilibria that arise under different institutional arrangements and their welfare properties. The first-best equilibrium can only be achieved by eliminating the distortions directly, which is not assumed to be feasible; pre-commitment to a monetary rule that specifies how monetary policy should responds to shock delivers the second best; discretionary monetary policymaking results in fourth-best equilibria because of time-consistency problems. Rogoff (1985a) and Svensson (1996) find that third- or even second-best equilibria can be obtained by delegating monetary policy to a conservative central bank. Alternatively, an optimal central bank contract as suggested by Walsh (1995) and Persson and Tabellini (1993) can lead to the second best. All these papers do not consider fiscal policy or simply assume it as exogenous or constant. Our work explicitly models fiscal policy and studies the equilibria that emerge when the fiscal authority is a strategic player; it shows that monetary-fiscal interactions are indeed suboptimal when the monetary and fiscal authorities have conflicting goals.

There are some works that analyze the interaction of monetary and fiscal policies and our work is related to two of them. Alesina and Tabellini (1987) consider a closed-economy model where the monetary authority chooses the inflation rate and the fiscal authority chooses the tax rate to finance government expenditures; both authorities have identical, explicit targets for inflation, output and the level of government expenditures but different tradeoffs among the targets. They find that monetary commitment may fail to improve welfare when the authorities have different tradeoffs among the goals because the loss of seignorage stemming from lower inflation induces an increase in taxes and a reduction in output that more than compensate the gain from lower inflation. Our paper differs from Alesina and Tabellini in many respects, but most notably in three. First, we do not consider public spending targets and rather focus on the welfare effects stemming from output and inflation stabilization. Second, we consider the case where the monetary and fiscal authorities have different targets *and* weights, which is a crucial difference as shown in Dixit and Lambertini (2003a). Third, when they briefly consider the case with no government spending targets, their model does not have a time-consistency problem, so that output and inflation are at their common targeted levels.

Debelle and Fischer (1994) study the case where the monetary authority has no explicit preferences about the level of public expenditures and consider Nash and Stackelberg equilibria, but do not consider state-contingent monetary rules.

Dixit and Lambertini (2003a, 2001) study monetary-fiscal interactions for the case of a monetary union. Both papers consider a model where monetary and fiscal policies do not have a time-consistency problem because optimal fiscal policy closes any output gap. The first work focuses on the case where the common central bank and all countries' fiscal authorities have identical output and inflation targets but possibly different tradeoffs between they objectives. It shows that the ideal output and inflation outcomes can be achieved for any order of moves, whether policies are discretionary or pre-committed. The second paper considers the more general case where the common central bank and the fiscal authorites have different goals and tradeoffs among them; it finds that this conflict of objectives leads to output and inflation outcomes that are extreme and different from what the authorities want. Our paper differs from Dixit and Lambertini (2003a, 2001) in two related respects. We derive social welfare and assume that fiscal policy is chosen so as to maximize it. This has three consequences: first, monetary and fiscal policies have a time-consistency problem; second, our analysis can rank equilibria from a social welfare point of view; third, we can address the issue of what design of monetary and fiscal institutions maximizes social welfare.

Dixit and Lambertini (2003b) study monetary-fiscal interactions in a model where fiscal policy consists of a production subsidy that creates deadweight losses. In their paper, a fiscal expansion is a higher subsidy that raises production and reduces prices. In our model fiscal policy consists of public spending financed by lump-sum taxes; a fiscal expansion is an increase in public spending that raises production and prices. The equilibria in our paper and their welfare implications are different from those in Dixit and Lambertini (2003b). For example, in our paper time inconsistency makes fiscal policy more expansionary than optimal at the Nash equilibrium; as a result, monetary policy is more contractionary than optimal and the Nash equilibrium has lower inflation and higher output than optimal. The opposite occurs in Dixit and Lambertini. The characteristics of the leadership equilibria also differ substantially.

Our work shows that if both monetary and fiscal policies have a time-consistency problem, making the monetary, but not the fiscal, authority conservative may make things worse. The general idea behind our result – reminiscent of the second best theory – is present in other works on policymaking with multiple strategic interactions. Rogoff (1985b) and Kehoe (1989), for example, show that cooperation between two governments with an incentive toward time-inconsistent behavior may reduce welfare as lack of policy cooperation acts as a disciplining device.

3 The Model

3.1 Consumers

We consider a general equilibrium model with differentiated goods, monopolistic competition and staggered prices.[1] The representative household maximizes the discounted sum of utilities of the form

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U_s, \tag{1}$$

where the period utility function is

$$U_s = \left[\log C_s + \chi \log \frac{M_s}{P_s} - \frac{d}{1+\eta} \int_0^1 N_s(i)^{1+\eta} di + \frac{\alpha}{1-1/\rho} G_s^{1-1/\rho} \right], \tag{2}$$

with $d > 0, \chi > 0, \eta \geq 0$ and $\rho > 1$. $0 < \beta < 1$ is a discount factor, C is consumption, M/P are real balances, $N(i)$ is the quantity of labor of type i supplied by the representative individual and G is public spending. Hence, it is assumed here that period utility depends positively on public good provision, with the parameter α measuring the relative importance of public versus private consumption for welfare. Because we are interested in studying how monetary and fiscal polices can better stabilize output and inflation in response to shocks, our welfare analysis will be based on the limiting economy as α goes to zero and public spending ceases to raise social welfare *per se*.[2]

[1] For an excellent survey of this literature, see Woodford (2003).

[2] We do not need to assume that $\alpha \to 0$. In fact, our results hold true if $\alpha > 0$ as long as $\alpha < 1/(\theta - 1)$; since most estimates of θ suggest a value around 11, we focus on economies where $\alpha < 0.1$ and public spending raises private utility considerably less than private consumption. Intuitively, if $\alpha < 1/(\theta - 1)$ government spending is a public good that raises social welfare but, in a sense, not enough; the natural rate of output is still suboptimally low so that monetary and fiscal policies are time inconsistent. See Appendix C for a detailed proof.

Our utility function assumes that private and public consumption are separable; this allows us to consider the case where public spending does not enter the utility function as the limiting economy with $\alpha \to 0$ and to easily compare our results with the existing literature on monetary-fiscal interactions, which typically assumes that government spending does not raise social welfare *per se*. A simplifying but restrictive feature implied by separability is that the marginal utility of private consumption at date t is independent of public spending at date t and $t+1$. The assumption that individual preferences are logarithmic in private while more generally isoelastic in public consumption is completely inconsequential: for $\rho = 1$ the last term in the period utility function is replaced by its limit, $\alpha \log G_s$.

C_t is the real consumption index

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\theta > 1$ is the constant elasticity of substitution among the individual goods available in the economy. The corresponding price index P_t is

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{4}$$

which is the minimum cost of a unit of the aggregate consumption good defined by (3), given the individual goods prices $P_t(i)$. There is a continuum of differentiated goods distributed over the interval $[0, 1]$. In (1), $N_t(i)$ is the quantity of labor of type i supplied by the representative household and it is assumed that each differentiated good uses a specialized labor input in its production. The assumption of differentiated labor inputs is not necessary but convenient, as households with identical initial assets supply the same quantities of labor and receive the same labor income. $\eta > 0$ is the elasticity of the marginal disutility of labor with respect to labor supply. The parameters d, θ, η are stochastic.

If all households begin with the same amount of financial assets, as we assume, they will have the same intertemporal budget constraints and will therefore choose the same sequences of consumption, real balances and efforts. Hence, our model is truly a representative household one. The budget constraint for such agent is

$$\frac{B_{t+1}}{1+r_t} + \frac{M_t}{P_t} + C_t = B_t + \frac{M_{t-1}}{P_t} + \int_0^1 \frac{W_t(i)}{P_t} N_t(i) di + \int_0^1 \frac{\Pi_t(i)}{P_t} di + T_t - \tau_t. \tag{5}$$

Here B_{t+1} is the purchase of a riskless bond that pays one unit of aggregate consumption at time $t+1$; since all household are the same, this asset is redundant and, in equilibrium, there will be no trade in it. $W_t(i)$ is the nominal wage of labor of type i in period t and $\Pi_t(i)$ are nominal profits of the firm producing good i. We assume that each household owns an equal share of all the firms in the economy. T_t represents transfers received from the household at time t and τ_t is a lump-sum tax levied by the government at time t.

Households face four decisions. First, how to allocate consumption across the differentiated goods. Taking prices as given, the optimal consumption of each good i is given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} C_t. \tag{6}$$

Second, the household must decide the optimal amount of riskless bonds to purchase, B_{t+1}. The first-order condition delivers the Euler equation

$$\frac{1}{C_t} = \beta(1 + r_t)E_t\frac{1}{C_{t+1}}. \tag{7}$$

Third, the household must decide the optimal level of money balances to carry into next period, M_t. After making use of (7), the first-order condition for optimal money balances is given by[3]

$$\frac{M_t}{P_t} = \chi C_t \frac{1}{E_t \frac{i_{t+1}}{1+i_{t+1}}}, \tag{8}$$

where i^e_{t+1} is the expected nominal interest rate between period t and $t + 1$:

$$(1 + i^e_{t+1}) = (1 + r_t)E_t\frac{P_{t+1}}{P_t}. \tag{9}$$

Finally, the household must decide the optimal quantity of each type of labor to supply, taking wages and prices as given. The related first-order condition is given by

$$N_t(i) = \left[\frac{W_t(i)}{P_t C_t d}\right]^{\frac{1}{\eta}}. \tag{10}$$

3.2 Policymakers

There is a central bank that runs monetary policy and a government that runs fiscal policy. The central bank is instrument-independent in the sense that it chooses monetary policy freely and it does not share the government budget constraint. We also assume that the central bank is conservative in the sense that it maximizes a utility that is more conservative than society's – this will be explained in detail in Section 4. The budget constraint for the central bank is

$$T_t = \frac{M_t - M_{t-1}}{P_t}. \tag{11}$$

Hence, the central bank rebates seignorage back to households.

Fiscal policy consists of public spending financed with lump-sum taxes τ_t. The budget constraint for the government is

$$\tau_t = G_t. \tag{12}$$

For simplicity, we assume that government allocates its consumption across goods like households do so that

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} G_t. \tag{13}$$

[3] To obtain (8), we have assumed that C_{t+1} and i_{t+1} are independent.

The government is benevolent and chooses G_t so as to maximize the utility function of the representative individual. Optimal public spending is given by

$$G_t = (\alpha C_t)^\rho. \tag{14}$$

Optimal public spending becomes negligible as α goes to zero.

3.3 Firms

Each good has a production function that makes only use of labor

$$Y_t(i) = A_t N_t(i), \tag{15}$$

where A_t is an exogenous stochastic technological factor common to all firms, i.e. a supply-side aggregate shock. Nomimal profits at time t for firm i are given by

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t(i)N_t(i). \tag{16}$$

The first term on the right hand side of (16) represents revenues from selling the good; the second term is the cost of producing, which is the nominal wage bill for employed labor. We have assumed that the firm takes the nominal wage as given. Firms maximize the present value of current and future profits

$$\sum_{s=t}^{\infty} \beta^{s-t} E_t \, Q_{t,s} \Pi_s(i), \tag{17}$$

where $Q_{t,s}$ is the stochastic discount factor

$$Q_{t,s} = \frac{u'(c_s)P_t}{u'(c_t)P_s} = \Pi_{j=t}^{s} Q_{j,j+1}. \tag{18}$$

From equation (6) and (11) we can obtain the demand for good i

$$Y_t(i)^d = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} (C_t + G_t). \tag{19}$$

If prices are flexible, firms choose prices every period to maximize current profits. The first-order condition is:

$$P_t(i) = \frac{\theta}{\theta - 1} \frac{W_t(i)}{A_t}. \tag{20}$$

With flexible prices, it is optimal for the firm to set its price as a markup over the marginal cost. The markup $\theta/(\theta-1)$ falls as θ grows, i.e. as goods become better substitutes and the monopolistic power of the firm becomes smaller.

3.4 Equilibrium

Aggregate output is defined as

$$Y_t \equiv \int_0^1 \frac{P_t(i)}{P_t} Y_t(i)\, di. \tag{21}$$

Making use of (11), (12), (16), (21) and the fact that the bonds are in zero net supply, the consolidated budget constraint becomes

$$C_t = Y_t - G_t. \tag{22}$$

Private consumption is equal to aggregate production minus government spending.

3.5 Steady State

At the randomless steady state, all differentiated goods have identical prices and wages across labor types are identical. Hence

$$\frac{W}{P} = \frac{\theta - 1}{\theta} A, \tag{23}$$

where variables without a time subscript indicate steady-state values. Labor is also equalized across different types and is given by

$$N = \left[\frac{(\theta - 1)A}{\theta d C} \right]^{1/\eta}. \tag{24}$$

The price level is

$$P = \frac{M(1-\beta)}{\chi C} \tag{25}$$

and steady-state output is

$$Y = AN = A \left[\frac{(\theta - 1)A}{\theta d (Y - G)} \right]^{1/\eta}, \tag{26}$$

where government spending is given by

$$G = (\alpha C)^{\rho}. \tag{27}$$

An increase in government spending raises output in the steady state. Because higher government spending requires higher taxes, households reduce consumption and substitute out of leisure into work, thereby raising production. Hence, government spending does not crowd out private spending completely. If public spending is zero, steady-state output is given by

$$Y = A \left[\frac{\theta - 1}{\theta d} \right]^{\frac{1}{1+\eta}}. \tag{28}$$

Steady state output is suboptimally low due to the monopolistic power of producers. As the degree of substitutability among goods becomes large, i.e. $\theta \to \infty$, output approaches its efficient level

$$Y^* = A\left[\frac{1}{d}\right]^{\frac{1}{1+\eta}}. \tag{29}$$

The efficient level of output can also be achieved by an appropriate production subsidy that offsets the distortion due to market power;[4] here we abstract from such subsidy.

3.6 Staggered Pricing

We assume a discrete-time variant of the Calvo (1983) model of staggered-price setting that has been extensively used in the literature. A fraction $\phi \in (0,1)$ of prices remain unchanged in each period, while new prices are chosen for the remaining $1-\phi$ fraction of goods. The probability that any given price will be changed in any given period is assumed to follow a Poisson process with arrival rate $1-\phi$, which is independent of time elapsed since the price was last changed. This assumption about the dynamics of prices implies that

$$P_t^{1-\theta} = \left[\phi P_{t-1}^{1-\theta} + (1-\phi)\tilde{P}_t^{1-\theta}\right]. \tag{30}$$

All suppliers that set new prices at t face exactly the same decision problem; hence, the newly set price $\tilde{P}_t$ is the same for all of them (and is therefore not a function of i).

A supplier that sets a new price at t chooses it so as to maximize

$$\sum_{s=t}^{\infty}(\phi\beta)^{s-t}E_t Q_{t,s}\Pi_s(i), \tag{31}$$

where the factor ϕ^{s-t} indicates the probability that the price chosen at t will still be charged in period $s > t$. Appendix A solves the firm's problem with staggered pricing and Appendix B log-linearizes the model around the steady state. Aggregate inflation is as a function of monetary and fiscal policies and current shocks:

$$\pi_t = m_t + cg_t + \omega_t + \gamma\beta\pi_{t+1}^e, \tag{32}$$

where small letters indicate percent deviations from the steady state of the capitalized-letter variable and

$$\pi_t \equiv p_t - p_{t-1},\;\; m_t = \frac{\psi}{1+\psi}\mu_t,\;\; c = \frac{\lambda\eta}{1+\psi} > 0,\;\; \psi = (1-\beta)\lambda(1+\eta),\;\; \lambda = \frac{(1-\phi\beta)(1-\phi)}{\phi},$$

$$\omega_t = -\frac{\lambda(1+\eta)}{1+\psi}a_t - \frac{\psi}{1+\psi}p_{t-1} + \frac{\beta\lambda(1+\eta)}{1+\psi}y_{t+1}^e,\;\; \gamma = \frac{1+\lambda(1+\eta)}{1+\psi} > 0.$$

Aggregate inflation is a sum of several components. First, of the component m_t, which is the controlled part of monetary policy and it is an increasing function of money supply. Second,

[4] The appropriate production subsidy is $1/(\theta-1)$.

of a contribution arising from fiscal policy g_t; $c > 0$: an increase in government spending financed by lump-sum taxes raises inflation. Third, of the term ω_t that captures the effect of technological shocks, past price changes and the expected future real marginal cost.[5] Fourth and last, of expected future inflation. The condition of rational expectations is

$$\pi_{t+1}^e = E_{z_t}[\pi(z_{t+1})] \equiv \int \pi(z_{t+1}), \tag{33}$$

where the integral is taken over the distribution of z_t, and is four-dimensional since all the components of z_t are functions of four underlying stochastic structural parameters. In words, π_{t+1}^e is the firms' rational expectation of π_{t+1} as of time t.

Output is given by

$$y_t = \tilde{y}_t + a g_t + b(\pi_t - \beta \pi_{t+1}^e), \tag{34}$$

where

$$\tilde{y}_t = a_t, \quad a = \frac{1}{1+\eta} > 0, \quad b = \frac{1}{\lambda(1+\eta)} > 0.$$

The derivation of the coefficients is spelled out in Appendix B. The explanation of the parameters in the output equation (34) is as follows: [1] $\tilde{y}_t$ is percentage deviation of the natural rate of output at t from its steady-state value and it depends on the technological shock. The natural rate of output is the level of production that arises in the economy with steady-state monetary and fiscal policy; this is suboptimally low because of monopolistic competition. [2] The scalar a is the direct effect of fiscal policy on GDP. An increase in government spending raises demand and has an expansionary effect on GDP; hence $a > 0$. [3] π_{t+1}^e is firms' rational expectation of π_{t+1} as of time t. [4] The last term on the right-hand side of equation (34) is the usual supply effect of an unexpected increase in inflation; thus $b > 0$. [5] The overall effect of a fiscal expansion on output is $a + bc > 0$.

4 Preferences of Policymakers

The central bank chooses a policy variable m_t, which stands for the base money supply, and determines a component of inflation; thus higher m_t means a more expansionary monetary policy. The fiscal authority chooses a policy variable g_t; a larger g_t means higher government spending and therefore a more expansionary fiscal policy. These policies affect the GDP level y_t and aggregate inflation π_t in the country according to equations (34) and (32) above.

The fiscal authority is benevolent and chooses g_t every period to maximize social welfare, which is the utility of the representative individual. We approximate it by a second-order Taylor series expansion to the level of expected utility of the representative consumer in the rational expectations equilibrium associated with given monetary and fiscal policies – this is shown in Appendix C. We are interested in the welfare effects of output and inflation stabilization; for this reason, we consider the second-order approximation to the utility of

[5] This is shown in Appendix B.

the representative household as $\alpha \to 0$ and the direct welfare effect of public goods becomes very small.

The fiscal authority minimizes the loss function

$$V_{F,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} U_s, \tag{35}$$

where the period utility U_t is approximated by

$$U_t = -\Omega L_{F,t},$$

where Ω is a positive constant and $L_{F,t}$ is the quadratic loss function

$$L_{F,t} = \frac{1}{2}\left[(\pi_t - \pi_F)^2 + \theta_F (y_t - y_F)^2 + 2\delta g_t\right]. \tag{36}$$

$\pi_F = 0$ and it is socially optimal to minimize price level dispersion. The GDP that minimizes social losses is y_F, which is the GDP that would arise in an economy with flexible prices and without monopolistic power by the firms; hence, $y_F \geq \tilde{y}_t$ and extra output is desirable. Fiscal policy can raise output above its natural rate, but it creates social losses $\delta > 0$ because government spending is financed by lump-sum taxes that reduce private consumption. $\theta_F > 0$ parameterizes the social preference for the output versus the inflation goals. All parameters are spelled out in Appendix C.

Monetary policy is chosen by a monetary authority that is independent and conservative in a way that encompasses both Rogoff's and Svensson's definition, and minimizes a loss function

$$V_{M,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} L_{M,s}, \tag{37}$$

where

$$L_{M,t} = \frac{1}{2}\left[\theta_M (y_t - y_M)^2 + (\pi_t - \pi_M)^2\right], \tag{38}$$

where y_M is the output target, π_M the inflation target and θ_M the preference for the output versus the inflation goal for the monetary authority.

Sargent and Wallace (1981) show that inflation is a fiscal phenomenon if the central bank cannot choose how much of the budget deficit to monetize. In models where fiscal policy is absent or chosen non-strategically, welfare-maximizing discretionary monetary policy generates an inflation bias with no output gain because of its time-consistency problem. To eliminate this inflation bias, Svensson (1997) suggests delegating monetary policy to a central bank with an inflation target more conservative than society, i.e. with $\pi_M \leq \pi_F$; Rogoff (1985a), on the other hand, suggests delegating monetary policy to a the central bank with a greater concern for inflation than society, i.e. $\theta_M \leq \theta_F$. These academic contributions have played an important role in shaping monetary institutions in the last two decades. In recent years, many countries have made their central banks increasingly independent of political control and/or have instituted explicit inflation targeting regimes for their monetary policies.

We take central bank independence and conservatism as a starting point of our analysis. The monetary authority in our model has full control over monetary policy and does not share the government budget constraint. The central bank is more conservative than society in the sense that $\theta_M \leq \theta_F$ and/or $\pi_M \leq 0, y_M \leq y_F$; moreover, we choose the weights on the output target and the inflation target in (38) optimally in the sense that they are consistent with the second-best outcome – this is explained in greater detail in Section 6.

The natural rate of output $\tilde{y}_t$, the scalar parameter a summarizing the fiscal policy effect on GDP, the scalar parameter b for the supply effect of surprise inflation, the scalar parameter c of the effect of fiscal policy on inflation, the scalar parameter δ for the deadweight loss of fiscal policy, the scalar parameter θ_F for the social preferences, the efficient level of output y_F, the central bank's output target y_M and inflation target π_M and the scalar parameter θ_M for the central bank's preferences, are all stochastic shocks because they depend on the four stochastic preference and technology parameters of our structural model. We denote the whole vector of these shocks by $z_t = (\tilde{y}_t, a, b, c, \delta, \theta_F, y_F, y_M, \pi_M, \theta_M)$. The policy variables m_t and g_t are implemented after the shocks are observed, and therefore are written as functions $m(z_t)$ and $g(z_t)$ (although the functional form may be fixed before the shocks are observed in regimes where policies are precommitted). The resulting outcomes of GDP and inflation are then also realization-specific or functions $y(z_t)$ and $\pi(z_t)$.

The literature in this area usually considers only linear policy rules – it restricts the form of the function $m(z_t)$ to be linear, and then finds the optimal values of the coefficients in this function. Since linear rules are not in general optimal, it becomes necessary to restrict the stochastic shocks; only additive shocks like our $\tilde{y}_t$ are considered. Our stochastic structure is richer and we allow $m(z_t)$ and $g(z_t)$ to be arbitrary, and find the fully optimal rules.

4.1 Timing of Actions

We will consider various possible policy regimes. In absence of commitment, the two policies may be simultaneous (Nash) or one of them may be first (leadership). If monetary policy is precommitted, then it has leadership with respect to setting the rule, and fiscal policy is the follower in each state of the world (realization of the shocks); if fiscal policy is precommitted, it has leadership with respect to setting the rule, and monetary policy is the follower in each state of the world. There is no conclusive evidence about the correct choice from among the possibilities. It may be argued on the one hand that the central bank's reputational considerations give it an advantage in making and keeping commitments, and on the other hand that the lags in fiscal policy enable commitment there. Similar conflicting arguments can be made for the order of moves under discretion. Therefore we consider all possibilities.

Hence, the timing of events is as follows:

1. We consider three possible scenarios of commitment:

    (a) If there is joint commitment of the two policies, this is done in a coordinated manner using the fiscal authority's objective function, which coincides with social welfare.

(b) If the fiscal policy regime is one of commitment, the fiscal authority chooses its policy rule $g = g(z_t)$; this specifies how fiscal policy will respond to the stochastic shocks. If the fiscal regime is one of discretion, nothing happens at this step.

(c) If the monetary policy regime is one of commitment, the central bank chooses its policy rule $m = m(z_t)$. If the monetary regime is one of discretion, nothing happens at this step.

2. The stochastic shock vector z_t is realized.

3. The private sector forms expectations π^e_{t+1}.

4. (a) If the monetary policy regime is one of discretion, the central bank chooses m_t. If the monetary regime is one of commitment, the central bank simply implements the monetary rule m_t that was chosen at step 1.

   (b) If the fiscal regime is one of discretion, the fiscal authority chooses fiscal policy g_t. If the fiscal regime is one of commitment, the fiscal authority simply implements the fiscal rule g_t that was chosen at step 1.

   When monetary and fiscal policies are discretionary, the relative timing of step 4 (a) and 4 (b) raises some questions. In fact, monetary and fiscal policies may be chosen simultaneously or their order may be reversed.

We proceed to consider the different cases of commitment and sequence of moves.

5 Optimal Committed Policies

First we study the equilibrium with joint commitment of monetary and fiscal policies. This is done when both authorities can precommit so as to minimize social losses. This delivers the socially optimal and feasible allocation that we refer to as second best; hence, it is the natural benchmark against which to compare all other equilibria.

Let both the monetary and fiscal authorities minimize the social loss function (35) and recognize the rational expectations constraint. Since this is a separable problem, at step 1 the two authorities choose the whole functions $m(\cdot), g(\cdot)$ to minimize

$$\sum_{s=t}^{\infty} \beta^{s-t} \frac{1}{2} \int \left[\pi(z_s)^2 + \theta_F (y(z_s) - y_F)^2 + 2\delta g(z_s) \right] . \tag{39}$$

Substituting π^e_{t+1} into the objective complicates the algebra, because it then involves one integration inside another. We avoid this by regarding the authorities as if they had another choice variable, namely π^e_{t+1}, but their choice was subject to the constraint (33). The common Lagrangean for this problem is:

$$\mathcal{L}^{JC}_{F,t} = \sum_{s=t}^{\infty} \beta^{s-t} \int \left\{ \frac{1}{2} \left[\theta_F (y(z_s) - y_F)^2 + \pi(z_s)^2 + 2\delta g(z_s) \right] + \lambda^c_s \pi(z_{s+1}) \right\} - \lambda^c_s \pi^e_{s+1}, \tag{40}$$

where λ_t^c is the Lagrangean multiplier. The first-order condition with respect to the function $g(z_t)$ is given by

$$\left(\pi(z_t) + \frac{\lambda_{t-1}^c}{\beta}\right) c + \theta_F (a + bc)(y(z_t) - y_F) + \delta = 0. \tag{41}$$

The first-order condition with respect to the function $m(z_t)$ is given by

$$\left(\pi(z_t) + \frac{\lambda_{t-1}^c}{\beta}\right) + \theta_F b(y(z_t) - y_F) = 0. \tag{42}$$

The first-order condition with respect to π_{t+1}^e is given by

$$-\lambda_t^c + \int_{z_t} \theta_F (y(z_t) - y_F) b\beta \left(-1 + \gamma + \frac{1}{1+\psi}\right) + \pi(z_t)\beta \left(\gamma + \frac{1}{1+\psi}\right) +$$

$$\int_{z_{t-1}} \frac{\lambda_{t-1}^c}{\beta} \beta \left(\gamma + \frac{1}{1+\psi}\right) - \left[\pi(z_{t-1}) + \theta_F (y(z_{t-1}) - y_F) b\right] \beta \frac{1}{1+\psi} + \int_{z_{t-2}} \frac{\lambda_{t-2}^c}{\beta} \beta \frac{1}{1+\psi}. \tag{43}$$

In deriving (43) we have taken into account that $\pi(z_t), y(z_t), \pi(z_{t-1}), \pi(z_{t-2})$ and $y(z_{t-1})$ all depend on π_{t+1}^e. Using (42) and (41) we find

$$y(z_t) = \tilde{y}_t = y_F - \frac{\delta}{a\,\theta_F}, \tag{44}$$

which we can substitute back into (42) to obtain

$$\pi(z_t) + \frac{\lambda_{t-1}^c}{\beta} = \frac{\delta\, b}{a}. \tag{45}$$

Using (45) and (44), the first-order condition (43) simplifies to

$$\lambda_t^c = \int_{z_t} \beta \frac{\delta b}{a}, \tag{46}$$

so that

$$\pi(z_t) = \frac{\delta\, b}{a} - \int_{z_t} \frac{\delta b}{a}. \tag{47}$$

Notice that our monetary and fiscal rules are not a linear function of the shocks; the reason is that even though the model is linear-quadratic, the stochastic shocks are not in general additive. If government spending did not create deadweight losses, i.e. $\delta = 0$, joint commitment would deliver the first best allocation

$$y_t = y_F, \quad \pi_t = \pi_F = 0.$$

Government spending, however, must be financed by current taxes and it therefore negatively affects private consumption and social welfare.[6]

Joint commitment yields the second best. At the second best, there is no inflation bias; public spending could raise output at its efficient level, but it is not optimal to do so. As α becomes small and goes to zero, the "public good" effect of government spending disappears while the negative impact of taxes on consumption and welfare remains. The output gap, $\delta/(a\theta_F)$, is higher the larger the welfare cost of public spending δ, the less important is output in social preferences θ_F, and the smaller the direct impact of fiscal policy on output a.

The rational expectations constraint is binding when all the m, g are chosen ex-ante optimally. More precisely, λ_t^c is the (discounted) average inflation reduction achieved the next period by joint commitment.[7]

6 Discretionary Policies: Nash equilibrium

In this policy regime, after each realization of the stochastic shock vector z_t, the fiscal authority chooses g_t, taking m_t as given, so as to minimize the loss function $L_{F,t}$; the monetary authority chooses m_t, taking g_t as given, so as to minimize its loss function $L_{M,t}$. The two authorities act non-cooperatively and simultaneously; however, when their choices are made, the private sector's expectations π_{t+1}^e, y_{t+1}^e are fixed. After completing the analysis of the policy equilibrium and economic outcome for an arbitrarily given state z_t, we can find π_{t+1}^e from the rational expectations condition (33).

The first-order condition for fiscal policy is obtained by differentiating (36) with respect to g_t, recognizing the dependence of π_t on g_t; this gives

$$\pi_t = -\theta_F \left(\frac{a}{c} + b\right)(y_t - y_F) - \frac{\delta}{c}. \tag{48}$$

This defines the reaction function of the fiscal authority (FRF) in the (y_t, π_t) space. One can obtain the reaction function in terms of the policy variables (m_t, g_t) by substituting y_t and π_t into (48) using (34) and (32). Since $c > 0$, FRF is negatively sloped.

The first-order condition for monetary policy is obtained by differentiating (38) with respect to m_t, which gives

$$\pi_t = \pi_M - \theta_M b\,(y_t - y_M). \tag{49}$$

This defines the reaction function for the monetary authority (MRF) in the (y_t, π_t) space. Since $b > 0$, the MRF is negatively sloped.

The Nash equilibrium outcomes y_t and π_t are found by solving (48), (49) and (33) together and the solution is given in Appendix D. Making use of (34) and (32) and (33), we can find

[6] Our qualitative findings would not be affected if we assumed that current government spending is deficit financed because Ricardian equivalence holds in our setup. With non lump-sum taxes, however, the financing of government spending would matter.

[7] In fact, $\delta b/a$ is the bias in inflation that arises with discretionary monetary and fiscal policies. This result is shown in Section 6.

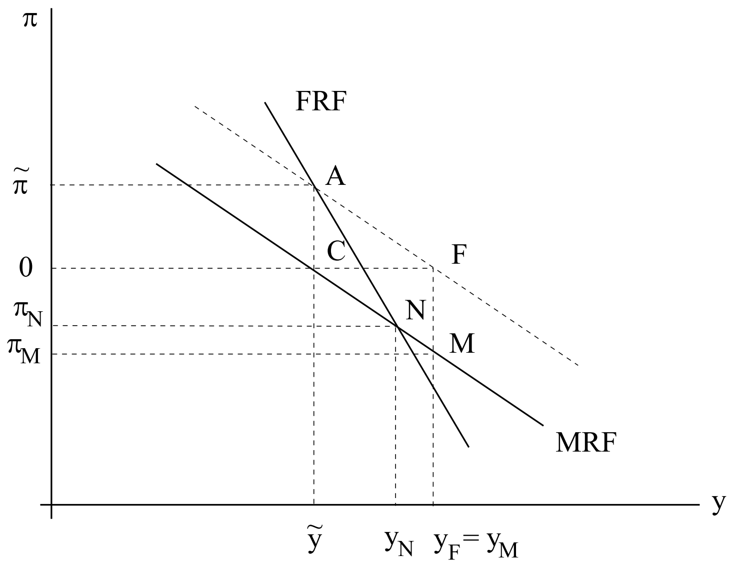

Figure 1: Nash Equilibrium

the policy variables m_t and g_t that emerge in the Nash equilibrium. This is also done in Appendix D.

Figure 1 depicts the MRF (49) and the FRF (48) in the (y_t, π_t) space. The MRF is the solid line through point M, the bliss point for the conservative monetary authority; FRF is the solid line below point F, the bliss point for society and for the fiscal authority. With $\delta > 0$, FRF does not pass through point F because it is suboptimal to raise public spending so as to raise output to y_F. The second best allocation is point C, where $y_t = \tilde{y} < y_F$ and $\pi_t = 0$. It is easy to check that FRF is steeper than MRF. The Nash equilibrium occurs at the intersection of the two reaction functions MRF and FRF, and it is labeled N. We denote output and inflation at the Nash equilibrium by (y_N, π_N).

When $c > 0$, the Nash equilibrium has the following characteristics:

$$y_N > \tilde{y}, \quad 0 > \pi_N.$$

The Nash equilibrium does not coincide with the second best. In fact, the Nash equilibrium has higher output and lower inflation than optimal. How the Nash equilibrium compares with the goals of the two authorities depends on the preferences of the monetary authority and on the realization of the shocks. Figure 1 depicts the case where the Nash equilibrium is on the left of both point M and point F so that $y_N < y_M = y_F$ and $0 > \pi_N > \pi_M$, namely output is below and inflation is in between the goals of the authorities. However, it may well be the case that the Nash equilibrium occurs on the right of point M; in this case the Nash equilibrium is suboptimal and extreme because output is above and inflation below what either authority wants.

A Nash equilibrium with high output and low inflation may sound good; however, output is higher and inflation is lower than optimal. In words, household work too much in this equilibrium and they would happily substitute labor for leisure; at the same time, lower-than-optimal inflation implies price dispersion that distorts consumption choices.

Why does the Nash equilibrium fail to achieve the second best? It is the time-inconsistency of policies *and* the conflict of objectives between the policymakers. With some prices preset, the fiscal authority runs a more expansionary fiscal policy than under joint commitment because it believes this will boost demand and therefore output. In fact, FRF is on the right of point C in Figure 1, which implies higher-than-optimal public spending. This incentive, however, is perfectly anticipated by rational firms and it results in higher output and higher inflation. The monetary authority of Figure 1 is appropriately conservative in the sense that its monetary policy is consistent with the second best;[8] higher-than-optimal public spending, however, raises output and inflation, which makes monetary policy more contractionary than under joint commitment. Hence, at the Nash equilibrium output is higher and inflation lower than at the second best.

If the central bank is not conservative and minimizes the loss function (36), then the MRF is the dotted line through the first best, point F in Figure 1. In this case, the Nash equilibrium would be:

$$y_N = \tilde{y} = y_F - \frac{\delta}{a\,\theta_F}, \qquad \pi_N = \tilde{\pi} = \frac{\delta b}{a}. \tag{51}$$

This is point A of Figure 1. The inflation bias, $\delta/(ba)$, is higher the stronger the time-consistency problem of monetary (higher b) and fiscal policy (higher a and δ).

Dixit and Lambertini (2003b) find that the Nash equilibrium has output lower and prices higher than optimal and than what either authority wants. Hence, their Nash equilibrium is suboptimal and extreme, but in a different way than here. Their Nash equilibrium lies above and to the left of point C in Figure 1, while our Nash equilibrium lies below and to the right of point C. In Dixit and Lambertini, time inconsistency makes fiscal policy tighter than optimal so that production subsidies are too low in the Nash equilibrium; since output is lower than optimal, monetary policy is more expansionary than optimal, thereby raising inflation above its optimal level. Fiscal policy works differently here: time inconsistency makes fiscal policy more expansionary than optimal, thereby raising output and making monetary policy tighter than optimal. As a result, the Nash equilibrium has higher output and lower inflation than at the second best.

[8] Given the output goal y_M and the weight on it θ_M, the central bank is appropriately conservative (so that its policy is consistent with the second best) when

$$\pi_M = \pi_M^{AC} = \int \theta_M b(\tilde{y}_t - y_M). \tag{50}$$

In drawing Figures 1, we have assumed $\theta_M = \theta_F, y_M = y_F$ and $\pi_M = -\delta b/a$.

7 Discretionary Policies: Leadership Equilibria

Now we consider the case where monetary and fiscal policies are discretionary, that is, chosen at step 4 without any commitment to a rule, but one of the policies is announced and fixed before the other, so one policymaker is the leader and the other the follower in the two-move subgame of step 4. It is not clear what leadership game describes reality better. On one hand, people argue that monetary policy can be changed quite quickly while changes in fiscal policy usually take a long time to be approved by the legislature; hence monetary policy has first-mover advantage. On the other hand, one can argue that the fiscal budget is decided at the beginning of the period while monetary policy is chosen afterward; hence, fiscal policy has first-mover advantage. We consider both cases.

7.1 Monetary Leadership

Here we consider the case of monetary leadership. Monetary policy is chosen at step 4 (a); when fiscal policy is chosen at step 4 (b), m_t is known. Private sector's expectations π^e_{t+1} are set before and known when m_t and g_t are chosen.

Fiscal policy is exactly as described in Section 6. The fiscal authority minimizes (36) with respect to g_t taking m_t and π^e_{t+1} as given. Hence, the fiscal authority's reaction function is still described by (48).

The monetary authority minimizes the loss function (38) with respect to m_t taking into account the reaction function of the fiscal authority. We can use (48) and the defining equations (34), (32) to solve for the fiscal response g_t in terms of m_t, and substitute back into (34) and (32) to find y_t and π_t as functions of m_t incorporating the fiscal reaction. This gives

$$\pi_t = \pi_M + \frac{\theta_M}{\theta_F\,(\,b + a/c)}\,(y_t - y_M). \tag{52}$$

The outcome under monetary leadership is then found by solving (52) and the fiscal reaction equation (48) jointly. This gives the solution for y_t and π_t, whose derivation is spelled out in Appendix E.

Figure 2 depicts the monetary leadership equilibrium. MRFL is the first-order condition (52), which defines an upward-sloping line through point M because $a + bc > 0$. This looks quite different from the first-order condition under Nash, MRF, which is downward sloping. The reason is that the central bank now anticipates the reaction of the fiscal authority. Graphically, MRFL is the locus of the tangency points of the central bank's iso-loss ellipses and FRF. Monetary leadership equilibrium occurs at the intersection of MRFL and FRF, which is point ML. In the monetary leadership equilibrium output is higher and inflation lower than in the Nash equilibrium. The monetary authority knows that the fiscal authority prefers higher inflation and it therefore contracts policy more than under Nash; this lowers output and inflation, causing a stronger expansion of public spending than under Nash.

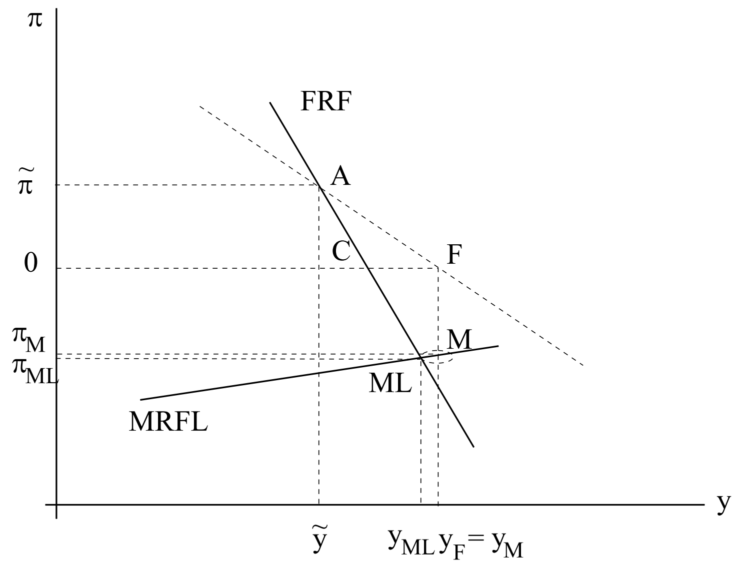

Figure 2: Monetary Leadership Equilibrium

7.2 Fiscal Leadership

Now we consider the case of fiscal leadership. After the shocks are realized, the government acts first and chooses g_t; with g_t fixed, the monetary authority chooses m_t. As usual, we solve this game by backward induction; we start from the last-moving player, the government, and we then consider the first-moving one, the monetary authority.

The central bank minimizes the loss function (38) with respect to monetary policy m_t with firms' expectations and fiscal policy already fixed; hence, the first-order condition is exactly the MRF (49) of Section 6.

The fiscal authority minimizes the loss function (36) with respect to g_t, subject to the MRF (49). The first-order condition for fiscal leadership is:

$$\pi_t = \frac{\theta_F}{\theta_M\, b}\,(y_t - y_F)\; + \frac{\delta(1+\theta_M b^2)}{\theta_M\, b\, a}\,. \tag{53}$$

The outcome is then found by solving the MRF (49) and the fiscal first-order condition (53) together. This gives the solution for y_t and π_t that we derive in Appendix F.

The first-order condition (53) defines an upward sloping line FRFL passing through point A of Figure 3 that can be steeper or flatter than FRF depending on the realization of the stochastic parameters. Hence, fiscal leadership has lower output and higher inflation than Nash.

Why would a government with first-mover advantage choose an equilibrium with lower output and higher inflation than Nash? Ex-post, namely once private expectations are set, the fiscal authority prefers fiscal leadership over Nash: a discretionary fiscal authority with first-mover advantage chooses the allocation along the MRF that minimizes social losses. Since these allocations include the Nash equilibrium, fiscal leadership is necessarily preferred,

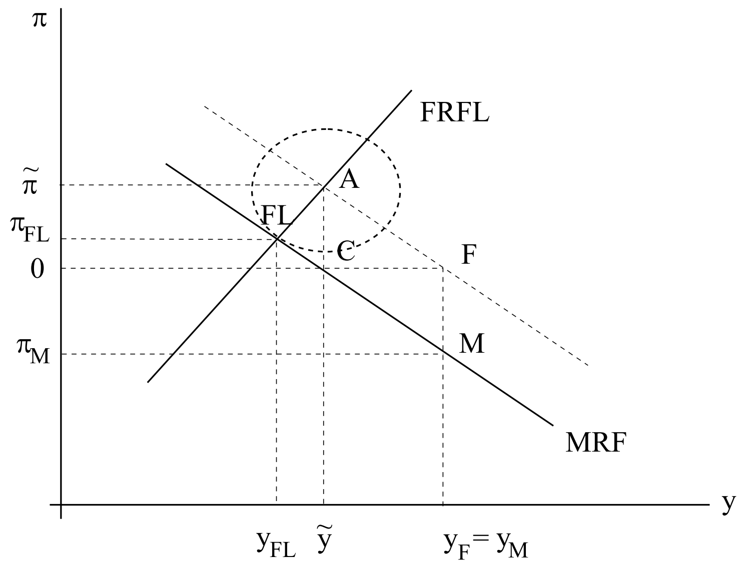

Figure 3: Fiscal Leadership Equilibrium

at least weakly, over Nash from an ex-post point of view. The fiscal authority with first-mover advantage anticipates that the central bank has a lower inflation goal and runs a tighter fiscal policy than under Nash to avoid a contractionary monetary policy; as a result, fiscal policy is tigher and monetary policy looser than under Nash, making output lower and inflation higher than under Nash.

7.3 Discretionary Regimes: a Welfare Comparison

What discretionary regime performs best in terms of social welfare? This section addresses this issue by comparing social welfare under the discretionary regimes (Nash, monetary and fiscal leadership) from an ex-ante point of view. We are going to run a Monte Carlo simulation of the structural model to obtain the paramaters of the log-linearized model. In fact, different draws of the four underlying stochastic parameters d, θ, η, A necessarily imply changes in the elements of the vector z, which are jointly distributed. Our steady state is calibrated using parameter values typically used in the literature; these are summarized in Appendix G; also, see Galí (2001) and Chari, Kehoe and McGrattan (2000). We then assume preference shocks that deliver output fluctuations within the range of +/- 6 percent of steady-state output, which are roughly consistent with the fluctuations of U.S. output around a quadratic trend.[9]

Table 1 reports the outcome of our comparison among discretionary regimes. We take 4,000 random draws of the stochastic parameters d, θ, η, A and, for each draw, we simulate our economy under joint commitment and under the three discretionary regimes: Nash, fiscal leadership and monetary leadership. The first two columns show the comparison between

[9]See Appendix G for details.

	Fiscal Leadership versus Nash		Monetary Leadership versus Fiscal Leadership		Nash versus Monetary Leadership	
	average $(L_F^N - L_F^{FL})$	output equivalent	average $(L_F^{FL} - L_F^{ML})$	output equivalent	average $(L_F^{ML} - L_F^N)$	output equivalent
Bench-mark	0.0106 (4.1113)	-0.57	-0.0116 (-14.7917)	-0.74	-0.0009 (-0.3537)	0.58
low π_M	0.005 (3.1193)	-0.31	-0.0097 (-15.0963)	-0.74	0.0048 (2.8468)	1.32
low θ_M	0.0012 (3.178)	-0.08	-0.0026 (-8.4513)	-0.21	-0.0014 (-3.1428)	1.13

Note: FL: fiscal leadership, N: Nash, ML: monetary leadership

Table 1: Welfare Comparison among Discretionary Regimes

fiscal leadership and Nash. The first column reports the average difference in social losses between Nash and fiscal leadership; the figure in parentheses is the associated $t-$statistic indicating whether the difference is statistically significant. The second column reports the output-equivalent difference in welfare between fiscal leadership and Nash; this is the percentage change in output in the better-performing regime that is necessary to make social losses equal to those under the worse-performing regime, leaving inflation and government spending unchanged.[10]

In the benchmark economy the central bank is appropriately conservative (as defined in Section 6) with the same weight and target for output as the fiscal authority, namely $\theta_M = \theta_F$ and $y_M = y_F$. In this economy, output is highest and inflation lowest under monetary leadership while output is lowest and inflation highest under fiscal leadership. Both Nash and monetary leadership are more expansionary in terms of output than optimal; fiscal leadership, on the other hand, is contractionary. Inflation is below optimal under Nash and monetary leadership and above optimal under fiscal leadership.

Fiscal leadership is the best discretionary regime from an ex-ante point of view. Time inconsistency leads to excessive public spending in the Nash equilibrium. But the government with first-mover advantage anticipates that a strong fiscal expansion will be met by a strong monetary tightening because the central bank is inflation-conservative; as a result, fiscal policy is expanded less than under Nash. Output should fall by 0.57 percent in the fiscal leadership equilibrium for this regime to be welfare-equivalent to Nash. Monetary leadership and Nash deliver similar output-inflation combinations and social welfare is not significantly different under the two regimes.

The second row reports the results of the simulation for the same 4,000 draws of the benchmark economy assuming that the central bank is more inflation-conservative than

[10]More precisely, leaving inflation and fiscal policy unchanged, we calculate: a) the certainty-equivalent output in the better-performing regime; b) the certainty-equivalent output in the better-performing regime that is necessary to make social losses equal to those in the worse-performing regime. Table 1 reports the percentage difference between these two output measures.

appropriate: the inflation target for the central bank is 20 percent lower than its appropriate level, $\pi_M = 0.8 * \pi_M^{AC}$. Because the central bank has a more conservative inflation target, the discretionary equilibria have lower inflation than in the benchmark economy. Fiscal leadership is still the best performing discretionary regime from an ex-ante point of view and monetary leadership is outperformed by Nash; in fact, output should increase by 1.32 percent for the Nash equilibrium to be welfare-equivalent to monetary leadership.

The last row of table 1 shows the results of the simulation when the central bank has a lower weight on the output target than the fiscal authority does. More precisely, $y_M = y_F$ and $\pi_M = -\delta b/a$ but $\theta_M = \theta_F/3$. The weight-conservativeness of the monetary authority lowers inflation in the Nash equilibrium but raises inflation in the monetary leadership one. The central bank with leadership anticipates that the fiscal authority will expand fiscal policy in response to monetary tightenings and therefore runs a more expansionary policy than under Nash. As a result, monetary leadership is now welfare-superior to Nash. Fiscal leadership remains the best-performing discretionary regime.

The results of this section can be summarized as follows. Fiscal leadership is the best-performing discretionary regime because leadership diminishes the time-consistency problem of fiscal leadership that generates higher-than-optimal public spending. Monetary leadership and Nash do not restrain fiscal policy and lead to higher social losses; their ranking depends on the central bank's preferences.

8 Committed Policies

8.1 Monetary Commitment

Here we analyze the case where the central bank credibly commits to a monetary rule at step 1 of the game while fiscal policy is chosen discretionary at step 4. Once the shocks are realized, monetary policy is fully predictable even if it has not been taken yet. The government acts knowing what monetary policy will be and it is therefore a follower, even if its action may come before that of the central bank in calendar time.

One would think that first-mover advantage and pre-commitment to a rule would deliver a better outcome for the central bank than first-mover advantage only; but this is not the case. Monetary commitment turns out to be equivalent to monetary leadership for every realization of the shocks. Hence, the advantage of monetary commitment is eliminated by fiscal discretion. With fiscal policy discretionary, the FRF of the government acts as a constraint on the monetary authority and the best the central bank can do for itself is to act as a leader state-by-state and ensure the equivalent of first-mover advantage, if that exists, in the game at step 4.

This game is solved by backward induction. At step 4 (b), the government minimizes its loss function (36) with respect to g_t knowing what m_t is or will be; hence, the government's behavior is described by the FRF (48) of Section 6. We can solve for fiscal policy as a function of the shocks, the monetary rule and the firms' expectations by substituting equations (34) and (32) into (48) and then solving for output and inflation that take into account the government's reaction function. This is done in Appendix H.

At step 1, the monetary authority chooses the whole function $m(\cdot)$ to minimize

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\int\left[\theta_M(y(z_s)-y_M)^2+(\pi(z_s)-\pi_M)^2\right], \tag{54}$$

where $y(z_t)$, $\pi(z_t)$ and π^e_{t+1} are given by (H.26), (H.27), and (33) respectively. Once again, rather then substituting π^e_{t+1} directly into the objective, we regard the monetary authority as having an additional choice variable, π^e_{t+1}, subject to the constraint (33). The Lagrangean for this problem is

$$\mathcal{L}^M_t=\sum_{s=t}^{\infty}\beta^{s-t}\int\left\{\frac{1}{2}\left[\theta_M(y(z_s)-y_M)^2+(\pi(z_s)-\pi_M)^2\right]+\lambda^m_t\pi(z_{s+1})\right\}-\lambda^m_s\pi^e_{s+1}, \tag{55}$$

where λ^m_t is the Lagrangean multiplier.

The first-order condition with respect to the function $m(z_t)$ is given by

$$\theta_M(y(z_t)-y_M)-\theta_F\left(\frac{a}{c}+b\right)\left(\pi(z_t)-\pi_M+\frac{\lambda^m_{t-1}}{\beta}\right)=0. \tag{56}$$

The first-order condition with respect to π^e_{t+1} is given by

$$\begin{aligned}-\lambda^m_t+\int_{z_t}\beta\left[-\theta_M(y(z_t)-y_M)+\left(\pi(z_t)-\pi_M+\frac{\lambda^m_{t-1}}{\beta}\right)\frac{\theta_F}{\Omega}\left(\frac{a}{c}+b\right)\right]\left[b+\frac{a}{c}\left(\gamma+\frac{1}{1+\psi}\right)\right]+\\ \frac{1}{\beta}\int_{z_{t-1}}\left[-\theta_M(y(z_{t-1})-y_M)+\left(\pi(z_{t-1})-\pi_M+\frac{\lambda^m_{t-2}}{\beta}\right)\frac{\theta_F}{\Omega}\left(\frac{a}{c}+b\right)\right]\beta\,\frac{a}{c}\,\frac{1}{1+\psi}=0,\end{aligned} \tag{57}$$

where

$$\Omega=\frac{1}{c\left[\theta_F(b+a/c)^2+1\right]}.$$

Using (56), the first-order condition (57) simplifies to

$$\lambda^m_t=0,$$

for all t. The rational expectations constraint is on the borderline of not binding. Using $\lambda^m_t=0$, (56) becomes

$$(y(z_t)-y_M)-\frac{\theta_F}{\theta_M}\left(\frac{a}{c}+b\right)(\pi(z_t)-\pi_M)=0, \tag{58}$$

which is equivalent to (52), the first-order condition for m_t in the case where monetary policy is discretionary with monetary leadership. The outcome for each realization of the shocks can be found by solving (58) together with (48), which is done in Appendix E. The outcome under monetary commitment is therefore exactly the same as the outcome under monetary discretion with monetary leadership for every realization of the shocks.

This is also the finding in Dixit and Lambertini (2003b). In fact, this result does not depend on the specifics of fiscal policy. The point is that discretionary fiscal policy eliminates the gains of monetary commitment. With discretionary fiscal policy, monetary commitment must lie on the fiscal reaction function and, as long as the fiscal reaction function is not vertical, a reduction in inflation is necessarily accompanied by a change in output. In our case, a fall in inflation is accompanied by an increase in GDP because the FRF is negatively sloped. But the monetary authority, even if it can commit, does not want to pursue a policy any tighter than that chosen under discretion with monetary leadership; in fact, inflation is already below the central bank's goal with discretionary monetary leadership. But this implies that the monetary authority has no incentive to influence expectations (relative to what they are in the discretionary solution with monetary leadership) either. Thus, the Lagrangean multiplier on expectations is just equal to zero.

This result has important implications for the design of central banks. Recent policy discussions have put a lot of emphasis on monetary commitment. Our work shows that monetary commitment combined with fiscal discretion will not bring any reduction of inflation with respect to the case of discretionary monetary leadership.

Monetary commitment is equivalent to discretionary monetary leadership if the central bank does not internalize the welfare losses created by fiscal policy, i.e. the cental bank's loss function does not include the term $2\delta g_t$. If that were the case, however, the optimal monetary rule would recognize the time inconsistency of fiscal policy and it would pursue a more expansionary monetary policy than under monetary leadership to induce a less expansionary fiscal policy.

8.2 Fiscal Commitment

Now we consider the case where fiscal policy is committed at step 1 whereas monetary policy is discretionary and chosen at step 4 (a). The way to solve this case is the same as in the previous section for monetary commitment, except that fiscal policy is now committed and leader while monetary policy is discretionary and follower.

The monetary authority minimizes the loss function (38) with respect to m_t with g_t and π^e_{t+1} fixed. The first-order condition with respect to m_t is, once again, the MRF (49). Then, one can solve for monetary policy as a function of the stochastic shocks, the fiscal rule and private sector's expectations by substituting (34) and (32) into (49), and then output and inflation, as of step 1 and taking into account the choice of the monetary authority. This is done in detail in Appendix I.

The Lagrangean for the problem of the fiscal authority is as follows:

$$\mathcal{L}_t^F = \sum_{s=t}^{\infty} \beta^{s-t} \int \left\{ \frac{1}{2} \left[\theta_F (y(z_s) - y_F)^2 + \pi(z_s)^2 + 2\delta g(z_s) \right] + \lambda_s^f \pi(z_{s+1}) \right\} - \lambda_s^f \pi_{s+1}^e, \tag{59}$$

where λ_t^f is the Lagrangean multiplier, $y(z_t), \pi(z_t)$ and π^e_{t+1} are given by (I.29), (I.30) and (33), respectively.

The first-order condition with respect to the function $g(z_t)$ is given by

$$(y(z_t) - y_F)\theta_F - \left(\pi(z_t) + \frac{\lambda_{t-1}^f}{\beta}\right)\theta_M b + \frac{\delta}{a}(1 + \theta_M b^2) = 0. \tag{60}$$

The first-order condition with respect to π_{t+1}^e is given by

$$-\lambda_t^f + \int_{z_t} - \frac{b\beta}{1 + \theta_M b^2}\left[-\theta_F(y_t(z) - y_F)\right] + \left(\pi(z_t) + \frac{\lambda_{t-1}}{\beta}\right)\frac{b^2\beta\theta_M}{1 + \theta_M b^2} = 0. \tag{61}$$

Using (60), the first-order condition (61) simplifies to

$$\lambda_t^f = \int_{z_t} \beta\frac{\delta b}{a} > 0. \tag{62}$$

Using (60), (49) and (62), one can solve for output and inflation; this is done in Appendix I.

As in Dixit and Lambertini (2003b), fiscal commitment leads to a reduction in inflation even if monetary policy is discretionary. Technically, the Lagrangean multiplier of the rational expectations constraint λ_t^f is positive and exactly equal to λ_t^c of equation (46) for the case of joint commitment. Because the fiscal authority is benevolent and cares about the welfare losses caused by public spending, fiscal commitment eliminates the time inconsistency of fiscal policy; and because the fiscal authority has first-mover advantage, it anticipates the reaction of the central bank and therefore eliminates the time inconsistency of monetary policy too! In fact, if the central bank is appropriately conservative as defined in Section 6, fiscal commitment with monetary discretion delivers the second best.

Notice that fiscal commitment is equivalent to fiscal leadership for all realization of shocks without time inconsistency of fiscal policy. The first-order condition with fiscal commitment (60) is the same as the first-order condition with fiscal leadership (53) except for the term λ_{t-1}^f, which is the inflation bias stemming from time inconsistency at $y_t = \tilde{y}$. Figure 4 shows the equilibrium with fiscal commitment and the equilibrium with fiscal leadership. The first-order condition with discretionary fiscal leadership FRFL goes through point A; FCOM, the first-order condition with commitment, is parallel to FRFL but shifted down by λ_{t-1}^f, the Lagrangean multiplier of the rational expectations constraint. Fiscal commitment occurs at the intersection of FCOM and MRF; this is the second best when the central bank is appropriately conservative, as depicted in Figure 4.

9 Optimal Design of Monetary and Fiscal Institutions

Commitment to a policy rule is difficult in practice. This raises the question of how monetary and fiscal institutions should be designed to guarantee optimal macroeconomic stabilization when policies are discretionary. We find that two different designs can achieve that: Identical goals and complete separation.

One optimal design calls for both authorities to share identical targets. This makes a lot of sense as conflicting goals lead to bad interactions between monetary and fiscal policies. Since

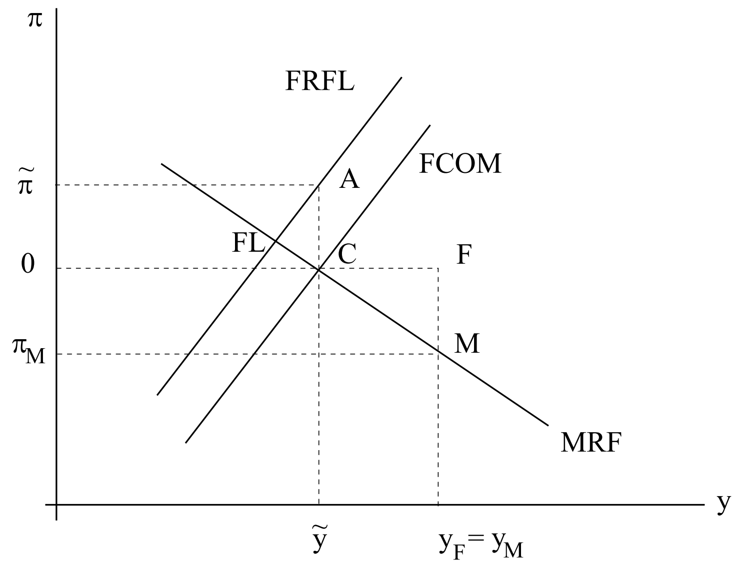

Figure 4: Fiscal Commitment

both policies suffer a time-consistency problem, the inflation goal should be appropriately conservative; the output goal, on the other hand, should be the socially efficient GDP level. To be more precise, the monetary and fiscal authorities should share the period loss function:

$$\tilde{L}_{F,t} = \frac{1}{2}\left[(\pi_t - \tilde{\pi}_F)^2 + \theta_F(y_t - y_F)^2 + 2\delta g_t\right],$$

with

$$\tilde{\pi}_F = -\frac{\delta b}{a}.$$

The Nash equilibrium as well as any leadership equilibria deliver the second best. Intuitively, this design of monetary and fiscal authorities shifts the bliss point of the government from F to M *and* makes the central bank internalize the welfare losses stemming from fiscal policy. This is shown in Figure 5.

The alternative design commands a complete separation of goals between monetary and fiscal policies. The logical division of labor calls for the central bank to care only about inflation and for the government to care only about output and the welfare losses stemming from public spending. More precisely, the central bank's period loss function should be

$$L^*_{M,t} = \frac{1}{2}\,\pi_t^2. \tag{63}$$

With Nash or monetary leadership, the fiscal authority should minimize the period loss function

$$L^{*N}_{F,t} = \frac{1}{2}\left[\theta_F(y_t - y^*_F)^2 + 2\delta g_t\right], \tag{64}$$

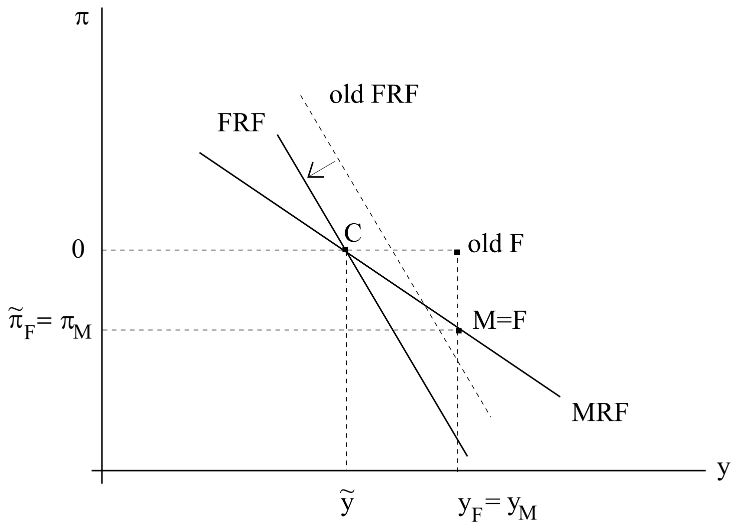

Figure 5: Identical Goals

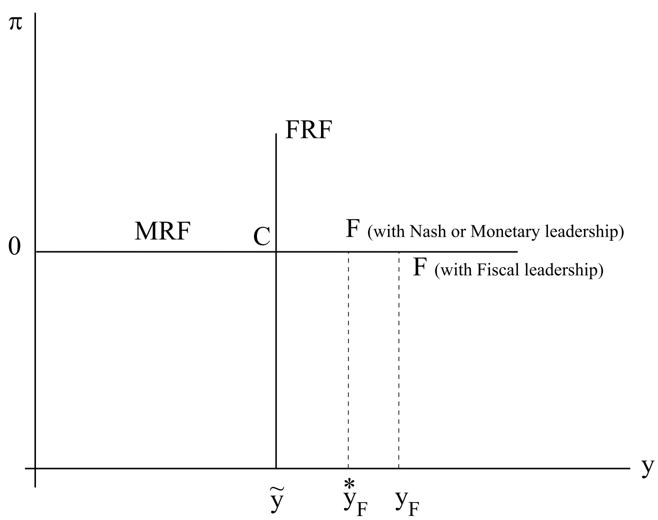

Figure 6: Complete Separation of Goals

where

$$y_F^* = y_F - \frac{\delta b}{a\theta_F(a/c + b)} < y_F.$$

Intuitively, the output target of the government must be appropriately conservative to eliminate the (expansionary) bias stemming from time inconsistency of fiscal policy. The equilibrium is shown in Figure 6: the MRF becomes the horizontal line $\pi_t = 0$, the FRF becomes the vertical line $y_t = \tilde{y}_t$, and their intersection is point C, which is the Nash and monetary leadership equilibrium.

If fiscal policy has leadership over monetary policy, the fiscal authority's period loss function should be

$$L_{F,t}^{*FL} = \frac{1}{2}\left[\theta_F (y_t - y_F)^2 + 2\delta g_t\right]. \tag{65}$$

The government with first-mover advantage anticipates that the central bank is ultraconservative and that any expansion of public spending that brings inflation above zero is met by a monetary contraction to bring inflation back to zero. Hence, fiscal leadership coupled with the fact that the central bank is ultraconservative eliminates the time inconsistency of fiscal policy.

10 Concluding Comments

Monetary and fiscal policies interact via their effects on GDP and inflation; at the same time, both policies suffer a time-inconsistency problem. If the central bank is more conservative than the fiscal authority, the non-cooperative game among them can result in a Nash equilibrium that is suboptimal and possibly more extreme than the goals of either authority.

Fiscal leadership improves welfare over Nash not only ex-post but also ex-ante; however, leadership equilibria fail to deliver the second best.

Much emphasis has been put lately on the benefits of monetary pre-commitment to a rule that specifies the policy action to be taken as a function of the realization of the shock. The merits of commitment to a monetary rule are well understood from models that consider monetary policy in isolation. Our work shows that fiscal discretion destroys monetary commitment: monetary commitment is negated by the fact that the monetary rule must recognize the fiscal reaction function as a constraint in each state of the world. In fact, monetary commitment is equivalent to monetary leadership in each state of the world.

These findings have important implications for the design of monetary and fiscal institutions. Commitment is useful only if it can be extended to both monetary and fiscal policies. If fiscal policy is discretionary, it is not worth putting in place any mechanisms of monetary commitment. If monetary policy is discretionary, fiscal commitment is useful but only if the central bank's preferences are specified so that monetary policy is consistent with the second best.

If monetary and fiscal policies are discretionary, then welfare gains can be achieved by appropriately assigning output-inflation goals to the policymakers. Such assignment should eliminate any conflict of objectives among the monetary and fiscal authorities and, at the same time, it should eliminate the bias stemming from time inconsistency. Hence, the monetary and fiscal authorities should either have identical output and inflation goals, with the inflation goal appropriately conservative, or they should have separate goals, with the central bank caring only about inflation and the government caring only about output and the welfare losses due to fiscal policy.

Many countries have recently made their central banks increasingly independent of the treasuries and accountable for their inflation outcomes. We have accordingly assumed central bank independence and we regard cooperation between the monetary and fiscal authorities as unrealistic and in contrast with recent institutional reforms.

Changes in the degree of price inertia affect our results in a number of ways. A higher degree of price stickyness worsens the time-consistency problem of monetary and fiscal policies. To attain the second-best outcome when the authorities have identical goals, the inflation target needs to be more conservative; if the authorities have separate goals and play Nash or monetary policy has leadership, the outcome target for the fiscal authority needs to be made more conservative.

Having public spending financed by fiscal deficits would leave our results unchanged because we have assumed that taxes are lump sum and Ricardian equivalence holds. That would not be the case with distortionary taxation. This is an interesting extension that we leave to future work.

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Appendix

A Optimal Price

The maximization problem of a firm is

$$\sum_{s=t}^{\infty}(\phi\beta)^{s-t}E_t Q_{t,s}\Pi_s(i). \tag{A.1}$$

The first-order condition with respect $P_t(i)$ is

$$(\theta-1)E_t\sum_{s=t}^{\infty}(\phi\beta)^{s-t}Q_{t,s}Y_s\left(\frac{P_t(i)}{P_t}\right)^{-\theta}=\theta E_t\sum_{s=t}^{\infty}(\phi\beta)^{s-t}Q_{t,s}Y_s\frac{W_s}{A_sP_s}\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}.$$

Let $\tilde{P}_t \equiv P_t(i)/P_t$ and notice that all new prices are equal; the first-order condition can be rewritten as

$$\tilde{P}_t(\theta-1)E_t\sum_{s=t}^{\infty}(\phi\beta)^{s-t}Q_{t,s}Y_s\Pi_{v=t+1}^{s}\Pi_v^{\theta}=\theta E_t\sum_{s=t}^{\infty}(\phi\beta)^{s-t}Q_{t,s}Y_s\frac{W_s}{A_sP_s}\Pi_{v=t+1}^{s}\Pi_v^{1+\theta}, \tag{A.2}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate. Notice that the steady state value of $\tilde{P}$ is

$$\tilde{P}=\frac{\theta}{\theta-1}\frac{W}{AP}.$$

The relative price is a mark-up over the real marginal cost. Log-linearization of (A.2) around its steady state value gives

$$\frac{\tilde{p}_t}{1-\phi\beta}=E_t\sum_{s=t}^{\infty}(\phi\beta)^{s-t}\left[w_s-a_s-p_s+\sum_{v=t+1}^{s}\pi_v\right], \tag{A.3}$$

where small letters indicate the percentage deviation of the capital variable from its steady-state value, i.e. $a_t \equiv (A_t - A)/A$. Notice that (A.3) can be rewritten as

$$\tilde{p}_t=(1-\phi\beta)(w_t-a_t-p_t)+\phi\beta E_t(\pi_{t+1}+\tilde{p}_{t+1}). \tag{A.4}$$

Log-linearizing (30) gives

$$\tilde{p}_t=\frac{\phi}{1-\phi}\pi_t, \tag{A.5}$$

which can be substituted into (A.4) to obtain

$$\pi_t=\lambda(w_t-a_t-p_t)+\beta E_t\pi_{t+1}, \quad \lambda=\frac{(1-\phi\beta)(1-\phi)}{\phi}. \tag{A.6}$$

B Log Linearization around the Steady State

This section log linearizes around the steady state and solves for aggregate inflation and output. Log-linearizing the first-order condition with respect to labor (10) we obtain

$$n_t = \frac{1}{\eta}(w_t - p_t - c_t), \tag{B.7}$$

where, as usual, small letters indicate percent deviations from the steady state of the capitalized-letter variable. Also notice that (15) implies

$$y_t = a_t + n_t. \tag{B.8}$$

Substituting this into (B.8) and (B.7) into (A.6) we obtain

$$y_t = a_t + \frac{1}{1+\eta}g_t + \frac{1}{\lambda(1+\eta)}(\pi_t - \beta\pi^e_{t+1}), \tag{B.9}$$

which is equation (34) in the text.

Log-linearization of the demand for real balances (8) delivers

$$\mu_t + \beta i^e_{t+1} = c_t + p_t, \tag{B.10}$$

where μ_t is the percent deviation of money supply M_t from its steady state value. Using the Euler equation (7) we find

$$i^e_{t+1} = \frac{1}{1-\beta}E_t(c_{t+1} - c_t + \pi_{t+1}). \tag{B.11}$$

We consider i.i.d shocks; using the resource constraint and substituting the result above into (B.10) we obtain

$$y_t = (1-\beta)\mu_t + \beta(y^e_{t+1} + \pi^e_{t+1}) + g_t - (1-\beta)p_t, \tag{B.12}$$

which we then substitute into (B.9) to obtain

$$\pi_t = \frac{\lambda(1+\eta)}{1+(1-\beta)\lambda(1+\eta)}[-a_t + (1-\beta)(\mu_t - p_{t-1}) + \beta y^e_{t+1}] + \tag{B.13}$$

$$\frac{\lambda\eta}{1+(1-\beta)\lambda(1+\eta)}g_t + \beta\frac{1+\lambda(1+\eta)}{1+(1-\beta)\lambda(1+\eta)}\pi^e_{t+1},$$

which is equation (32) in the text. Notice that

$$y^e_{t+1} = \frac{1}{\lambda(1+\eta)}(\pi^e_{t+1} - \beta\pi^e_{t+2}) = \frac{1}{1+\eta}E_t(w_{t+1} - p_{t+1} - a_{t+1}).$$

In words, current inflation depends on current policies, monetary and fiscal, on the current technological shock, on expected future inflation as well as the expected future real marginal cost of producing.

C Social Welfare Function

We follow Woodford (2003) and consider a second-order Taylor series approximation to the objective

$$U_t = u(C_t; \epsilon_t) - \int_0^1 v(N_t(i); \epsilon_t) di + \alpha x(G_t; \epsilon_t) \tag{C.14}$$

with

$$u(C_t; \epsilon_t) = \log C_t, \quad v(N_t(i); \epsilon_t) = \left(\frac{d}{1+\eta}\right) N_t(i)^{1+\eta}, \quad x(G_t; \epsilon_t) = \frac{1}{1-1/\rho} G_t^{1-1/\rho}.$$

The approximation is made around the steady-state level of output Y for each good and the mean values for the exogenous shocks. Here we derive the welfare criterion that applies to a limiting cashless economy and therefore we abstract from the welfare consequences of monetary frictions.

We will proceed briefly; for details, see Woodford (2003). Let $\epsilon_t = (d, \eta, \theta, A_t)$ denote the complete vector of preference and technological shocks that we normalize so that $E(\epsilon_t) = 0$ and let a $^{-}$ denote steady-state value and, for simplicity, we drop time subscripts; a second-order expansion of the first and last term on the right-hand side of (C.14) is given by

$$\bar{u} + u_C \tilde{C}_t + u_\epsilon \epsilon_t + \frac{1}{2} u_{CC} \tilde{C}_t^2 + \frac{1}{2} \epsilon_t^{'} u_{\epsilon\epsilon} \epsilon_t + u_{C\epsilon} \tilde{C}_t \epsilon_t + \alpha \left[\bar{x} + x_G \tilde{G}_t + x_\epsilon \epsilon_t + \frac{1}{2} u_{GG} \tilde{G}_t^2 + \frac{1}{2} \epsilon_t^{'} u_{\epsilon\epsilon} \epsilon_t + u_{G\epsilon} \tilde{G}_t \epsilon_t \right],$$

where $\tilde{C}_t \equiv C_t - \bar{C}_t$ and $\tilde{G}_t \equiv G_t - \bar{G}_t$. At the steady state, $\bar{C} = \bar{Y} - \bar{G}$. We assume that $\tilde{G}$ is small enough, specifically of order $\mathcal{O}(||\epsilon||^2)$. After using Taylor expansion

$$\frac{Y_t}{\bar{Y}} = 1 + \hat{Y}_t + \frac{\hat{Y}_t^2}{2},$$

where $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$ (and similarly for other variables) and neglecting terms that are of order $\mathcal{O}(||\epsilon||^3)$ or higher) order, we obtain

$$u(C_t; \epsilon_t) + \alpha x(G_t; \epsilon_t) = \bar{Y} u_C \left(\hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) - u_C \tilde{G}_t + \frac{1}{2} u_{CC} \bar{Y}^2 \hat{Y}_t^2 + u_{C\epsilon} \epsilon_t \bar{Y} \hat{Y}_t + \alpha x_G \tilde{G}_t + \alpha u_{G\epsilon} \epsilon_t \tilde{G}_t. \tag{C.15}$$

Let

$$s_t \equiv -\frac{u_{C\epsilon} \epsilon_t}{u_{CC} \bar{Y}}, \qquad \frac{1}{\sigma} \equiv -\frac{\bar{Y} u_{CC}}{u_C},$$

where s_t is of order $\mathcal{O}(||\epsilon||)$. Taking the limit as $\alpha \to 0$, (C.15) simplifies to

$$u(C_t; \epsilon_t) + \alpha x(G_t; \epsilon_t) = \bar{Y} u_C \left[\hat{Y}_t \left(1 + \frac{s_t}{\sigma}\right) + \frac{1}{2} \hat{Y}_t^2 \left(1 - \frac{1}{\sigma}\right) - \frac{G_t}{\bar{Y}} \right]. \tag{C.16}$$

A second-order Taylor expansion of each $v(N_t(i); \epsilon)$, using the fact that $N_t(i) = Y_t(i)/A_t$, gives

$$\bar{Y}(i) v_Y \left\{ \hat{Y}_t(i) \left(1 + \frac{v_{Y\epsilon} \epsilon}{v_Y}\right) + \frac{\hat{Y}_t(i)^2}{2} \left(1 + \frac{v_{YY} \bar{Y}}{v_Y}\right) \right\}. \tag{C.17}$$

Let

$$q_t \equiv -\frac{v_{Y\epsilon}\epsilon_t}{v_{YY}\bar{Y}},$$

where q_t is of order $\mathcal{O}(||\epsilon||)$. Since $v_{YY}\bar{Y}/v_Y = \eta$, we have that

$$\int_0^1 v(N_t(i);\epsilon_t)di = \bar{Y}v_Y\left[E\hat{Y}_t(i) + \frac{1+\eta}{2}(E\hat{Y}_t(i)^2 + var\hat{Y}_t(i)) - \eta q_t E\hat{Y}_t(i)\right]. \tag{C.18}$$

Using the Taylor series approximation

$$\hat{Y} = E\hat{Y}_t(i) + \frac{1}{2}\frac{\theta-1}{\theta}var\hat{Y}_t(i),$$

with

$$var\hat{Y}_t(i) = \theta^2 var\log P_t(i) = \theta^2\frac{\phi}{1-\phi}\pi_t^2,$$

where $\pi_t \equiv p_t - p_{t-1}$ and substituting these expressions in (C.17) also using

$$\frac{v_Y}{u_C} = \frac{\theta-1}{\theta},$$

which is the monopolistic distortions that we assume to be of order $\mathcal{O}(||\epsilon||)$, we obtain

$$\int_0^1 v(N_t(i);\epsilon_t)di = \bar{Y}u_C\left\{\left(1-\eta q_t - \frac{\theta-1}{\theta}\right)\hat{Y}_t + \frac{1+\eta}{2}\hat{Y}_t^2 + \frac{1}{2}\theta^2\frac{\phi}{1-\phi}\left(\eta + \frac{\theta-1}{\theta}\right)\pi_t^2\right\}. \tag{C.19}$$

Next, we subtract (C.19) from (C.15) we obtain that U_t is approximated by

$$-\frac{\bar{Y}u_C}{2}\left\{\hat{Y}_t^2\left(\eta+\frac{1}{\sigma}\right) - 2\hat{Y}_t\left(\eta q_t + \frac{s_t}{\sigma} + \frac{\theta-1}{\theta}\right) + 2\frac{G_t}{\bar{Y}} + \theta^2\frac{\phi}{1-\phi}\left(\eta+\frac{\theta-1}{\theta}\right)\pi_t^2\right\}.$$

Notice that $\sigma = 1$ with $u(C) = \log C$. The output terms above (together with a constant) come from the term $[\hat{Y}_t - \hat{Y}_t^n - \log(Y_t^*/\bar{Y})]^2$, where $\hat{Y}_t^n = \log(Y_t^n - \bar{Y})$, where Y_t^n is the equilibrium level of output at t under complete price flexibility. Let y_t be gap between current output and output under complete flexibility, i.e. steady-state output, and let y_F be the gap between steady-state and efficient output. We can write

$$U_t = -\Omega L_{F,t},$$

where

$$L_{F,t} = \frac{1}{2}\left[\pi_t^2 + \theta_F(y_t - y_{F,t})^2 + 2\delta g_t\right], \tag{C.20}$$

where

$$\Omega = \frac{\phi\theta[\theta(1+\eta)-1]}{1-\phi} > 0,\ \ \theta_F = \frac{(1+\eta)}{\Omega} > 0,$$

$$\delta = \frac{1}{\Omega} > 0,\ \ y_F = \log\frac{Y^*}{\bar{Y}} = -\frac{1}{1+\eta}\log\frac{\theta-1}{\theta}.$$

Social welfare is lower: a) the larger the gap between actual and the efficient level of output; b) the higher price dispersion that materializes with changes in the price level; c) the larger public spending.

Finally, we briefly discuss the case where $\alpha > 0$. In this case, government spending is a public good that raises social welfare directly. A benevolent fiscal authority chooses government spending according to the first-order condition (14); steady-state output is

$$\bar{Y} = \left[\frac{(1+\alpha)(\theta-1)}{d\theta}\right]^{\frac{1}{1+\eta}}.$$

As long as $\alpha < \theta/(\theta-1)$, steady-state output is below efficiency and there is an output gap that fiscal policy can close but at the cost of over-providing the public good. U_t is approximated by

$$-\frac{\bar{Y}u_C}{2}\left\{\hat{Y}_t^2\left(\eta+\frac{1}{\sigma}\right)-2\hat{Y}_t\left(\eta q_t+\frac{s_t}{\sigma}+\frac{\theta-1}{\theta}\right)+\theta^2\frac{\phi}{1-\phi}\left(\eta+\frac{\theta-1}{\theta}\right)\pi_t^2\right\},$$

but $[\hat{Y}_t - \log(Y_t^*/\bar{Y})]^2 = \hat{y}^2 - 2\frac{\hat{Y}}{1+\eta}\log\frac{(1+\alpha)(\theta-1)}{\theta}$ + constant term $= -2\frac{\alpha}{1+\eta}\hat{G}$ if α is small. The period social loss function is as (C.20) with

$$\delta = \frac{\alpha}{\Omega} > 0, \qquad g_t \equiv \hat{G}_t.$$

D Nash equilibrium

Written in matrix notation, we have

$$\begin{bmatrix} \theta_F\,(b+a/c) & 1 \\ \theta_M b & 1 \end{bmatrix}\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \theta_F\,(b+a/c)\,y_F\, - \delta/c \\ \pi_M\, + \theta_M\, b\, y_M \end{bmatrix}$$

The determinant of the matrix on the left hand side is

$$\Omega \equiv \theta_F(\frac{a}{c}+b) - \theta_M b.$$

Then the solution exists as long as Ω is different from zero, which is the case almost surely (for probability one of realizations of shocks). The solution is given by

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = -\frac{1}{\Omega}\begin{bmatrix} 1 & -1 \\ -\theta_M\, b & \theta_F\,(b+a/c) \end{bmatrix} \times \begin{bmatrix} +\theta_F\,(b+a/c)\,y_F\, - \delta/c \\ \pi_M\, + \theta_M\, b\, y_M \end{bmatrix} \tag{D.21}$$

Write (34) and (32) also in vector-matrix notation:

$$\begin{bmatrix} a+bc & b \\ c & 1 \end{bmatrix}\begin{bmatrix} g_t \\ m_t \end{bmatrix} = \begin{bmatrix} y_t - \tilde{y}_t + b\,\pi_{t+1}^e \\ \pi_t - \omega_t - \gamma\beta\pi_{t+1}^e \end{bmatrix}$$

This has the solution

$$\begin{bmatrix} g_t \\ m_t \end{bmatrix} = \frac{1}{a}\begin{bmatrix} 1 & -b \\ -c & a+bc \end{bmatrix}\begin{bmatrix} y_t - \tilde{y}_t + b\,\pi_{t+1}^e \\ \pi_t - \omega_t - \gamma\beta\pi_{t+1}^e \end{bmatrix} \tag{D.22}$$

The values of g, m_t can then be obtained substituting y_t, π_t from (D.21).

E Monetary Leadership

Written in vector-matrix notation, we have

$$\begin{bmatrix} \theta_M & -\theta_F\,(b+a/c) \\ \theta_F\,(b+a/c) & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \theta_M\, y_M - \theta_F\,(b+a/c)\,\pi_M \\ \theta_F\,(b+a/c)\,y_F - \delta/c \end{bmatrix}$$

This has the solution

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{\theta_M + \theta_F^2\,(b+a/c)^2} \begin{bmatrix} 1 & \theta_F\,(b+a/c) \\ -\theta_F\,(b+a/c) & \theta_M \end{bmatrix} \times \begin{bmatrix} \theta_M\, y_M - \theta_F\,(b+a/c)\,\pi_M \\ \theta_F\,(b+a/c)\,y_F - \delta/c \end{bmatrix} \tag{E.23}$$

The values of g, m_t can then be obtained substitituting from (E.23) in (D.22).

F Fiscal Leadership

Written in vector-matrix notation, we have

$$\begin{bmatrix} \theta_M b & 1 \\ -\theta_F & \theta_M\, b \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \theta_M\, b\, y_M + \pi_M \\ -\theta_F\, y_F + \theta_M\, b + \delta(1+\theta_M\, b^2)/a \end{bmatrix}$$

This has the solution

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{\theta_M\, b}{\theta_M^2\, b^2 + \theta_F} \begin{bmatrix} \theta_M b & -1 \\ \theta_F & \theta_M\, b \end{bmatrix} \times \begin{bmatrix} \theta_M\, b\, y_M + \pi_M \\ -\theta_F\, y_F + \theta_M\, b + \delta(1+\theta_M\, b^2)/a \end{bmatrix} \tag{F.24}$$

G Simulation

We assume the mean of the parameter η to be 1, which implies a unit wage elasticity of labor supply in the steady state. $\phi = 0.5$; under the Calvo formulation this implies an average price duration of two quarters, which is in line with econometric estimates of the parameter as well as with survey evidence. The elasticity of substitution θ is set to be 11 on average, which is consistent with a 10% steady-state markup. The discount parameter β is set equal to .98. The mean of the parameter d is set to be equal to 1. This parameter is usually pinned down to match the labor share of income, which is equal to one in our setting; hence, the mean value of d is inconsequential here. The technological shock a_t is assumed to have zero mean. We then assume that the preference parameters d, θ, η and technology A are stochastic, i.i.d. with lognormal distribution with means as specified above. Their variances are calibrated to get output fluctuations in the range of +/- 6% of steady-state output, which are consistent with the fluctuations of U.S. output around a quadratic trend.

H Monetary Commitment

Substituting (34) and (32) into (48), we obtain

$$g(z_t) = \frac{1}{c\left[\theta_F(b+a/c)^2+1\right]}\left\{-\left[\theta_F b\ \left(\frac{a}{c}+b\right)+1\right]\left[m(z_t)+\omega_t+\gamma\beta\pi^e_{t+1}\right]\right.$$

$$\left.-\theta_F\left(\frac{a}{c}+b\right)\left[\tilde{y}_t-y_F-b\,\pi^e_{t+1}\right]-\frac{\delta}{c}\right\}. \tag{H.25}$$

Output and inflation, as of step 1 and taking into account the choice of the fiscal authority at step 4 (b), are

$$y(z_t) = \frac{1}{\theta_F(b+a/c)^2+1}\left\{-\frac{a}{c}\ \left[m(z_t)+\omega_t+\gamma\beta\pi^e_{t+1}\right]+\tilde{y}_t-b\ \beta\pi^e_{t+1}+\right. \tag{H.26}$$

$$\left.\theta_F\left(\frac{a}{c}+b\right)^2\left[y_F-\frac{\delta/c}{\theta_F\left(a/c+b\right)}\right]\right\},$$

and

$$\pi(z_t) = \frac{1}{\theta_F(b+a/c)^2+1}\left\{\theta_F\frac{a}{c}\left(\frac{a}{c}+b\right)\left[m(z_t)+\omega_t+\gamma\beta\pi^e_{t+1}\right]-\right. \tag{H.27}$$

$$\left.\theta_F\left(\frac{a}{c}+b\right)\left[\tilde{y}_t-y_F-b\beta\pi^e_{t+1}\right]-\frac{\delta}{c}\right\}.$$

Proceeding by backward induction, we now consider the private sector that sets its expectations rationally at step 2; this is done according to (33).

I Fiscal Commitment

Substituting (34) and (32) into (49), we obtain

$$m(z_t) = \frac{1}{1+\theta_M b^2}\left[\pi_M-\theta_M\,b(\tilde{y}_t-b\beta\,\pi^e_{t+1}-y_M+ag(z_t)\right]-(cg(z_t)+\omega_t+\gamma\beta\pi^e_{t+1}). \tag{I.28}$$

Output and inflation, as of step 1 and taking into account the choice of the monetary authority at step 4 (a), are

$$y(z_t) = \frac{1}{1+\theta_M b^2}\left[\tilde{y}_t+b(\pi_M-\beta\pi^e_{t+1})+a\,g(z_t)+y_M\theta_M b^2\right] \tag{I.29}$$

and

$$\pi(z_t) = \frac{1}{1+\theta_M b^2}\left[\pi_M-\theta_M\,b(\tilde{y}_t-y_M-b\beta\pi^e_{t+1}+a\,g(z_t))\right] \tag{I.30}$$

The private sector sets its expectations rationally at step 3 by taking the expected value of (I.30). Using (60), (49) and (62), we can solve for output and the price level as a function of the parameters of the model

$$y(z_t) = \frac{1}{\theta_F + \theta_M^2\, b^2} \left[\theta_F y_F + \theta_M^2\, b^2\, y_M + \theta_M\, b(\pi_M + \int \delta b/a) - \delta(1 + \theta_M\, b^2)/a \right] \qquad \text{(I.31)}$$

and

$$\pi(z_t) = \frac{1}{\theta_F + \theta_M^2\, b^2} \left[\theta_F \pi_M + \theta_M\, b\, \theta_F (y_M - y_F) - \theta_M^2\, b^2 \int \delta b/a + \delta\, \theta_M\, b(1 + \theta_M\, b^2)/a \right]. \qquad \text{(I.32)}$$

Assuming the monetary authority is appropriately conservative, as defined in Section 6, one obtains that

$$\int_{z_t} y_t = \int_{z_t} y_F - \frac{\delta}{\theta_F\, a}, \qquad \int_{z_t} \pi_t = 0.$$

If the central bank is appropriately conservative, fiscal commitment delivers on average the second-best allocation.